On magnetic-field induced non-geodesic corrections to the relativistic precession QPO model

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Abstract. Fitting the observational data of the twin peak kHz quasiperiodic oscillations (QPO) from low mass X-ray binaries (LMXBs) by the relativistic precession model gives a substantially higher neutron star mass estimate, $M \sim 2\,\mathrm{M}_{\odot}$, than the "canonical value", $M \sim 1.4\,\mathrm{M}_{\odot}$. Using a fully general relativistic approach we discuss the non-geodesic corrections to the orbital and epicyclic frequencies of slightly charged circularly orbiting test particles caused by the presence of a neutron star magnetic field. We show that consideration of such non-geodesic corrections can bring down the neutron star mass estimate and improve the quality of twin peak QPO data fits based on relativistic precession frequency relations.

Keywords: stars: neutron – X-rays: binaries – stars: magnetic fields
PACS: 95.30.Sf; 97.10.Gz; 97.10.Ld; 97.80.Jp

INTRODUCTION

X-ray timing measurements provided by the RXTE satellite have revealed existence of nearly periodic modulation of X-ray flux detected from several low-mass X-ray binaries (LMXBs), so called quasi-periodic oscillations (QPOs).

Particular, so called high frequency (kHz) QPOs often come in pairs consisting of the so called lower and upper QPO mode with frequencies $\nu_L,\ \nu_U$. Notably, the frequencies $\nu_L,\ \nu_U$ roughly correspond to Keplerian periods in the close vicinity of the binary compact object; see [1] for a review. Miscellaneous orbital QPO models have been proposed [see, e.g., 2–4]. In particular, relativistic precession (in next RP) model relates the upper and lower kHz QPOs to the Keplerian and periastron precession frequency on an orbit located in the inner part of the accretion disc [1]. Generally, for neutron star sources correlation between $\nu_U(\nu_L)$ is qualitatively well fitted by the RP model prediction [see, e.g., 5–7].

Nevertheless, there are difficulties when modelling QPO frequency relations from the RP model for individual sources. The mass and angular momentum relevant to the best fits are questionably high ($M \sim 2-3\mathrm{M}_{\odot},\ j \sim 0.2-0.4$); [see, e.g., 5, 7–9]. Also the quality of the fits is not satisfactory with chi-square indicating a systematic deviation between the expected and empirical trend. It has been discussed that the

[1] The same model relates another particular so called low frequency QPOs to the “Lense–Thirring” orbit precession.

above mentioned discrepancies could be connected to non-geodesic corrections to the orbital and epicyclic frequencies, most likely originating in the presence of a neutron star magnetic field [6, 7, 9].

In the present paper we discuss in detail non-geodesic perturbative corrections implied by a Lorentz force acting on a slightly charged circularly orbiting matter in the approximation of a spherically symmetric spacetime and intrinsic dipole magnetic field of the neutron star[2].

CIRCULAR ORBITAL MOTION IN A DIPOLE MAGNETIC FIELD ON THE SCHWARZSCHILD BACKGROUND

The line element in the Schwarzschild spacetime using geometric units, $c = G = 1$, has the familiar form

$$\mathrm{d}s^2 = -\eta(r)^2\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\eta(r)^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\,, \qquad \eta(r) \equiv \left(1 - \frac{2M}{r}\right)^{1/2}\,. \quad (1)$$

Solving the vacuum Maxwell equations on the background of the spacetime geometry (1) for a static magnetic dipole moment μ, parallel to the rotational axis of the star, one obtains formula for an exterior ($r > R$, where R is the neutron star radius) four-potential A_μ [e.g., 10, 11],

$$A_\mu = -\,\delta^\phi_\mu\, f(r)\frac{\mu\sin^2\theta}{r}\,, \qquad f(r) = \frac{3r^3}{8M^3}\left[\log\eta(r)^2 + \frac{2M}{r}\left(1 + \frac{M}{r}\right)\right]. \quad (2)$$

In case of potential (2), the Maxwell tensor $F_{\mu\nu}$ has only two independent nonvanishing components,

$$F_{r\phi} = B^\theta = \frac{\mu\sin^2\theta\,(f(r) - r f'(r))}{r^2}\,, \qquad -F_{\theta\phi} = B^r = \frac{\mu f(r)\sin 2\theta}{r}\,. \quad (3)$$

Throughout this paper we confine ourselves to studying only circular equatorial motion with appropriate four-velocity $U^\mu = (U^t, 0, 0, U^\phi)$[3]. Solving the radial component of equation of motion ($\tilde{\mathrm{q}} \equiv q/m$ is the specific charge of the particle)

$$\frac{\mathrm{d}U^\mu}{\mathrm{d}\tau} + \Gamma^\mu_{\alpha\beta}U^\alpha U^\beta = \tilde{\mathrm{q}}F^\mu_\nu U^\nu \quad (4)$$

together with the normalization condition $U^\mu U_\mu = -1$ for metric (1) and potential (2) we obtain the nonzero components of U^μ in the form

$$U^t = \sqrt{\frac{r - \tilde{\mathrm{q}}\,\mu\,\Phi(r)\,U^\phi}{(r - 3M)}}\,, \qquad U^\phi = \frac{\Upsilon(\mathrm{r}, \tilde{\mathrm{q}}, \mu)}{2r^3\,(r - 3M)}\,, \quad (5)$$

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[2] We restrict here ourselves to the following assumptions:
the frame-dragging effects are not considered; the neutron star magnetic field is fully dominant over the magnetic field generated by the currents in the disc.

[3] See [12] for a discussion of the existence of nonequatorial, so called "halo", orbits.

and the angular velocity defined as $\Omega = U^{\phi}/U^{t}$ then reads

$$\Omega = \frac{\Upsilon(\mathrm{r}, \tilde{\mathrm{q}}, \mu)}{r^{3/2}\sqrt{4r^4(r-3M) - 2\tilde{\mathrm{q}}\,\mu\Phi(r)\,\Upsilon(\mathrm{r}, \tilde{\mathrm{q}}, \mu)}}. \tag{6}$$

Here $\Phi(r)$, $\chi(r)$, $\Psi(r)$ and $\Upsilon(\mathrm{r}, \tilde{\mathrm{q}}, \mu)$ are given by

$$\Phi(r) \equiv f(r) - r f'(r), \qquad \chi(r) \equiv (r-2M)\,\Phi(r),$$

$$\Psi(r) \equiv \sqrt{4Mr^4(r-3M) + (\tilde{\mathrm{q}}\mu\,\chi(r))^2}, \qquad \Upsilon(\mathrm{r}, \tilde{\mathrm{q}}, \mu) \equiv \Psi(\mathrm{r}) - \tilde{\mathrm{q}}\,\mu\,\chi(r).$$

One may obtain the formulae for epicyclic frequencies by perturbing the particle's position around the stable circular orbit $(r, \theta) = (r_0, \pi/2)$, i.e., by presuming that $x^{\mu}(\tau) = z^{\mu}(\tau) + \xi^{\mu}(\tau)$ where $\xi^{\mu}(\tau)$ is a small perturbation [13, 14]. In the spacetime geometry (1) and magnetic field (2) the appropriate explicit expressions are given by

$$\begin{aligned} \omega_r^2 = {} & r^{-7}\left(U^t\right)^{-2}\Big\{\left(U^{\phi}\right)^2 r^6(3r-8M) + 2M(M-r)r^3\left(U^t\right)^2 \\ & + \tilde{\mathrm{q}}\,\mu\,\left[\Phi(r)\left(2U^{\phi}r^3(3r-7M) + \tilde{\mathrm{q}}\,\mu\,\chi(r)\right) + U^{\phi}r^5(r-2M)\,f''(r)\right]\Big\}, \end{aligned} \tag{7}$$

$$\omega_\theta^2 = \frac{U^{\phi}\left(U^{\phi}r^3 - 2\tilde{\mathrm{q}}\,\mu\,f(r)\right)}{\left(U^t\right)^2 r^3}. \tag{8}$$

MAGNETIC FIELD CORRECTIONS TO ORBITAL AND EPICYCLIC FREQUENCIES

We restrict our consideration to the approach of slowly rotating neutron star that posseses a dipole magnetic field and a thin accretion disc that is assumed to consist of test particles moving along nearly circular geodesics in the equatorial plane. As the Maxwell tensor projected into an orthonormal basis of observer located at the equator on the surface of the star with radius R has only $F_{\hat{r}\hat{\phi}}$ non-zero component, one may write

$$\mu = \frac{4M^3R^{3/2}\sqrt{R-2M}}{6M(R-M) + 3R(R-2M)\log\eta\,(R)^2}\,B_{surface}\,. \tag{9}$$

For a neutron star with a rather weak magnetic field strength, $B_{surface} = 10^7$ G $= 2.875\,\mathrm{x}\,10^{-16}\,m^{-1}$, mass $M = 1.5\,\mathrm{M}_{\odot}$ and radius $R = 4M$, we have $\mu = 1.06\,\mathrm{x}\,10^{-4}\,m^{-2}$.

We present here the resulted frequencies for the above value of μ and two different values of $\tilde{\mathrm{q}}$, $\tilde{\mathrm{q}} = 5.555\,\mathrm{x}\,10^{10}$ and $\tilde{\mathrm{q}} = 1.111\,\mathrm{x}\,10^{12}$. Both of these values are still very low in comparison with the value $\tilde{\mathrm{q}} = 1.111\,\mathrm{x}\,10^{18}$ corresponding to matter purely consisting of ions of hydrogen. The left panel of Fig. 1, made for $\tilde{\mathrm{q}} = 5.555\,\mathrm{x}\,10^{10}$, shows a high sensitivity of the radial epicyclic frequency keeping qualitatively the same profile that is however shifted to lower values and away from the central object.

The presence of the dipole magnetic field also violates the $\nu_K = \nu_\theta$ equality corresponding to spherical symmetry of the background Schwarzschild geometry. However this corrections are much less significant.

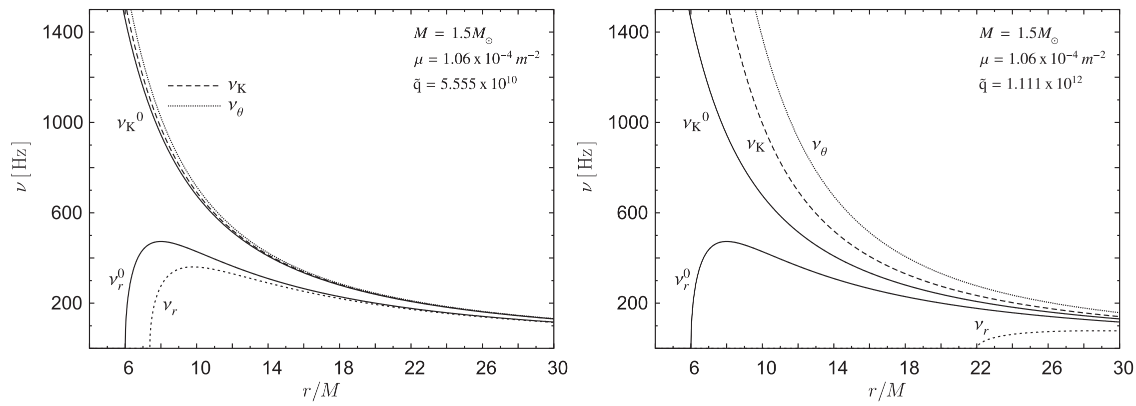

FIGURE 1. Left: An illustration of the radial epicyclic, $\nu_r^0 = \omega_r^0/(2\pi)$, vertical epicyclic, $\nu_\theta^0 = \omega_\theta^0/(2\pi)$, and orbital, $\nu_{\rm K}^0 = \Omega_{\rm K}/(2\pi) = \nu_\theta^0$, frequency behaviour in the Schwarzschild geometry in a pure geodesic case compared to case with a presence of an intrinsic external dipole magnetic field $B = 10^7$ G on the surface of the star with $M = 1.5\,{\rm M}_\odot$ and $R = 4M$ (quantities ν_K, ν_θ and $\nu_{\rm r}$ without a superscript). Right: The same comparison but for a higher value of the specific charge $\tilde{q}$.

Effective innermost stable circular orbit (EISCO)

The Lorentz force naturally alters the location of a charged test particle's effective innermost stable circular orbit (in next EISCO) given by the condition $\omega_r(r_{\rm EISCO}) = 0$. With growing values of q̃ it rapidly draws apart from the well-known radius of ISCO in the Schwarzschild geometry, $r_{\rm ISCO} = 6M$. In case of $\mu = 1.06 \,{\rm x}\, 10^{-4}\, m^{-2}$ corresponding to Fig. 1 we find that for $\tilde{q} = 5.555 \,{\rm x}\, 10^{10}$ there is $r_{\rm EISCO} = 7.39M$, while for $\tilde{q} = 1.111 \,{\rm x}\, 10^{12}$ we obtain $r_{\rm EISCO} = 22.16M$. For the extremal specific charge $\tilde{q} = 1.111 \,{\rm x}\, 10^{18}$ the location of EISCO orbit flies away onto $r_{\rm EISCO} = 177864.76M$.

IMPLICATIONS FOR THE RELATIVISTIC PRECESSION QPO MODEL AND DISCUSSION

The widely discussed RP QPO model identifies the frequencies of the lower and upper QPO peaks (ν_L and ν_U, respectively) as

$$\nu_L(r) = \nu_K(r) - \nu_{\rm r}(r), \quad \nu_U(r) = \nu_K(r), \tag{10}$$

where $\nu_K(r)$ and $\nu_{\rm r}(r)$ are the orbital and radial epicyclic frequencies [15][4]. It has been shown by [5] that these relations qualitatively well describe the trends presented in the observational data, but the characteristic mass of neutron stars in LMXBs obtained by such fits, $M \sim 2\,{\rm M}_\odot$, is high in comparison with the canonical value. Considering in the RP model the the corrected frequencies introduced above, the new fits can provide

[4] The orbital and epicyclic frequencies also play a significant role in the QPO models dealing with warped disc [e.g., 16] and tori [e.g., 17] oscillations. Our conclusion is therefore touching directly not only the hot spot kinematic QPO models, like the RP model, but also the "disc or torus oscillation - like" QPO models.

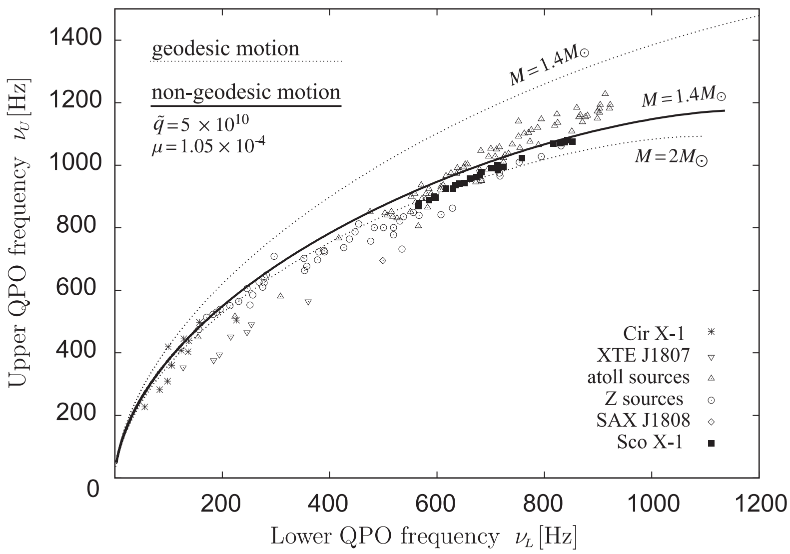

FIGURE 2. Inspired by [5]. The RP model rough fits of the observational twin peak kHz QPO data for a wide set of LMXBs. The thick solid curve refers to the case with $M = 1.4\,\mathrm{M}_\odot$ and the orbital and epicyclic frequencies being corrected by the presence of the Lorentz force induced by the specific charge of orbiting matter, $\tilde{q} = 5\,\mathrm{x}\,10^{10}$, and the star intrinsic magnetic dipole moment, $\mu = 1.06\,\mathrm{x}\,10^{-4}\,m^{-2}$. We also present fits corresponding to a pure geodesic case (thin dashed curves) for $M = 2\,\mathrm{M}_\odot$ that was discussed by [5] including data from [5, 8, 18, 19].

the characteristic neutron star mass close to the canonical value, $M \sim 1.4\,\mathrm{M}_\odot$. We illustrate this finding in Fig. 2 for the intrinsic magnetic dipole moment of the star, $\mu = 1.06\,\mathrm{x}\,10^{-4}\,m^{-2}$, and the specific charge of the orbiting matter, $\tilde{q} = 5\,\mathrm{x}\,10^{10}$, when the effective innermost stable circular orbit is shifted to $r_{\mathrm{EISCO}} \sim 7M$. Such a rough fit for a wide set of LMXBs is shown together with the fits for a pure Schwarzschild geodesic cases with $M = 2\,\mathrm{M}_\odot$ [5] and $M = 1.4\,\mathrm{M}_\odot$.

A natural implication of the RP model (and several other models) identifies the highest observed frequency of a particular source with the orbital frequency at ISCO. It is then possible to derive the mass of source using this direct identification [see, e.g, 3, 20]. Even here straightforward replacing the geodesic ISCO orbital frequency by the corrected EISCO one provides a significant decrease of the estimated mass. Moreover, it was shown by [9] that the lowering of the radial epicyclic frequency corresponding above discussed corections may in general significantly improve the quality of the fits based on the RP model.

It is widely expected [e.g, 1, 21] that magnetic field of the central compact objects in LMXBs should be given by an intrinsic exterior magnetic field, $B \sim 10^6 - 10^9$ G. There are also several indices supporting evidence of matter being accreted in the region with $r \leq 10M$ [see, e.g., 1]. Our results then imply that the specific charge related to the accreted plasma should not exceed $\tilde{q} \sim 1.86\,\mathrm{x}\,10^{12}$ $(1.87\,\mathrm{x}\,10^{11}, 1.90\,\mathrm{x}\,10^{10}, 1.91\,\mathrm{x}\,10^{9})$ for $B = 10^6$ G $(10^7, 10^8, 10^9$ G).

Discussed values of the specific charge are small in comparison to the charge of a fully ionized matter. Here we do not touch a problem of the (considerable) magnetic field induced by such a rotating charge. The full discussion of its role exceeds the framework of the paper. We however note that in principle its external exposure can be supressed by an influence of a corotating charge in a corona if the total assumed charge is approximately zero.

Finally we stress that also the diamagnetic effects should be considered in order to obtain coherent formulae describing approximately motion of a slightly charged accreted matter. We plan to include relevant corrections within a fully general relativistic approach in our consequent work.

ACKNOWLEDGMENTS

This work has been supported by the Czech grants LC 06014 (PB, ES) and MSM 4781305903 (ZS, GT). We thank to W. Kluzniak and D. Psaltis for comments.

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