WARPED AND ECCENTRIC DISCS AROUND BLACK HOLES

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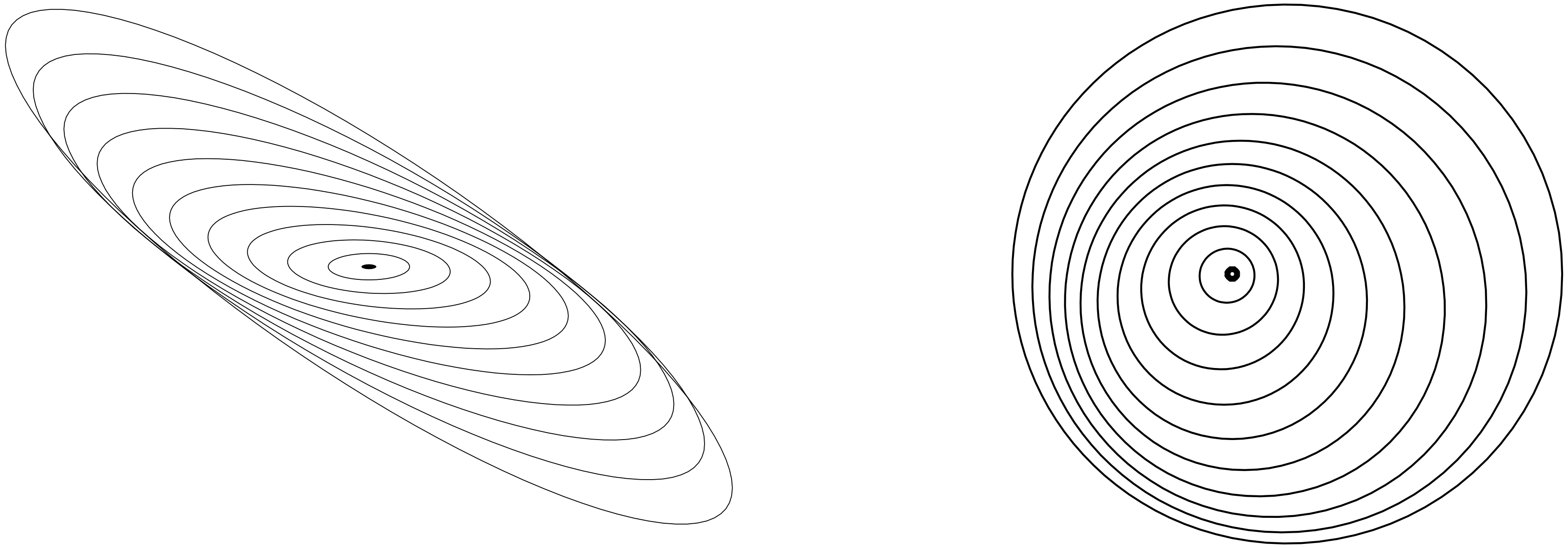
INTRODUCTION

Relevance of warped and eccentric discs :

- global precession and variability
- misalignments
- outburst dynamics
- precession of inner disc
- activity of inner disc and QPOs (Kato ; Ferreira)

GENERAL KEPLERIAN DISC

- variable inclination : warped disc
- (variable) eccentricity : eccentric disc



- collective effects : slow evolution
- in a binary : (coherent) precession
- inner disc : significantly non-Keplerian

LINEAR ANALYSIS

Warp or eccentricity as linear perturbation

Wave modes $\exp(\mathrm{i}m\phi - \mathrm{i}\omega t)$ $\hat{\omega} = \omega - m\Omega$

Local dispersion relation (isothermal case) :

$$(\hat{\omega}^2 - \kappa^2)(\hat{\omega}^2 - n\Omega_z^2) = \hat{\omega}^2 c_\mathrm{s}^2 k^2$$

- **radial wavenumber** k
- **vertical mode number** n
- **horizontal and vertical epicyclic frequencies** $\kappa,\ \Omega_z$

Warp : $m = 1,\ n = 1$

Eccentricity : $m = 1,\ n = 0$

SECULAR THEORIES

- **warp equations :**

$$\Sigma R^2\Omega\left[\frac{\partial W}{\partial t}-\mathrm{i}\left(\frac{\Omega^2-\Omega_z^2}{2\Omega}\right)W\right]=\frac{1}{R}\frac{\partial G}{\partial R}$$

$$\frac{\partial G}{\partial t}-\mathrm{i}\left(\frac{\Omega^2-\kappa^2}{2\Omega}\right)G+\alpha\Omega G=\frac{PR^3\Omega}{4}\frac{\partial W}{\partial R}$$

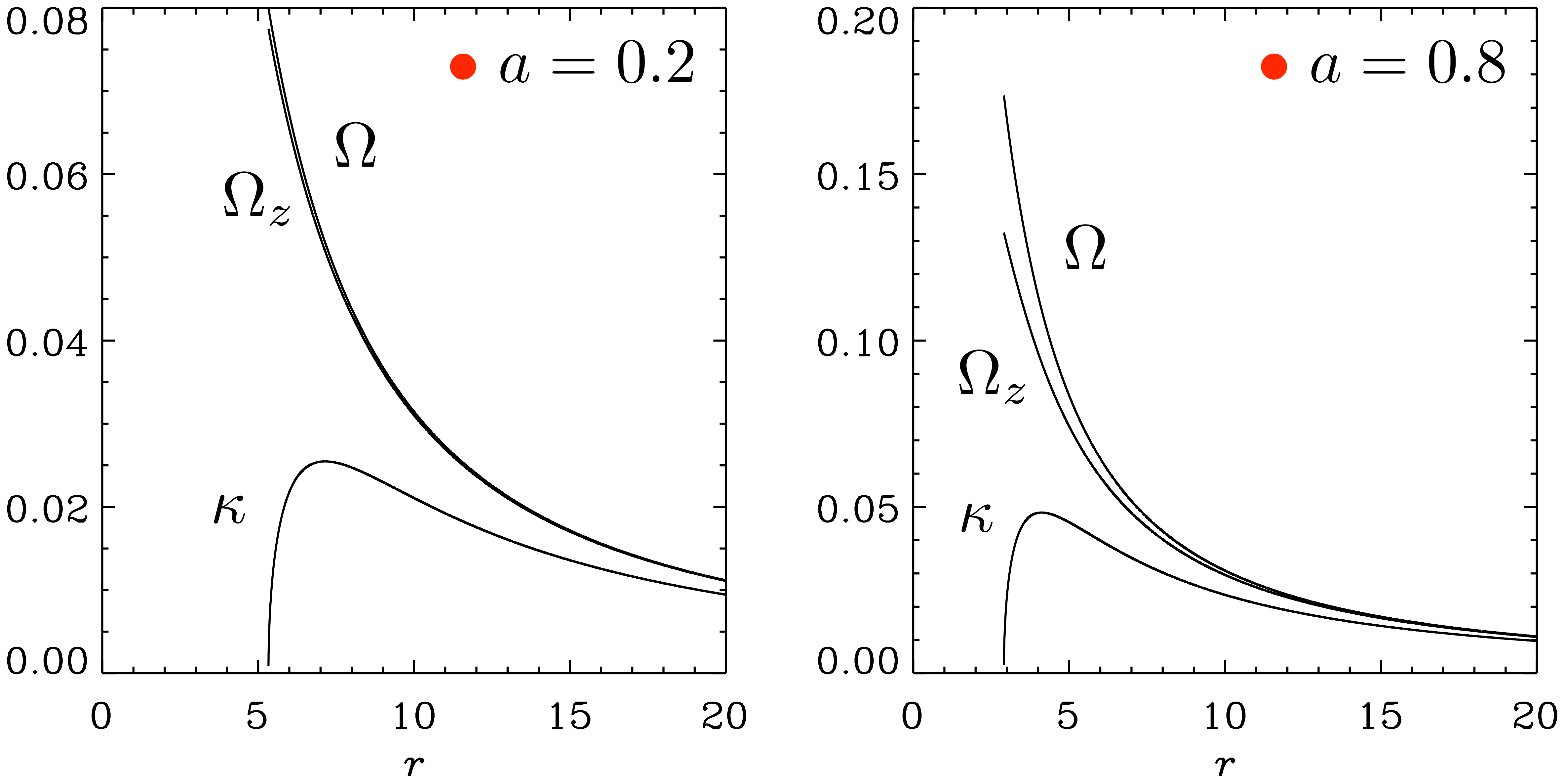
- **eccentricity equation (simplest version) :**

$$-2\mathrm{i}\Sigma R^2\Omega\frac{\partial E}{\partial t}=\frac{1}{R}\frac{\partial}{\partial R}\left[(\gamma-\mathrm{i}\alpha_\mathrm{b})PR^3\frac{\partial E}{\partial R}\right]+R\frac{dP}{dR}E$$
$$+\Sigma R^2(\Omega^2-\kappa^2)E$$

- **nonlinear theories (Ogilvie 1999, 2000, 2001)**

RELATIVISTIC DISCS

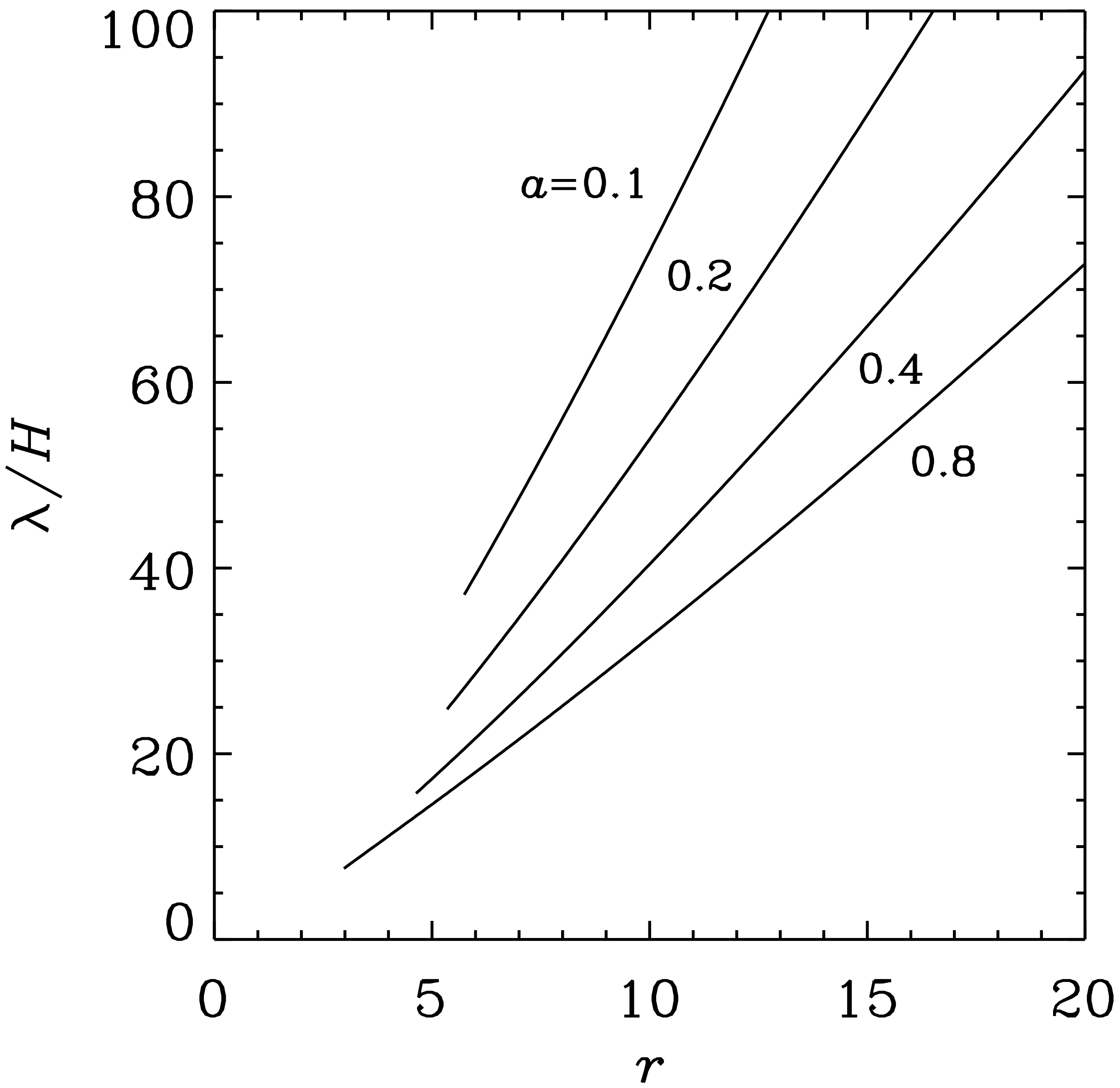
Characteristic frequencies in units of c^3/GM



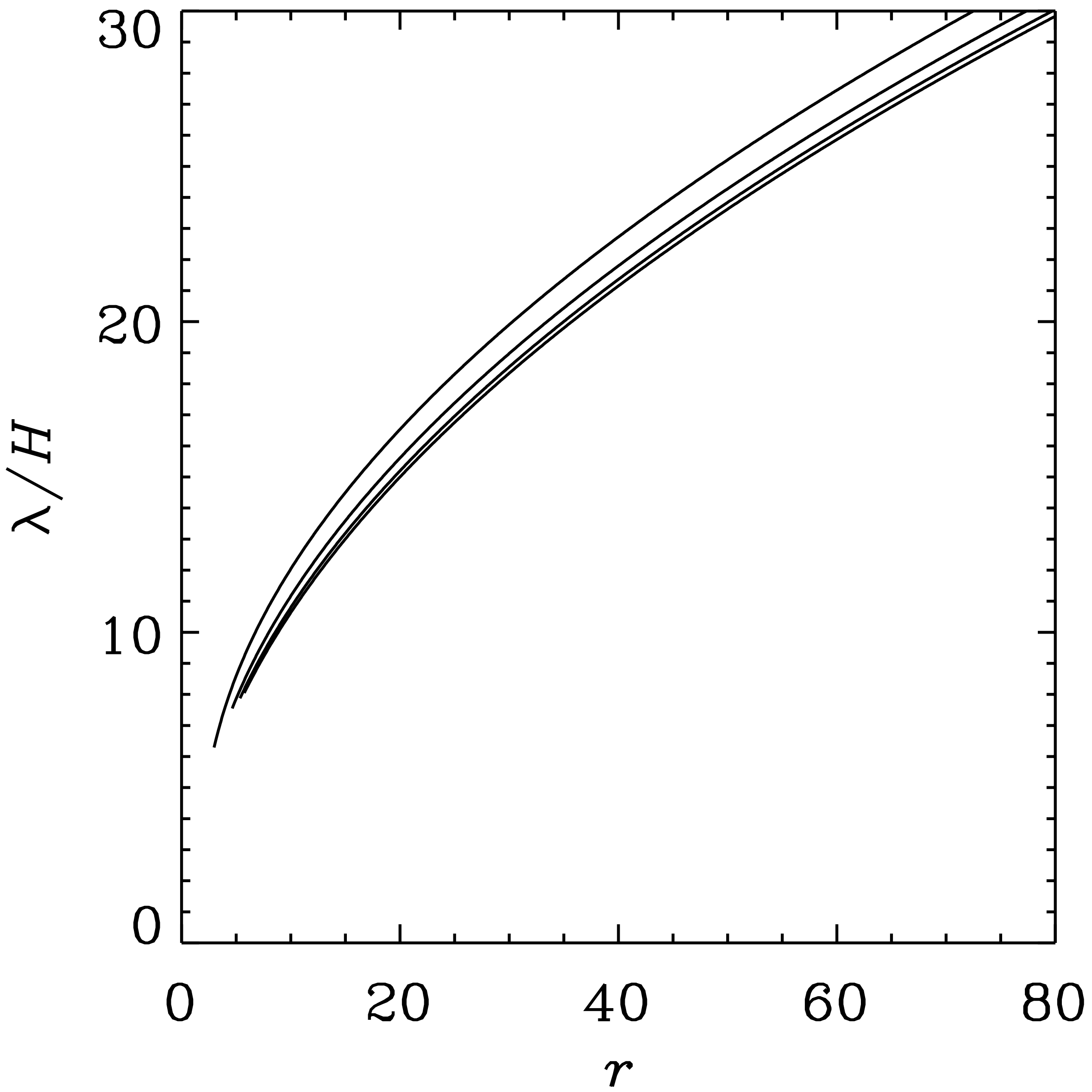
$$R = r(GM/c^2)$$

STATIONARY DEFORMATIONS

warp wavelength



eccentricity wavelength



$$(\hat{\omega}^2-\kappa^2)(\hat{\omega}^2-n\Omega_z^2)=\hat{\omega}^2c_{\mathrm{s}}^2k^2$$

WKB ANALYSIS (inviscid)

$$W \approx \frac{R^{1/8}}{(\Sigma H)^{1/2}} \exp\left[\pm \mathrm{i} \int \left(\frac{24a}{r^{5/2}}\right)^{1/2} \frac{dR}{H}\right]$$

$$E \approx \frac{R^{1/4}}{(\Sigma H)^{1/2}} \exp\left[\pm \mathrm{i} \int \left(\frac{6}{\gamma r}\right)^{1/2} \frac{dR}{H}\right]$$

Apply to Shakura-Sunyaev region (b) :

$$\frac{H}{R} = 0.012\,\alpha^{-1/10} m^{-1/10} \dot{m}^{1/5} f^{1/5} r^{1/20}$$

$$\Sigma \propto f^{3/5} R^{-3/5} \qquad f = 1 - \left(\frac{R_{\mathrm{in}}}{R}\right)^{1/2}$$

$$|W| \propto f^{-2/5} R^{-1/10} \qquad |E| \propto f^{-2/5} R^{3/20}$$

VISCOUS ATTENUATION

Warp attenuation : $\exp\left(-\mathrm{cst} \times a^{1/2} \frac{\alpha}{H/R}\right)$

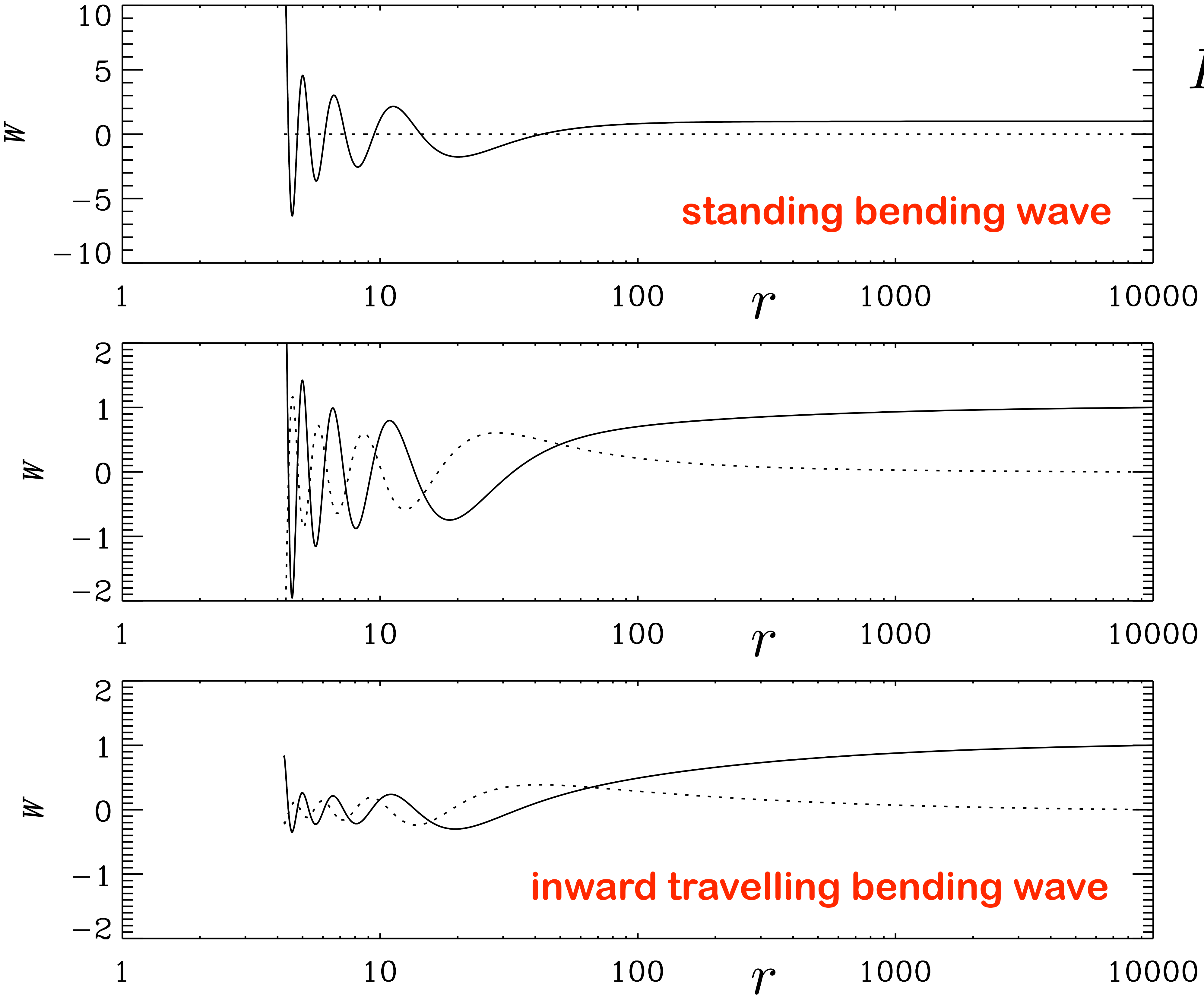
Eccentricity attenuation : $\exp\left(-\mathrm{cst} \times \frac{\alpha}{H/R}\right)$

- **potential for overstability**

Radiation pressure (more important for larger $\dot{m}$) :

- **thicker disc**
- **longer wavelength**
- **less attenuation**

NUMERICAL SOLUTIONS (warp)



$H/R \approx 0.02$

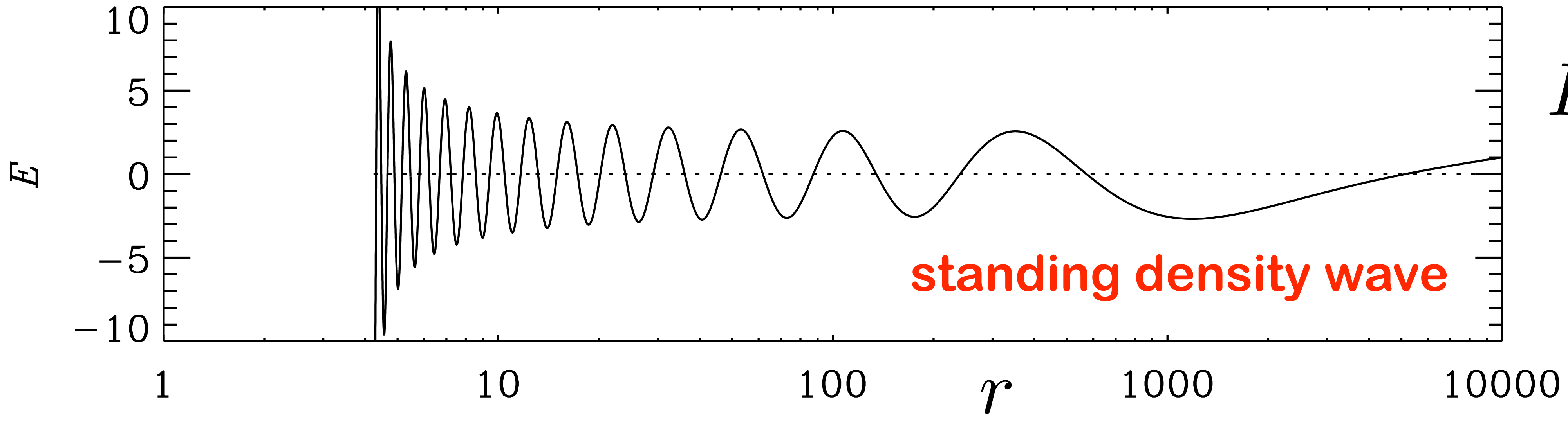
$a = 0.5$

$\alpha = 0$

$\alpha = 0.02$

$\alpha = 0.05$

NUMERICAL SOLUTIONS (ecc.)



$$H/R \approx 0.02$$
$$a = 0.5$$
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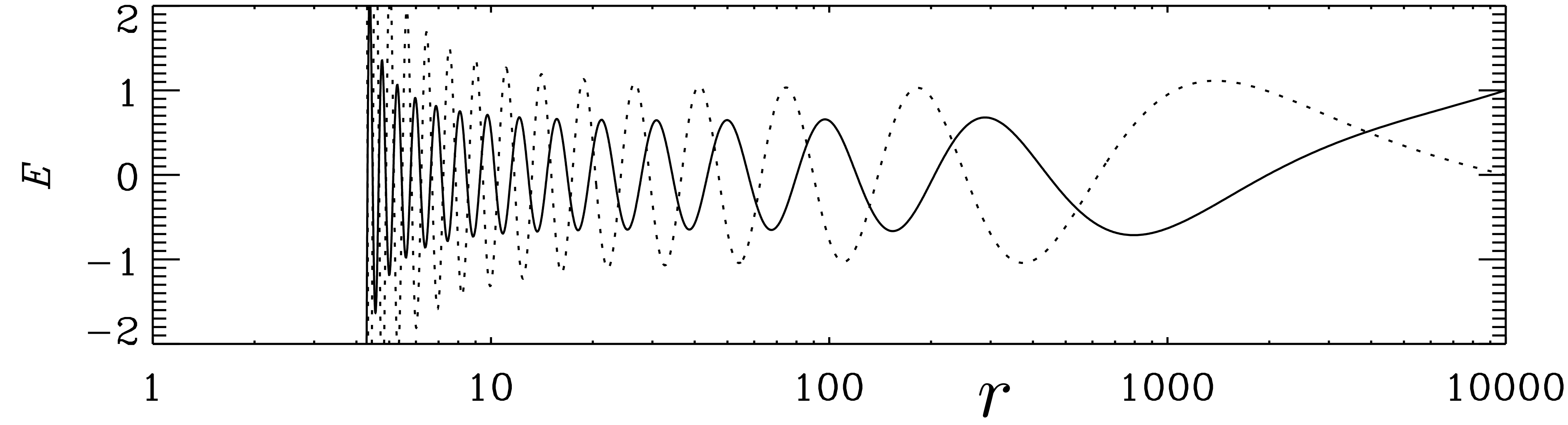
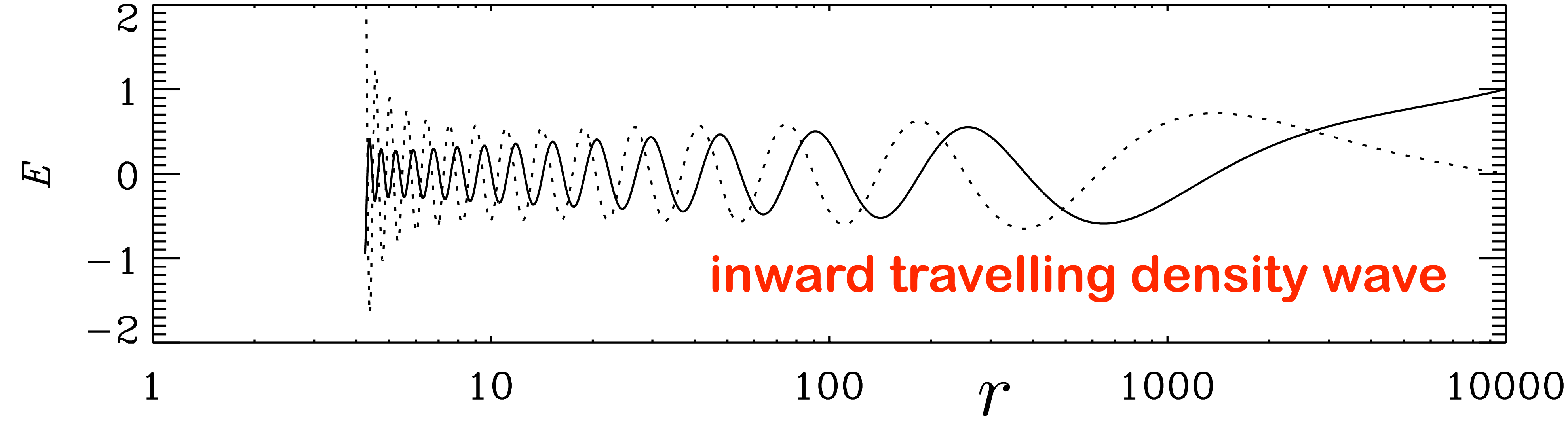


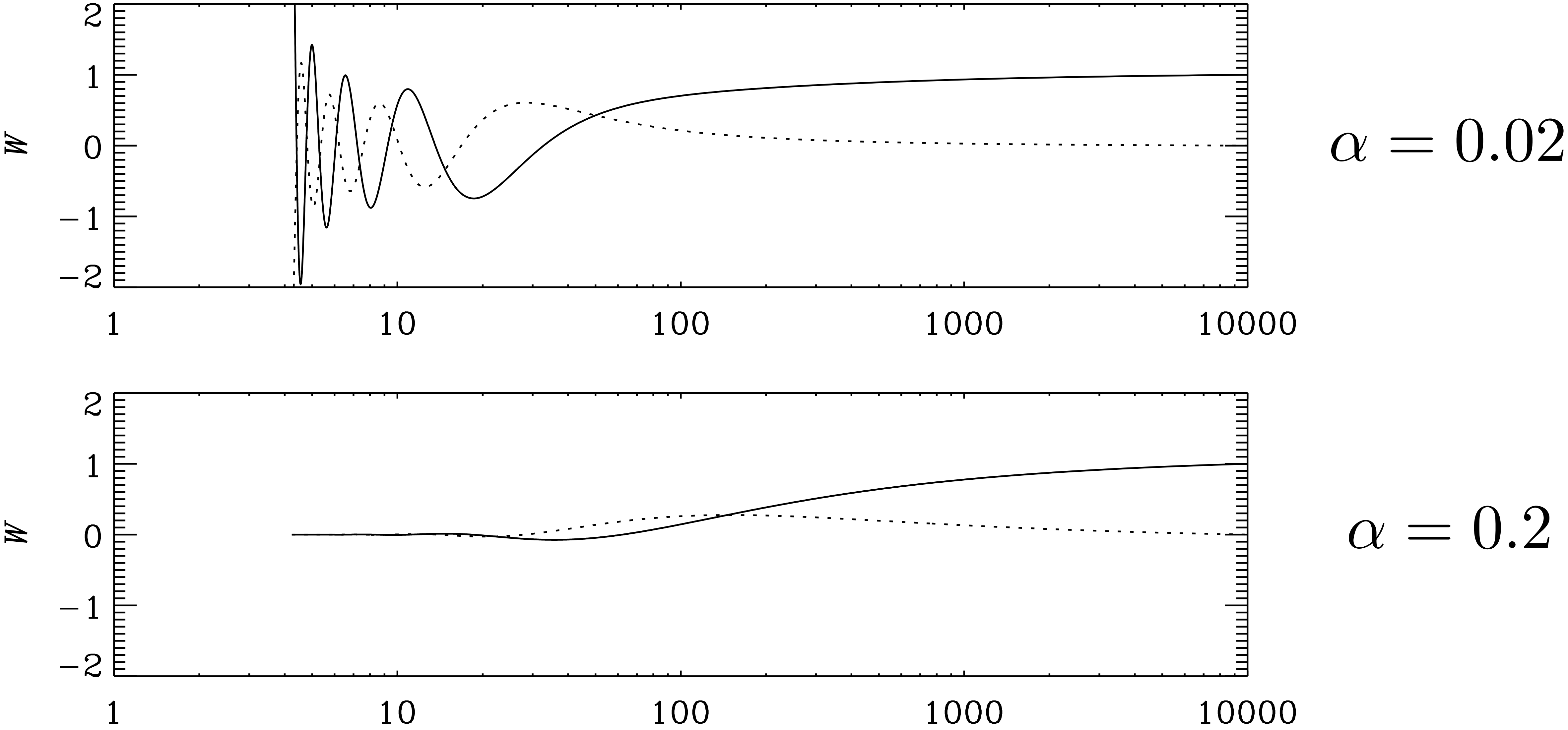
$$\alpha = 0.02$$



$$\alpha = 0.05$$

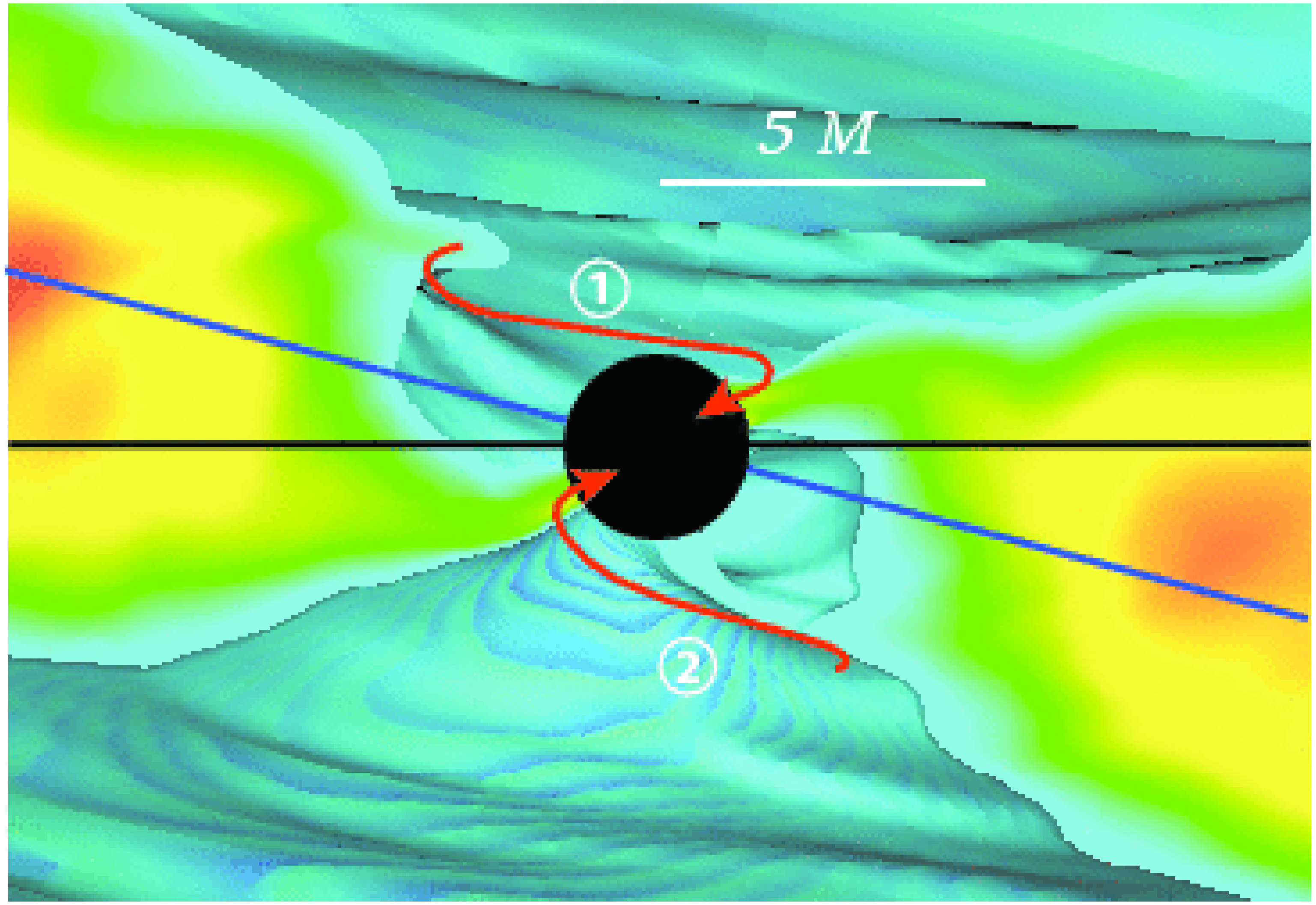
OSCILLATORY WARP

- **Ivanov & Illarionov 1997 ; Lubow et al. 2002**
- **contrast with Bardeen-Petterson effect**



OSCILLATORY WARP

- role of nonlinearity
- implications for jet orientation?
- support from simulations? (Fragile et al. 2008)



BH equator

disc plane

MISALIGNMENTS AND WARPS

Reported jet-binary misalignments :

- GRO J1655-40 } Maccarone 2002
- SAX J1819-2525
- what does jet direction signify?

Precessing tilted (warped) discs :

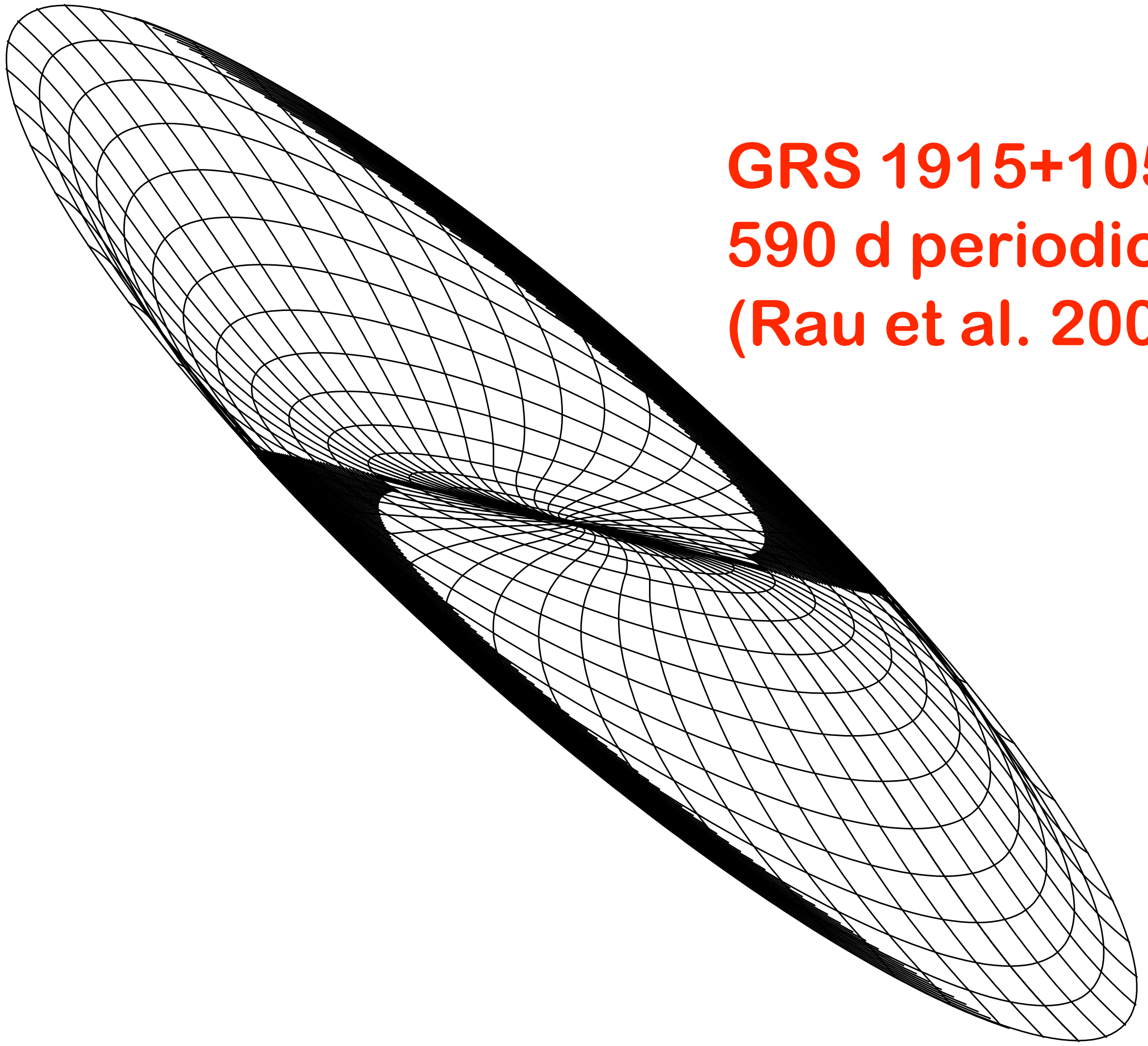
- Her X-1, SS 433, LMC X-4, ...

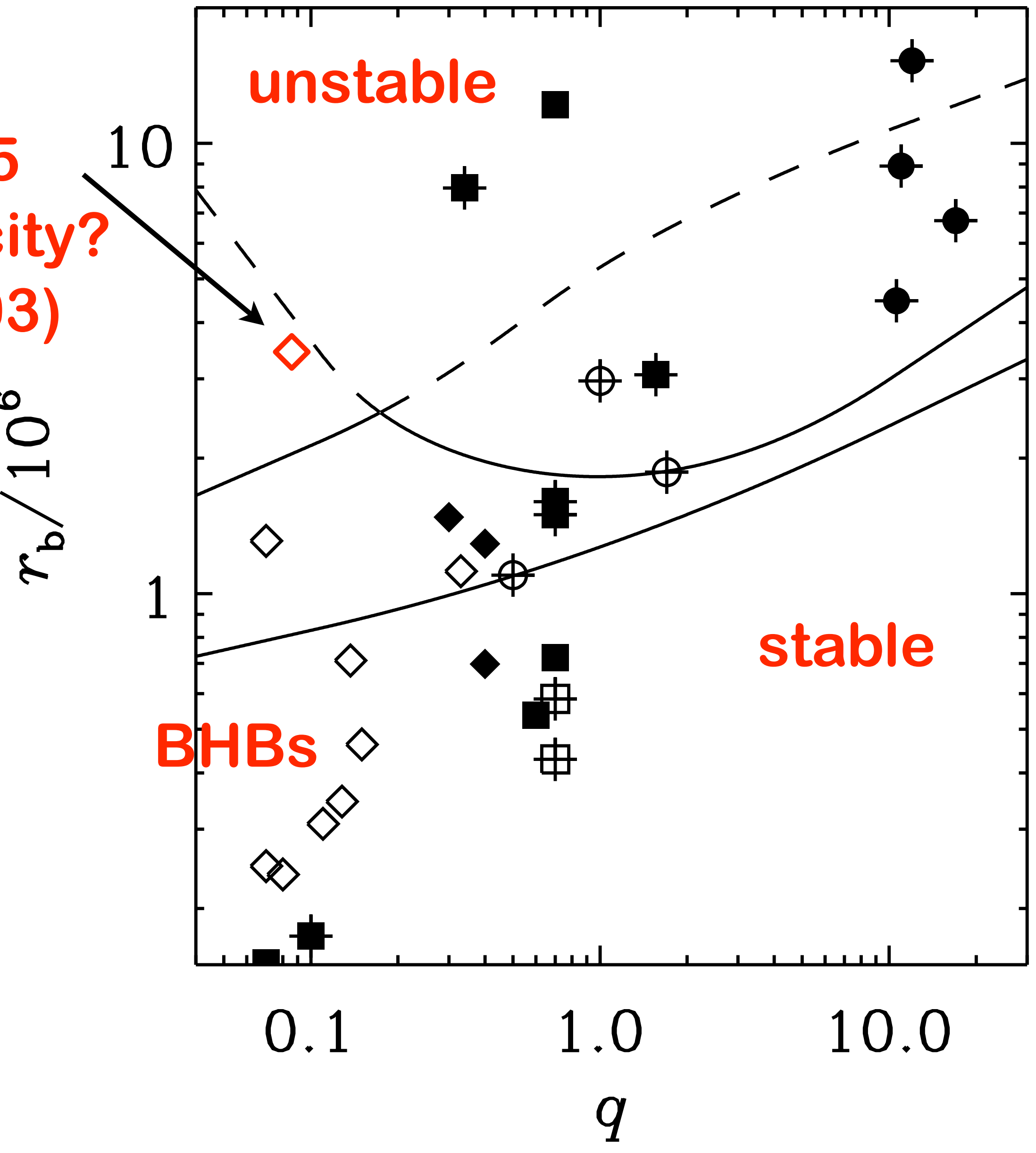
Various possibilities :

- radiation-driven warping (Pringle 1996)
- wind-driven warping (Schandl & Meyer 1994)
- magnetically driven warping (Lai 1999)

RADIATION-DRIVEN WARPING

Pringle (1996) ; Ogilvie & Dubus (2001)



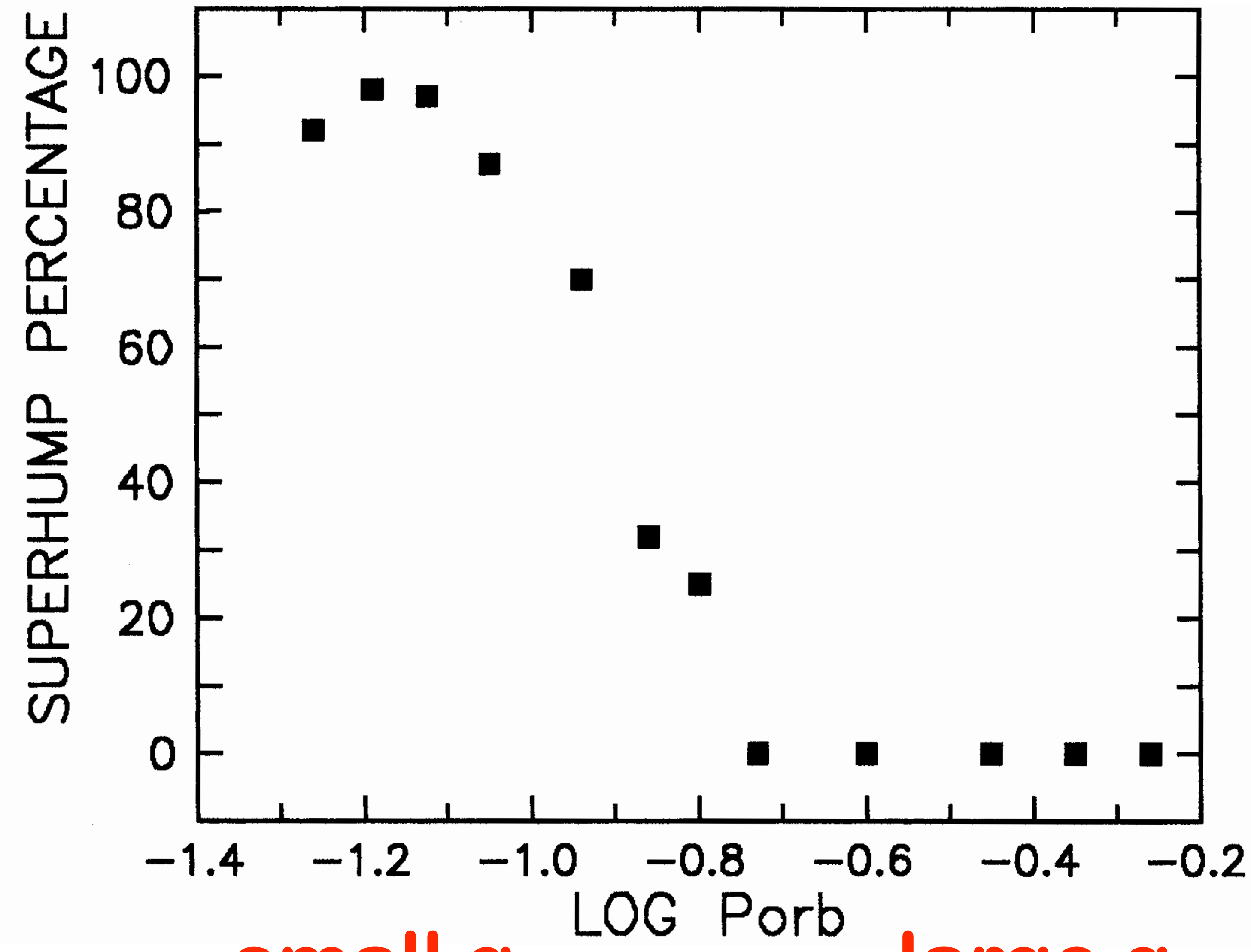


GRS 1915+105
590 d periodicity?
(Rau et al. 2003)

SUPERHUMPS IN CVs

Signature of precessing eccentric disc

Relation to superoutbursts



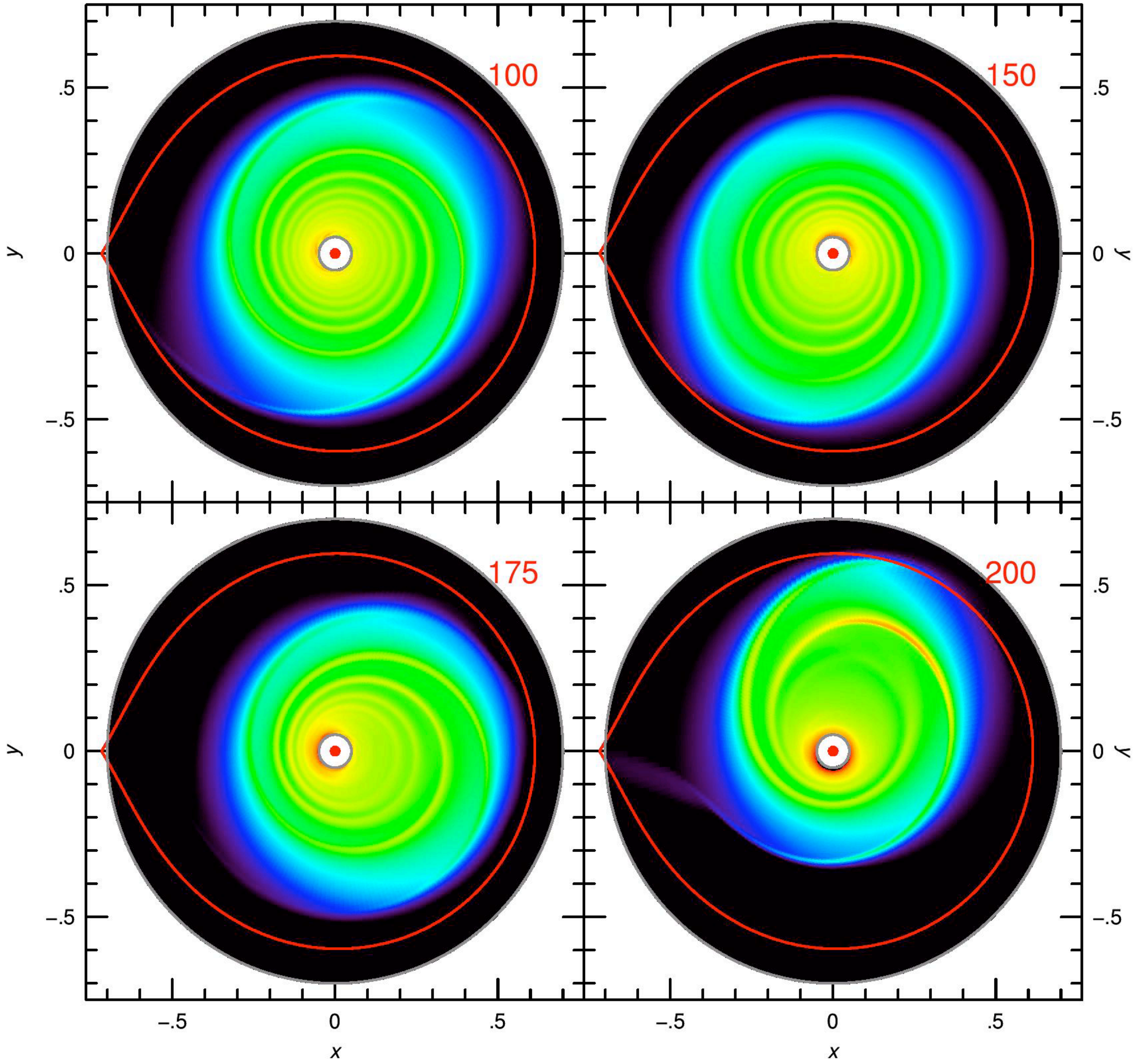
small q

large q

Patterson et al. (2005)

$$q_{\rm crit} \approx 0.3$$

ECCENTRIC DISCS IN BINARIES

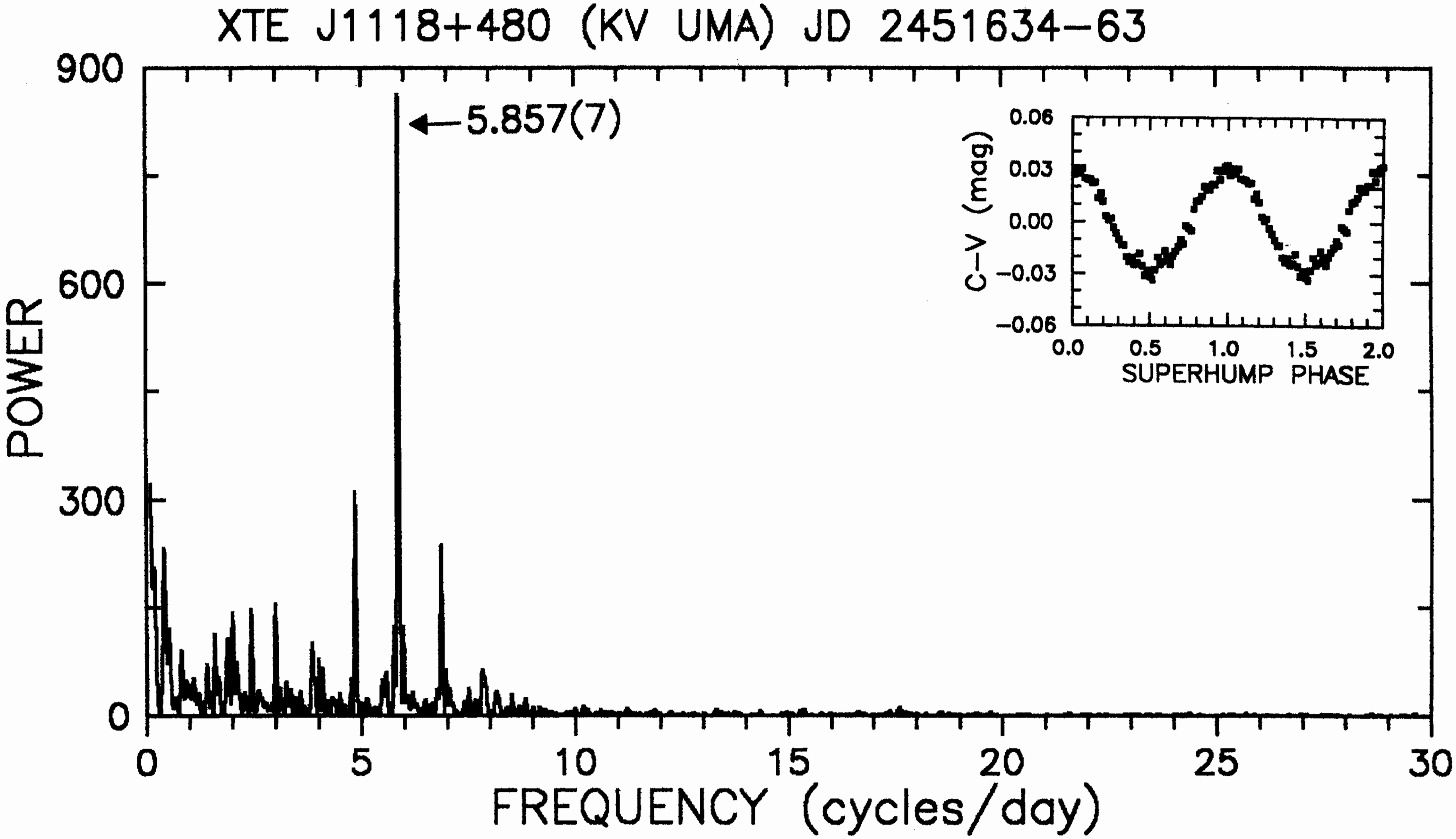


Kley, Papaloizou & Ogilvie (2008)

$q = 0.1$

SUPERHUMPS IN BHBs

Patterson et al. (2005)



SUPERHUMPS IN BHBs

Reported superhumps :

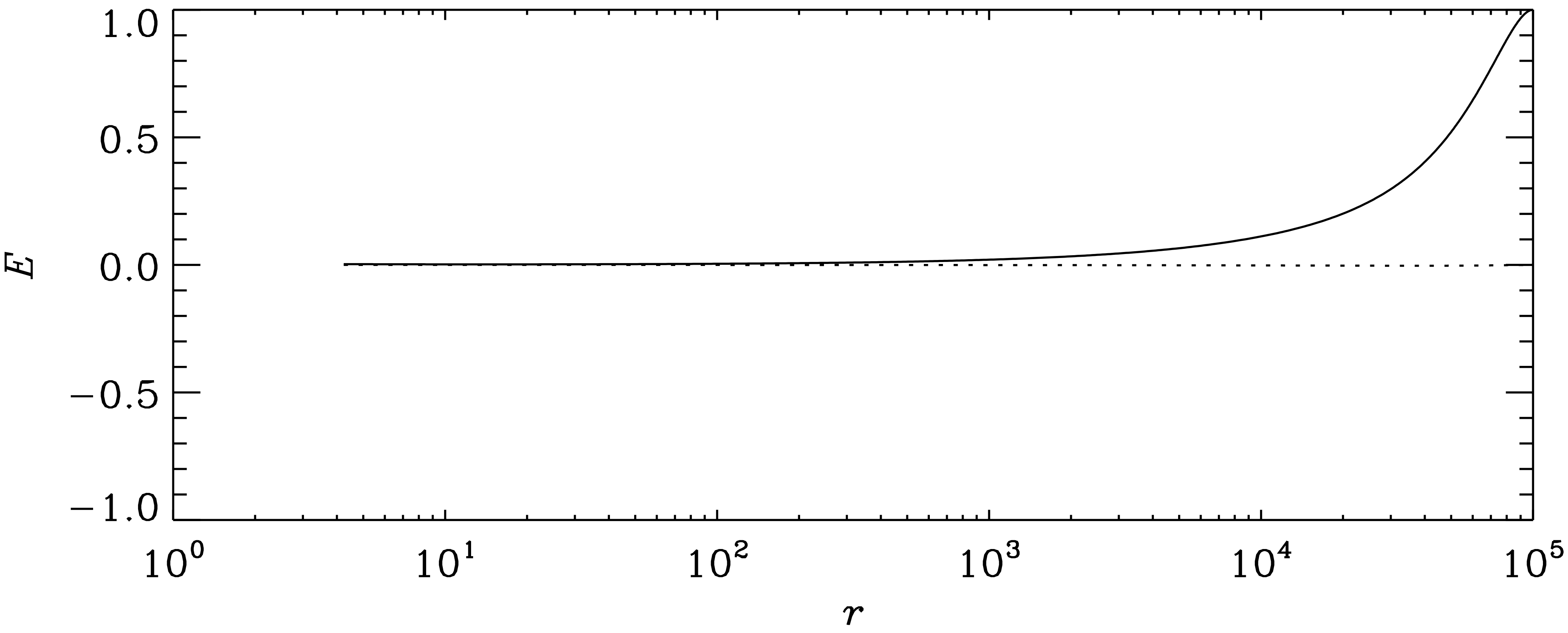
- Nova Mus 1991
- GRO J0422+32
- GS 2000+25 (?)

O’Donoghue & Charles 1996
Haswell et al. 2001

- XTE J1118+480 / KV UMa (Uemura et al. 2002)
- GRS 1915+105 (Neil et al. 2007)
- Swift J1753.5-0127 (Zurita et al. 2008)

GLOBAL ECCENTRIC MODE

With a Newtonian central mass :



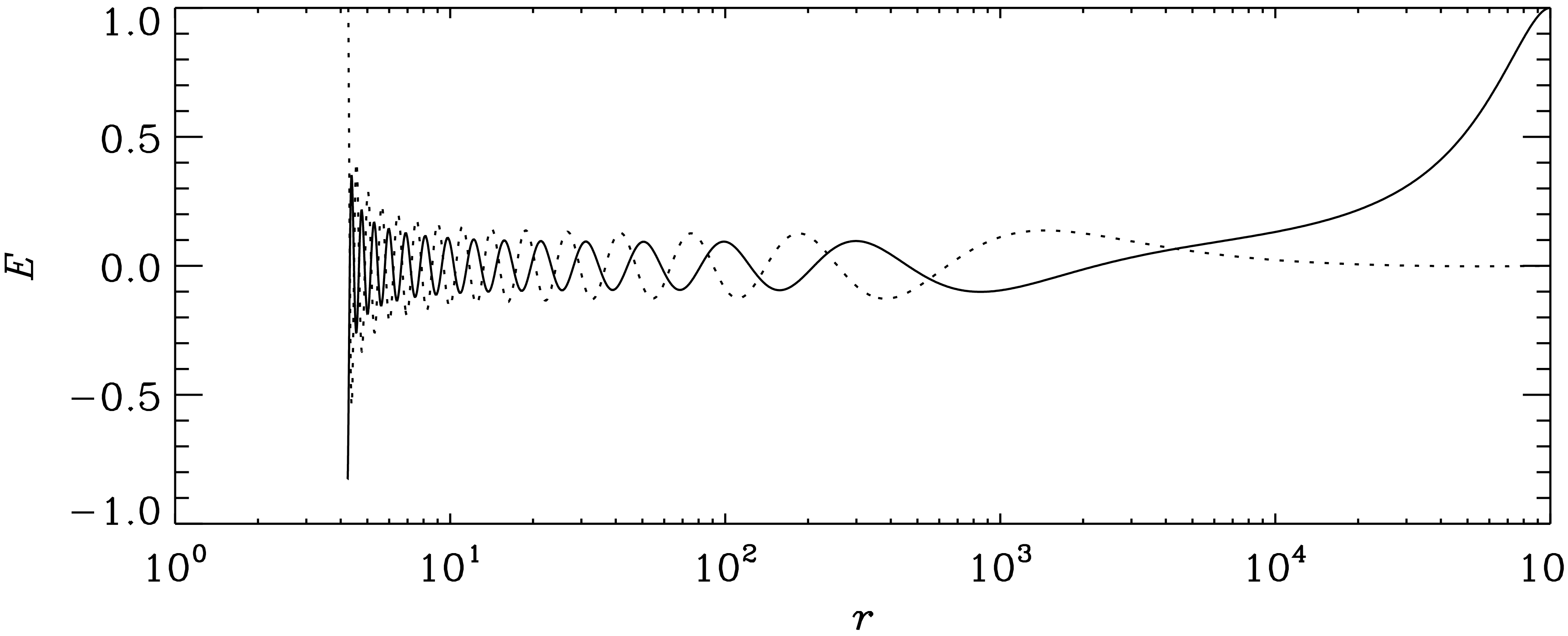
$$H/R \approx 0.02,\ a = 0.5,\ \alpha = 0.02,\ r_\mathrm{b} = 2 \times 10^5,\ q = 0.1$$

Precession frequency $0.0193\,\Omega_\mathrm{b}$

Growth rate $-0.00004\,\Omega_\mathrm{b}$ **(neglects resonant growth)**

GLOBAL ECCENTRIC MODE

Around a black hole :



$$H/R \approx 0.02,\ a = 0.5,\ \alpha = 0.02,\ r_{\rm b} = 2 \times 10^5,\ q = 0.1$$

Precession frequency $0.0195\,\Omega_{\rm b}$

Growth rate $-0.00006\,\Omega_{\rm b}$ **(neglects resonant growth)**

TIMESCALES

SU UMa stars :

- $P_{\rm orb} \lesssim 2\,{\rm hr}$
- superhumps appear from one night to next

Long-period LMXBs :

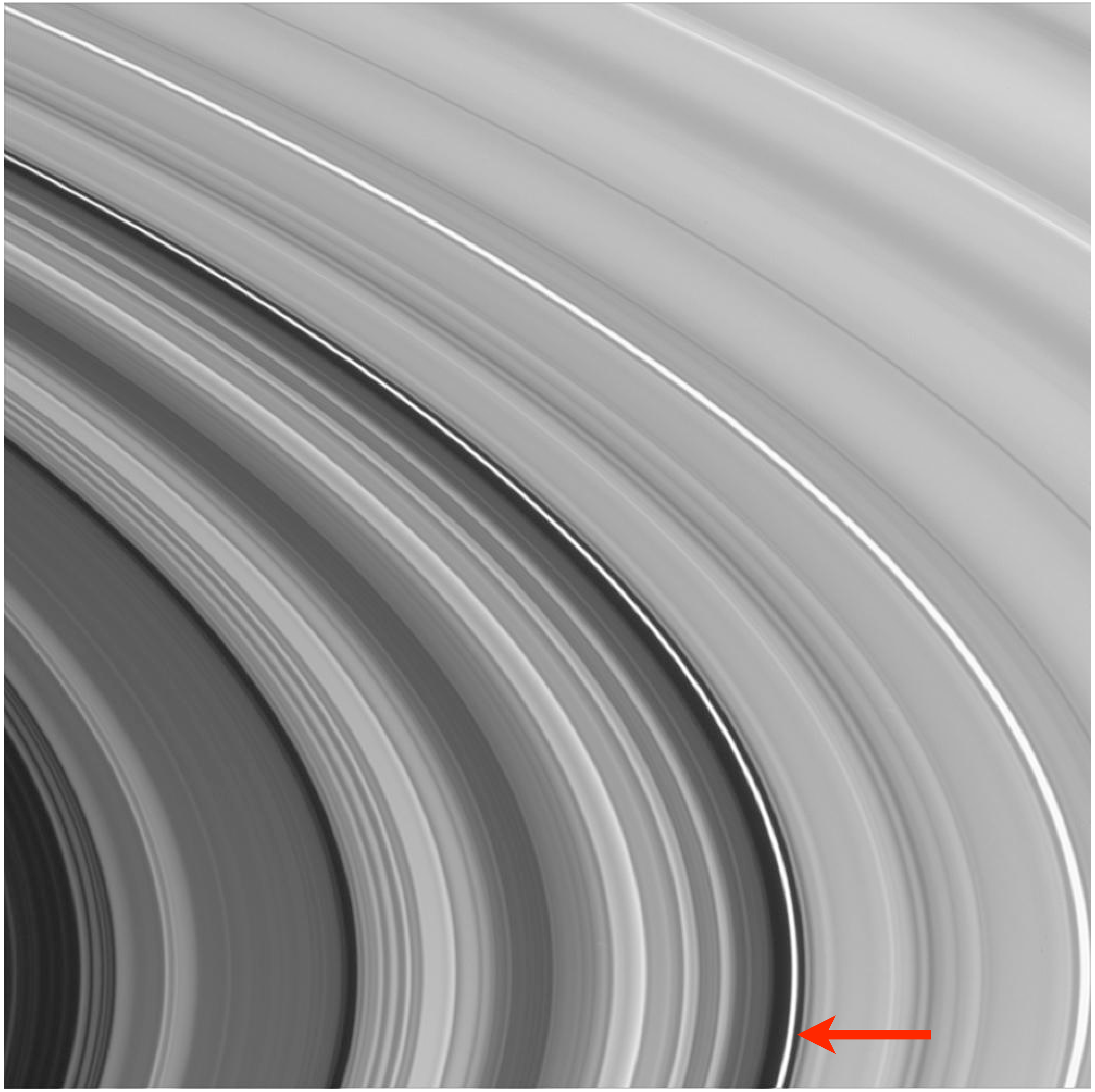
- growth / propagation times may be very long

FORCED ECCENTRICITY

Binary mean motion matches apsidal precession at

$$R \approx 1.6\, R_{\rm b}^{3/5} \left(\frac{GM}{c^2} \right)^{2/5}$$

COLOMBO RINGLET ANALOGY



Colombo ringlet

77,883 km

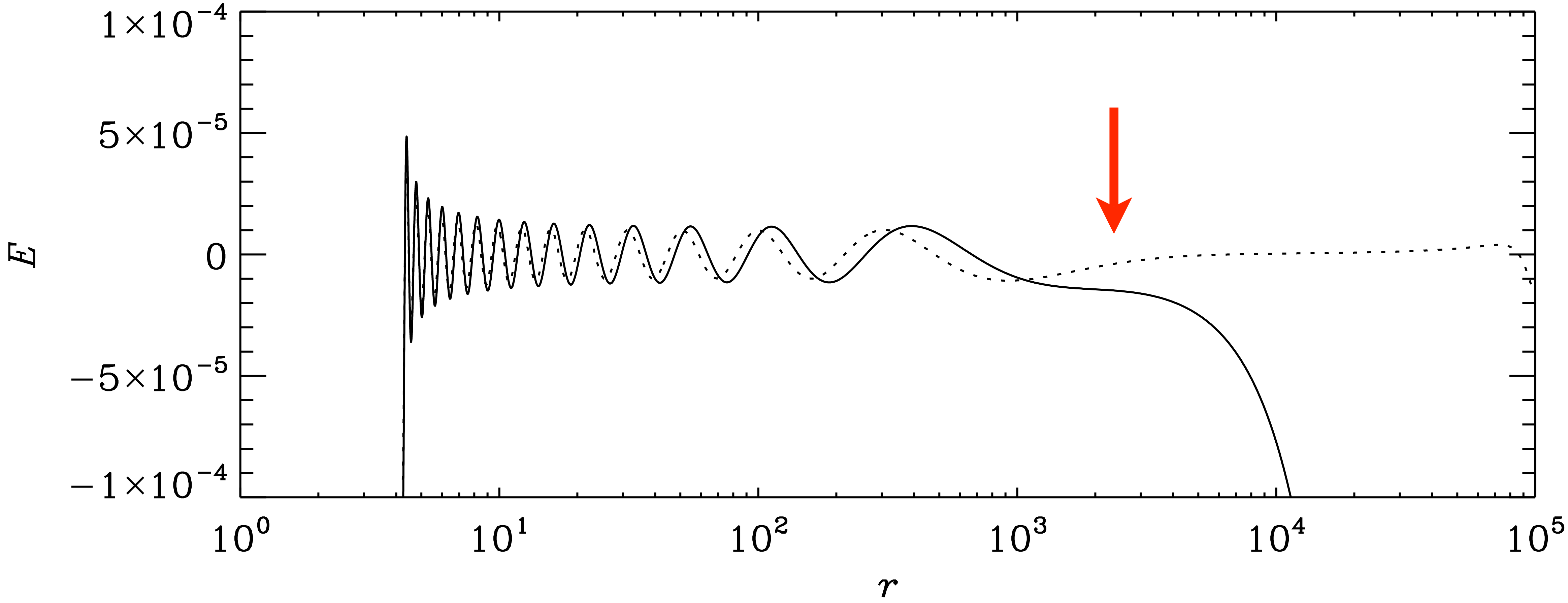
Titan

1,221,870 km

FORCED ECCENTRICITY

Binary mean motion matches apsidal precession at

$$R \approx 1.6\, R_{\mathrm{b}}^{3/5} \left(\frac{GM}{c^2}\right)^{2/5}$$



$$H/R \approx 0.02,\ a = 0.5,\ \alpha = 0.02,\ r_{\mathrm{b}} = 2 \times 10^5,\ q = 0.1$$

CONCLUSIONS

- Global warping and / or eccentricity may be common in X-ray binaries
- Both can propagate into the inner disc as stationary waves of reasonably long wavelength
- Viscous attenuation can be avoided if $\alpha \lesssim H/R$ or with the help of overstability
- Warp or eccentricity can excite other modes through nonlinear coupling in the inner disc