Short term aperiodic variability of X-ray binaries: its origin and implications

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Abstract. In this review I briefly describe the latest advances in studies of *aperiodic* variability of accreting X-ray binaries and outline the model which currently describes the majority of observational appearances of variability of accreting sources in the best way. Then I concentrate on the case of luminous accreting neutron star binaries (in the soft/high spectral state), where study of variability of X-ray emission of sources allowed us to resolve the long standing problem of disentangling the contribution of accretion disk and boundary/spreading layer components to the time average spectrum of sources. The obtained knowledge of the shape of the spectrum of the boundary layer allowed us to make estimates of the mass and radii of accreting neutron stars.

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INTRODUCTION

Emission of X-ray binaries is not stable. Already the first obervations of X-ray binaries showed that their emission varies on different time scales [e.g. 1]. Some X-ray sources appeared to be pulsars, whose emission is modulated by the rotation of an accreting neutron star [2], while variations of emission of some other sources were shown to be aperiodic [1, 3, 4]. Irrespective of the exact origin of this variability it was immediately realized that it brings us very important information about compact objects and the environment around them [1], therefore a lot of efforts were devoted to study it.

In this review I will briefly describe the latest advances in our understanding of aperiodic variability of accreting X-ray binaries and also will try to show how the study of spectral variability properties of the accreting systems helps us to disentangle different problems and to extract important information about compact objects.

SHOT NOISE AND PROPAGATING FLOW MODELS

Very soon after the discovery of aperiodic/flickering variability of X-ray binaries (in particular, of Cyg X-1) it was proposed that such light curves might be constructed out of a number of randomly occuring shots, which originate in the region of main energy release in the accretion flow and have some typical durations (i.e. “shot noise model”, [4]). For quite a long time this shot noise model was successfully applied to describe the observed broad band variability of accreting X-ray sources [5–7]. However, collection of more and more observational facts about variability of accreting sources (especially with the appearance of an X-ray observatory directly devoted to study the variations of X-ray emission of sources (the *Rossi X-ray Timing Explorer*), led to the point where original shot noise model should be modified in order to be more consistent with the data.

Among the main problems of the original shot noise model one can mention two issues:

1. Power spectra of some accreting sources (in particular, Cyg X-1 and the soft spectral state) can have the same power law shape over an enormous range of frequencies, from tens of Hz down to 10^{-5} Hz and lower. The same slope of the power spectrum over this wide range of frequencies suggests that variability at all these frequencies originates as a result of the operation of the same physical mechanism, but at the same time it is hard to imagine that all this huge range of time scales can originate in the innermost region of the accretion flow, where dynamical time scales are typically 5-8 orders of magnitude smaller than the longest timescale of the observed variability [8] (see Fig. 1).
2. The amplitude of the variability of the flux of accreting sources scales linearly with the value of their time average flux [9, 10], while in the original shot noise model, which constructs the light curve via summation of randomly occuring independent shots, the amplitude of variability can not be directly related with the time averaged flux of the source. The distribution of values of X-ray flux typically have log-normal distribution rather than normal (see Fig. 1), which should be expected if the flux of the source is constructed via simple summation of independent shots.

The solution of these problems was found in so called propagation flow models. In particular Lyubarskii [11] (some ideas were presented by Miyamoto et al. [12]) considered fluctuations of the mass accretion rate in the innermost region of the accretion flow associated with fluctuations of the viscosity parameter α of the flow at much larger radii. If the amplitude of fluctuations of α does not depend on the radius at which it occurs, the mass accretion rate at the innermost radius of the flow will have an f^{-1} power density spectrum. Thus in this model the mass accretion rate is modulated at different distances from accreting object, but the observed (modulated) X–ray flux is coming from the innermost region. The broad dynamic range of the variability time scales in the model of Lyubarskii [11] is naturally provided by the broad range of radii of accretion disk at which the viscosity is fluctuating.

This model was elaborated and successfully applied to the data of Cyg X-1 in Churazov et al. [8] (see Fig. 2). Subsequent application of this model to different objects showed that this model is able to reproduce the shape of the power spectra of sources, dependence of their variability amplitudes on the value of their flux, the distribution of value of fluxes of sources, Fourier frequency dependent time and phase lags and their dependence on energy [8, 10, 13, 14].

The main properties of variability of X-ray emission of accreting sources have the following interpretation in the propagation flow model:

1. Emission of an accreting object emerges from the region of the main energy release in the accretion flow, but the value of the mass accretion rate within this region at

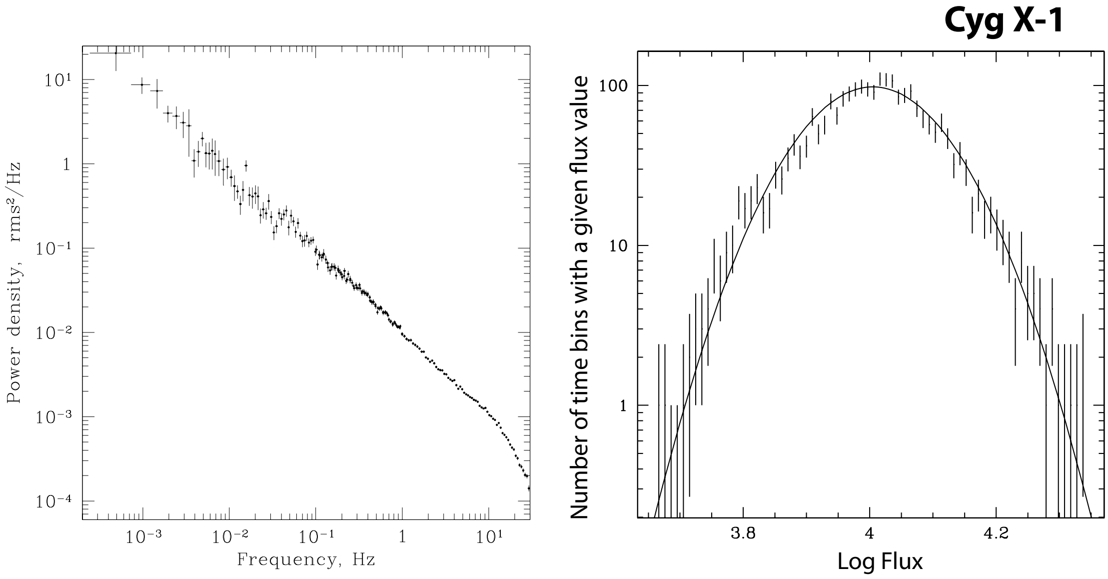

FIGURE 1. Problems of the original shot noise model. *Left:* Power spectrum of variability of the source (Cyg X-1 in the soft spectral state) on very wide range of time scales have a single slope. It is hard to imagine that all of these time scales are produced within small region of main energy release in the accretion flow. *Right:* Distribution of values of X-ray flux, constructed out of Cyg X-1 light curve, recorded with 1 sec time bins. Clear lognormal distribution of the flux values (solid curve) can not be constructed by additive summation of independent shots, implied by the original shot noise model.

any given time is determined by its modulations inserted in the flow at a broad range of distances from the compact object. If the structure of the accretion flow is the same at all distances, (as one might expect in the soft/high spectral state of a source, see e.g. Fig. 2), one should expect to see a power law shape of the power spectrum of the source variability from the shortest time scales, which can be created by instabilities in the innermost region of the accretion flow up to the longest time scales for which the outermost regions of the accretion flow are responsible [8].

2. As the mass accretion rate in the innermost region of the accretion flow is a result of *multiplicative* summation of fluctuations, created at different distances from the compact object, the distribution of the source flux values naturally becomes lognormal and the amplitude of short term variability is directly proportional to the current value of the source flux [10, 14].
3. At frequencies below which the outermost region of the accretion flow can not produce variability the power spectrum of a source light curve becomes flat [8, 15].
4. Under the assumption that the inner regions of the accretion flow emit slightly harder spectra of a power law shape, than the outer regions, the propagation flow model naturally predicts the logarithmic dependence of the value of phase lag of light curves in different energy channels [13] and the shape of the phase lag dependence on Fourier frequency [14].

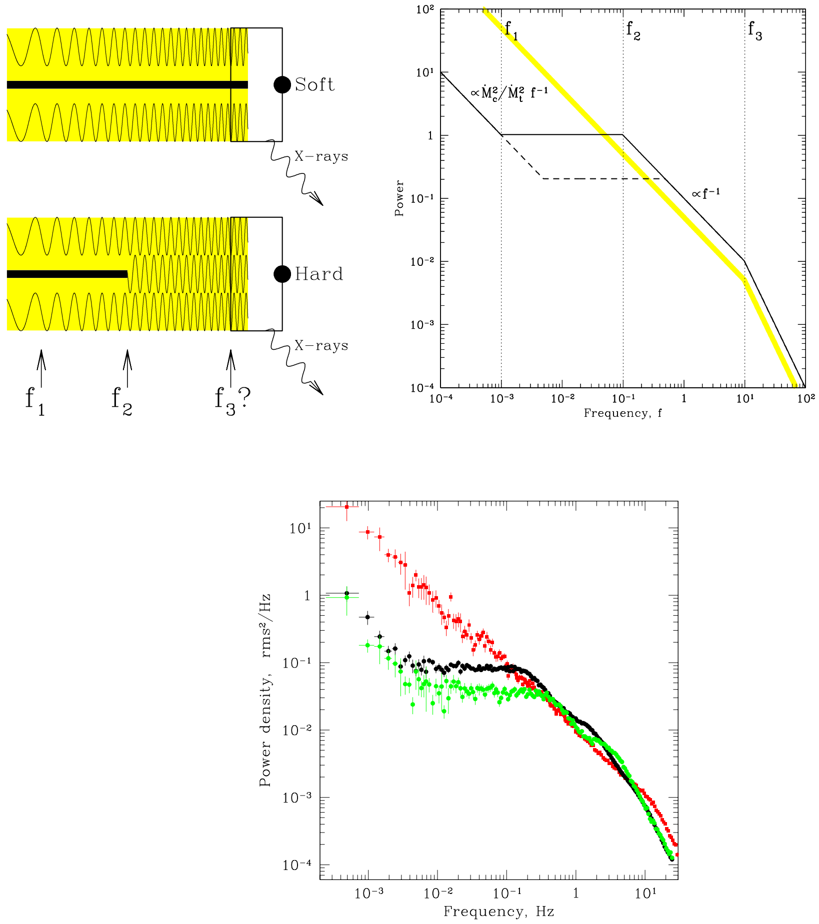

FIGURE 2. *Top–Left:* Sketch of the assumed geometry for the soft and hard states of Cygnus X-1. The solid circle marks a position of a black hole. The box shown by thin lines shows the area where most of the gravitational energy is released and where most of the X–ray radiation is emitted. The black “slab” shows the optically thick (geometrically thin) accretion disk. In the soft state the inner edge of the disk is close to the black hole, while in the hard state it is truncated far from the energy release region. Sandwiching the disk is an optically thin, geometrically thick corona (grey shaded regions), extending in the radial direction up to a large distance from the black hole. Oscillating curves show schematically that at different radii the mass accretion rate in the corona is modulated on different time scales. This modulated accretion flow reaches the innermost region and causes the fluctuations of the observed X–ray flux over the broad range of time scales. *Top–Right:* The overall shape of the power spectra expected in the simple geometry adopted here. In the hard state (thick solid line) there are three breaks (f_1, f_2, f_3 shown by thin vertical lines) in the power spectrum. f_2 is the characteristic frequency in the optically thin flow at the disk truncation radius. Anticipated changes in the power density spectrum associated with the inward motion of the disk truncation radius are shown by the dashed line. In the soft state the power spectrum (thick grey line) is a power law up to f_3. *Bottom:* Typical power density spectra of Cygnus X-1 in the hard (black and grey circles) and soft (squares) states. The power spectra are constructed from the RXTE data in the 6–13 keV energy range. From [8].

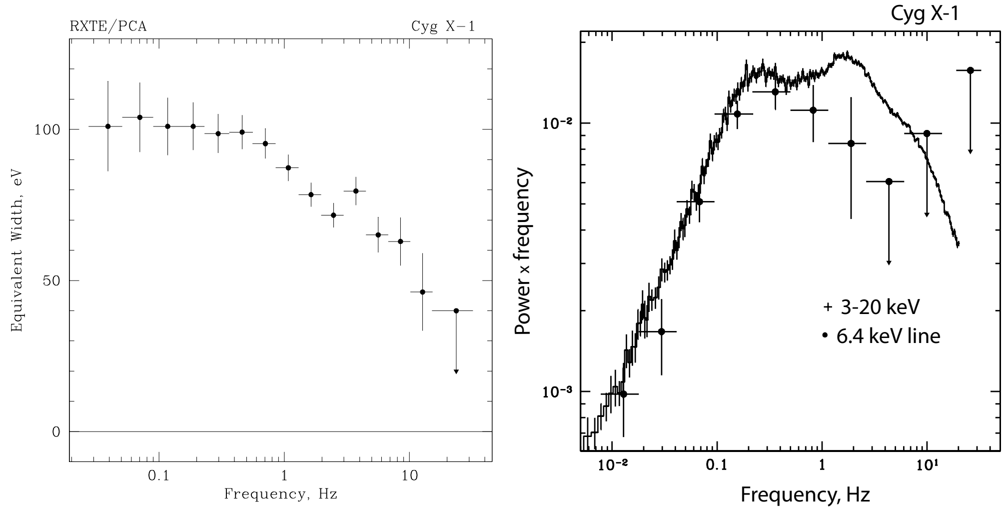

FIGURE 3. *Left:* Equivalent width of the fluorescent line as a function of Fouier frequency at which the variability spectrum is constructed (see details in [18]). *Right:* The power spectra of the total flux of Cyg X-1 (3–20 keV) and the power spectrum of the flux of the fluorescent line. The light curve of the fluorescent line was constructed via direct decomposition of the energy spectrum of the source, accumulated in 1/64 sec time bins.

FREQUENCY RESOLVED SPECTRAL ANALYSIS

Time variability of X-ray flux of sources contains a lot of information in addition to commonly used time averaged characteristics.

In particular, it may help to separate different components of the time averaged energy spectrum in a model independent way. It is very easy to understand: if, for example, the time averaged spectrum consists of two spectral components, one of which does not vary with time and another one varies as a whole at some Fourier frequency, one can separate these two spectral components from each other without any a priori assumptions about their shapes and without any spectral modelling (see e.g early work on this topic [16]).

The question about secure separation of spectral components is very important in a number of cases. As an example, one can mention the modelling of the fluorescent iron emission line in the spectra of accreting neutron stars and black holes (see e.g. [17]). For such spectral decomposition the model independent approach is essential.

It is likely that the most advanced approach to study the combined spectral and timing information in the emission of sources is the Fourier frequency resolved spectral technique [18]. In essence, it provides the energy spectrum of source variability in a given Fourier frequency band in instrumental units, which can be unfolded using knowledge of the response function of the instrument and standard packages like *XSPEC* (see details in [18, 19]).

For example, with the help of this method it was shown that the fluorescent iron line ($\sim$ 6.4 keV) which is often visible in the spectrum of accreting sources and originating, apparently, as result of reprocessing of the hard X-ray emission of the central regions of the accretion flow from a relatively cold surrounding accretion disk, is much less variable at high Fourier frequencies than the continuum emission [18]. Similar behavior of the variability of the iron line component in the emission of AGNs was revealed in [20–22]. This directly shows that these two spectral components (continuum and the line) originate in geometrically distinct regions, and one of the most probable explanations of such a behavior of the iron line is that the fastest innermost region does not produce the fluorescent line photons due to further distance from the reflector/reprocessor [18].

The Fourier frequency resolved spectral technique of separation of spectral components has significant advantages over other methods. For example, in order to see the variability of some particular spectral component one can decompose the spectrum of the source, collected in small time bins, construct the time history of the flux of the distinct spectral component and then obtain the power spectrum of this light curve. However, in this case (especially if the spectral model is not linear) the model fitting introduces significant additional noise in the resulting light curves and thus, the resulting power spectrum of the time history of some component will be more noisy than if we would determine the amplitudes of variability of the spectral component from Fourier frequency resolved energy spectra.

In Fig. 3 we present the strength of the fluorescent iron line in the spectrum of Cyg X-1 (in hard spectral state) as a function of Fourier frequency, obtained via the Fourier frequency resolved spectral technique (left panel) and via construction of the power spectrum of variations of the emission line component, which in turn was obtained with the help of fitting of the energy spectrum of the source collected in 1/64 sec time bins. It is seen that results obtained with these two methods are compatible, while the latter method gives much more noisy results.

In the following part of the paper I will concentrate on one example where application of the Fourier frequency resolved spectral technique allowed us to solve the long standing problem of separation of spectral components in the soft/high state of neutron star binaries and allowed for estimates of the mass and radii of accreting neutron stars.

EMISSION OF THE NEUTRON STAR BOUNDARY/SPREADING LAYER AND MASSES AND RADII OF NEUTRON STARS

Accreting neutron stars in low mass X-ray binaries (LMXB) are among the most luminous compact X-ray sources in the Galaxy. At least several of them have luminosities exceeding $\sim$ few $\times 10^{38}$ erg/s, and they presumably accrete matter at a level close to the critical Eddington accretion rate. Early observations of these sources [e.g. 23] revealed rather soft X-ray spectra, indicating that their X-ray emission is predominantly formed in optically thick media. Similar to accreting black holes, at lower X-ray luminosities (lower mass accretion rates), $L_{\rm X} < 5 \times 10^{36}$ erg/s, neutron stars undergo a transition to the hard spectral state. The energy spectra in this state point toward low optical depth in the emission region.

In the soft spectral state, the commonly accepted picture of accretion at values of the accretion rate that are not too extreme has two main ingredients – the accretion disk (AD) and the boundary layer (BL). While matter in the disk rotates with nearly Keplerian velocities, in the boundary layer it decelerates down to the spin frequency of the neutron star and settles onto its surface. For a typical neutron star spin frequency

($< 500-700$Hz), comparable amounts of energy are released in these two regions [24, 25]. This picture is based on rather obvious qualitative expectations, as well as on more sophisticated theoretical considerations and numerical modeling [24, 26]. This has been receiving, however, little direct observational confirmation. Due to the similarity of the spectra of the accretion disk and boundary layer, the total spectrum has a smooth curved shape, which is difficult to decompose into separate spectral components [16, 27–29]. This made it difficult to apply physically motivated spectral models to the description of observed spectra of luminous neutron stars, in spite of a very significant increase in the sensitivity of X-ray instruments. A possible solution was suggested by early results by Mitsuda et al. [16], who demonstrated the potential of using the combined spectral and variability information.

Recently, Gilfanov et al. [30] analyzed spectral variability in luminous LMXBs and showed that in these sources aperiodic and quasi-periodic variability on $\sim$ sec – msec time scales is primarily caused by variations in the luminosity of the boundary layer, while the flux of the accretion disk remains almost perfectly stable at these frequencies (note that a similar result about high stability of the X-ray flux of the accretion disk in the case of the accreting black hole Cyg X-1 in the soft spectral state was obtained previously in [8]). The spectral shape of the boundary layer component remains nearly constant in the course of the luminosity variations and is represented by the Fourier-frequency resolved spectrum (see Fig. 4). Moreover, in the considered range $\dot{M} \sim (0.1-1)\dot{M}_{\rm Edd}$ ($\dot{M}_{\rm Edd}$ is the critical Eddington mass accretion rate), it depends weakly on the global mass accretion rate and in the limit $\dot{M} \sim \dot{M}_{\rm Edd}$ is close to the Wien spectrum with $kT \sim 2.4$ keV. In the work of Revnivtsev and Gilfanov [31] it was shown that such a behavior is universal for all sources, for which the data allowed construction of frequency resolved energy spectra and the spectra of the boundary layer in all these cases are remarkably similar to each other in spite of almost an order of magnitude difference in total luminosities of sources (see Fig. 5, left panel).

Such behavior accords with the predictions of the model by Inogamov and Sunyaev [26], namely, that at sufficiently high accretion rates, $\dot{M} > 0.1\dot{M}_{\rm Edd}$, the boundary layer is radiation-pressure dominated, and thus *the local radiation flux is close to the critical Eddington value.* Increase of the mass accretion rate leads to an increase of the emitting area of the BL, while its vertical structure changes little [26].

It is important to notice that in this picture, the parameters of the BL emission can be used to determine the value of the Eddington flux limit on the surface of the neutron star. As the Eddington flux limit is uniquely determined by the neutron star surface gravity and by the photospheric chemical composition, the neutron star mass and radius can be constrained.

If the boundary layer emits true blackbody emission, the radiation flux of the unit area is determined only by its temperature. Therefore the observed shape of the BL spectrum, in particular the bestfit blackbody temperature, could be used to determine the value of the Eddington flux limit on the neutron star surface. This approach has been utilized in the context of the Eddington limited X-ray bursts [e.g. 32–35]. For fully ionized hydrogen atmosphere

$$\frac{\sigma T^4}{c}\frac{\sigma_{\rm T}}{m_{\rm p}} = \frac{GM(1-R_{\rm Sch}/R)^{3/2}}{R^2}$$

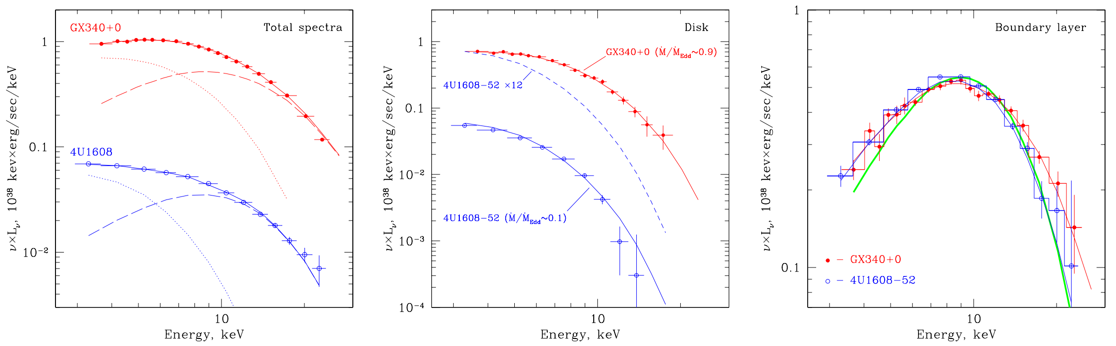

FIGURE 4. *Left:* Spectra of total emission of accreting neutron star binaries GX340+0 and 4U1608-52 decomposed into spectral components, originating in accretion disk around the neutron star (dotted curve) and the boundary layer on its surface (dashed curve). *Center:* Spectra of the accretion disk in these two cases. Note the change of the temperature of the accretion disk. *Right:* Spectra of the boundary layer in these two cases (the spectrum of the boundary layer of 4U1608-52 was scaled up to match those of GX340+0). It is clear that the shape of the BL spectrum virtually does not change in spite of more than an order of magnitude difference in its luminosity. From [30]

where σ is the Stefan-Boltzmann constant, σ_T is the Thomson cross-section, T is the blackbody temperature at infinity, m_p is the proton mass, M is the mass and R is the radius of the neutron star, $R_{Sch} = 2GM/c^2$ is the Schwarzschild radius of the neutron star. In addition, one would have to take into account that the value of the Eddington flux is somewhat (by $\sim 10-20\%$) reduced because of the action of the centrifugal force caused by the rotation of the boundary layer [26]. Rotation of the neutron star at this point is not very important unless it is very high (rotational frequency >800-1000 Hz).

What is very important in our case are the scatterings in the atmosphere of the neutron star, therefore, the boundary layer spectrum will differ from a black body (e.g. [34, 36]). The radiation transfer problem in the atmosphere of the neutron star has been intensively investigated, in particular in the context of X-ray bursts. Numerical calculations show that the effects of scatterings can be approximately accounted for by introducing the spectral hardening factor that relates the color and the effective temperatures of the emission [36–39]. Typical values of the hardening factor are about ~ 1.7. We used this result in order to make simple estimates of the gravity on the neutron star surface and to constrain its mass and radius. In these calculations we assumed the color temperature of the boundary layer emission $T = 2.4$ keV and considered the range of the hardening factor values of 1.6-1.8. We assumed that the centrifugal force reduces the Eddington flux limit in comparison with the non-rotating boundary layer by 20%. We also took the finite height of the boundary/spreading layer into account, which is about 1 km [26]. The calculations were performed for a Schwarzschild geometry, hydrogen atmosphere, and the result is shown in Fig. 5. The width of the shaded region is defined by the assumed range of the values of the hardening factor. Detailed modeling of the emergent spectrum of the BL allows one to strongly diminish the size of uncertainty region [40].

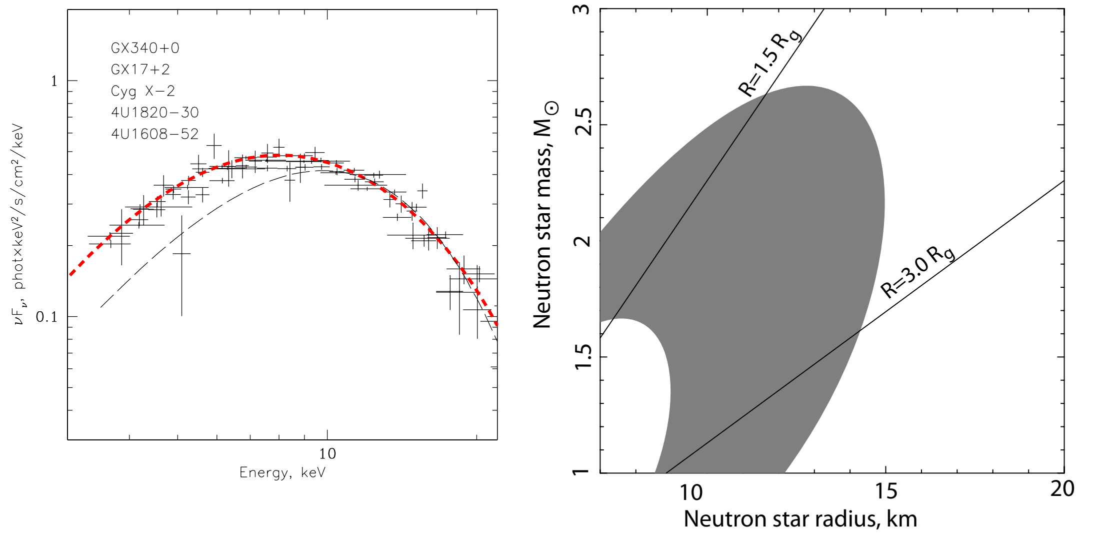

FIGURE 5. *Left:* Frequency resolved energy spectra (=spectra of the boundary layer, see text) of a number of accreting neutron star binaries in their soft spectral state. Thick dashed curve shows the best fit approximation of the spectra by a Comptonized emission model (see [31] for details). Thin dashed curve – the black body approximation of the spectra at energies $10-20$ keV. The color temperature of the black body model is $kT \sim 2.4$ keV. *Right:* Constraints on masses and radii of accreting neutron stars from the measured value of the Eddington flux emerging from the photosphere of their boundary layers (see text, details are presented in [31])

CONCLUSION

Summarizing all of the above one can say that over the last decade (with great help of high collecting area instruments aboard the *Rossi X-ray Timing Explorer*) we have achieved big progress in understanding aperiodic variability of accreting X-ray binaries. Existing models of propagating fluctuations allow us to explain the majority of observed properties of the aperiodic variability.

Now with the help of the Fouier frequency resolved spectral technique we are able to study combined spectral and timing variability of the accreting binaries, which proved to be a very efficient method of disentangling the contribution of geometrically different regions to the total time averaged spectrum of sources. Application of this method already allowed us to obtain separately the spectrum of the boundary/spreading layer at the surface of accreting neutron stars, thus emphasizing the importance of the surface effects in the case of the accretion flow around neutron star binaries. Our findings support the theoretical predictions that the boundary/spreading layer at the neutron star surface at high mass accretion rates should be radiation pressure dominated. This in turn allowed us to obtain important constrains on masses and radii of neutron stars. We hope that with the appearance of new timing instruments on the next generation of observatories (e.g. XEUS/HTRS, see [41]), we might probe even stronger gravity in the innermost regions of the accretion flow and variable phenomena on neutron stars surface under extreme conditions of high matter density and radiation pressure dominated environment.

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