Epicyclic Oscillations of Non-Slender Accretion Tori in Kerr Metric

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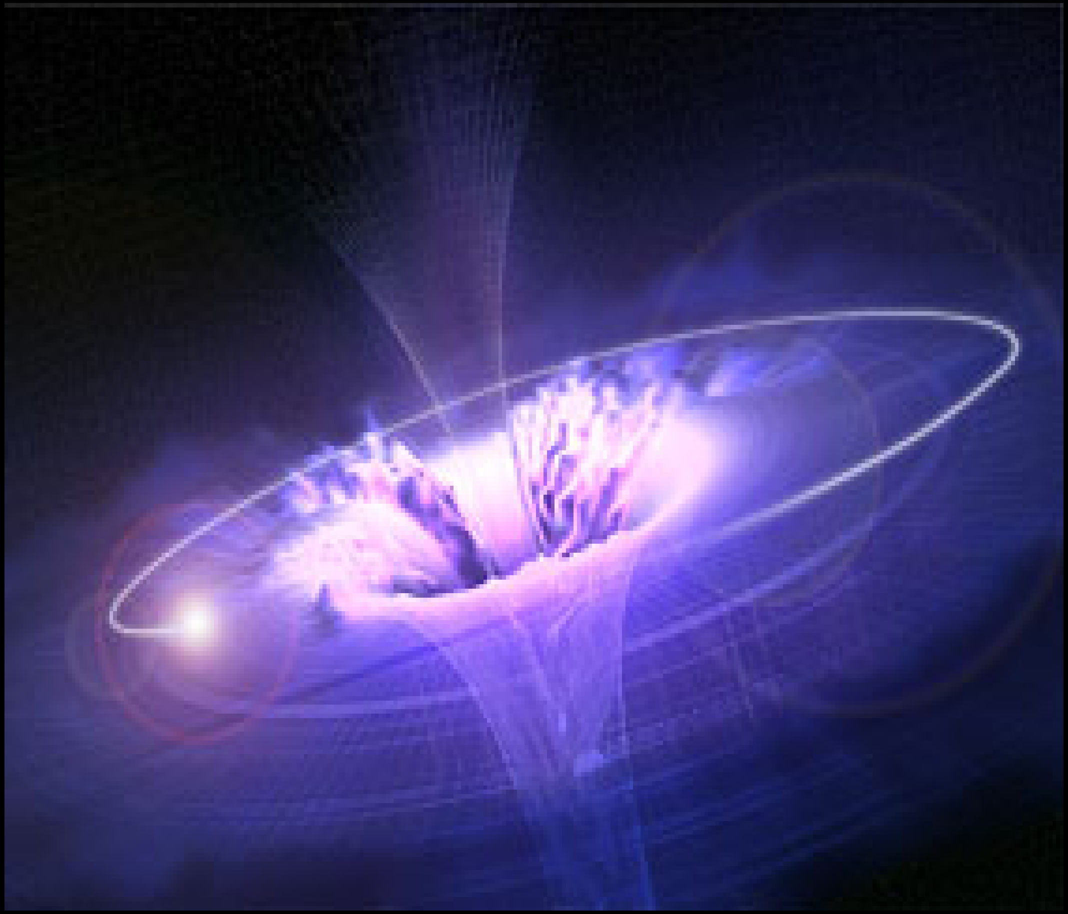

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Accretion Discs

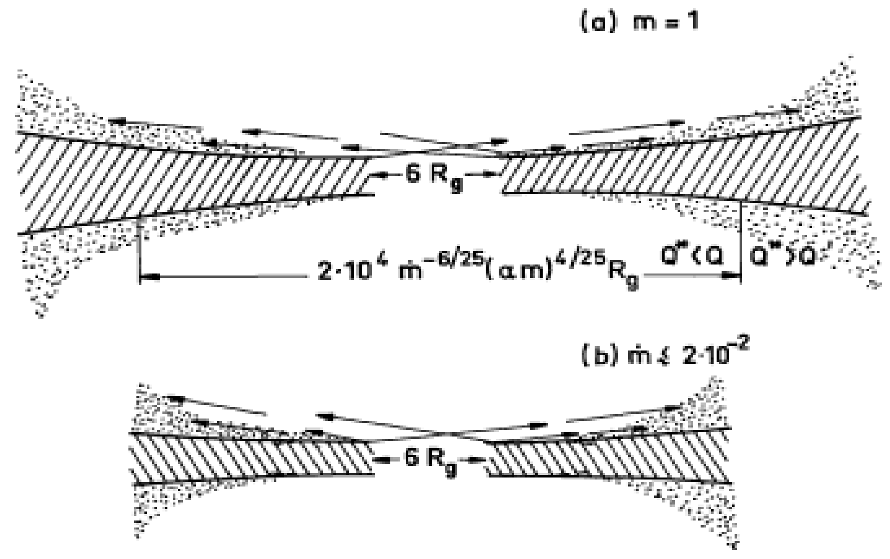
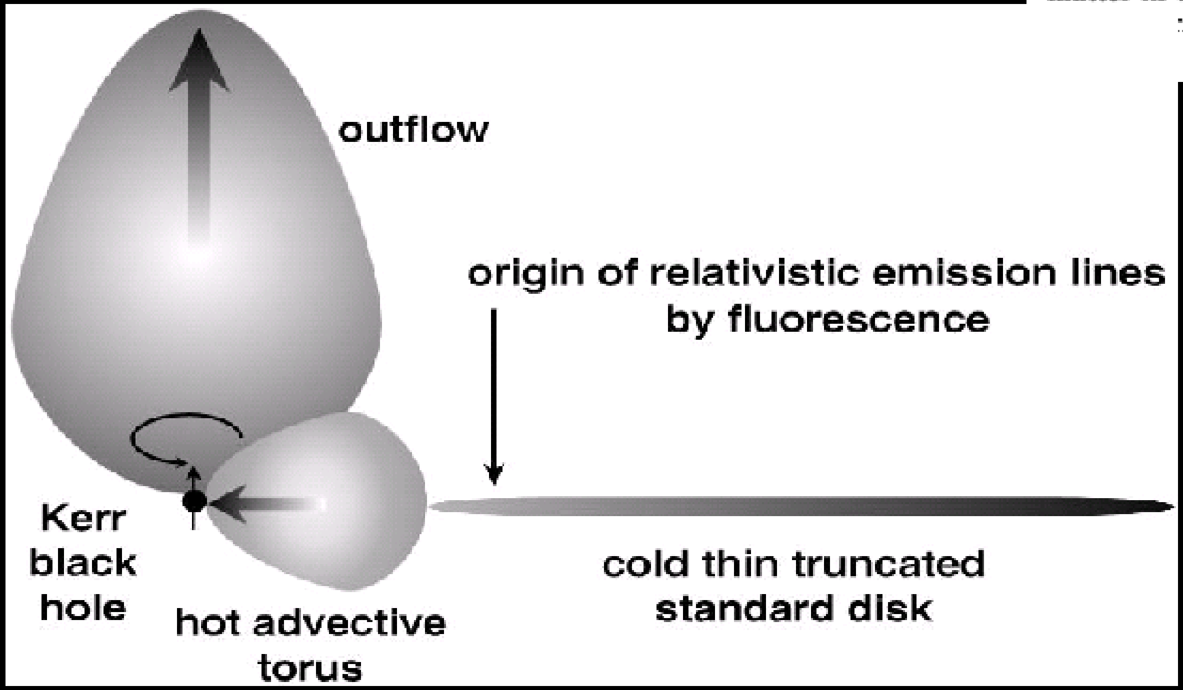
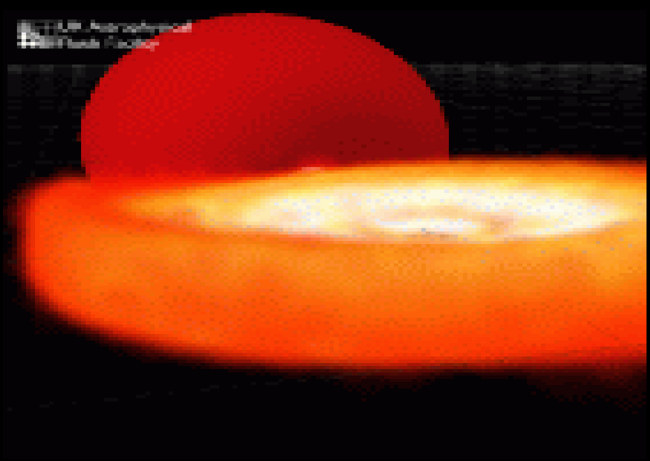


Fig. 4. The thickness of the disk as a function of the distance to the black hole: a) $\dot{M} = \dot{M}_{cr}$. b) $\dot{M} < 10^{-2} \dot{M}_{cr}$. In the central zone, $R < 3R_g$ newtonian mechanics is not applicable. Trajectories of X-ray and ultra-violet quanta which lead to evaporation and heating of the matter in the outer regions of the disk are shown by the arrows. The :med by the hot, evaporated matter is denoted by dots

(Shakura & Sunyaev, 1973)



(Müller & Camenzind, 2003)



Instabilities

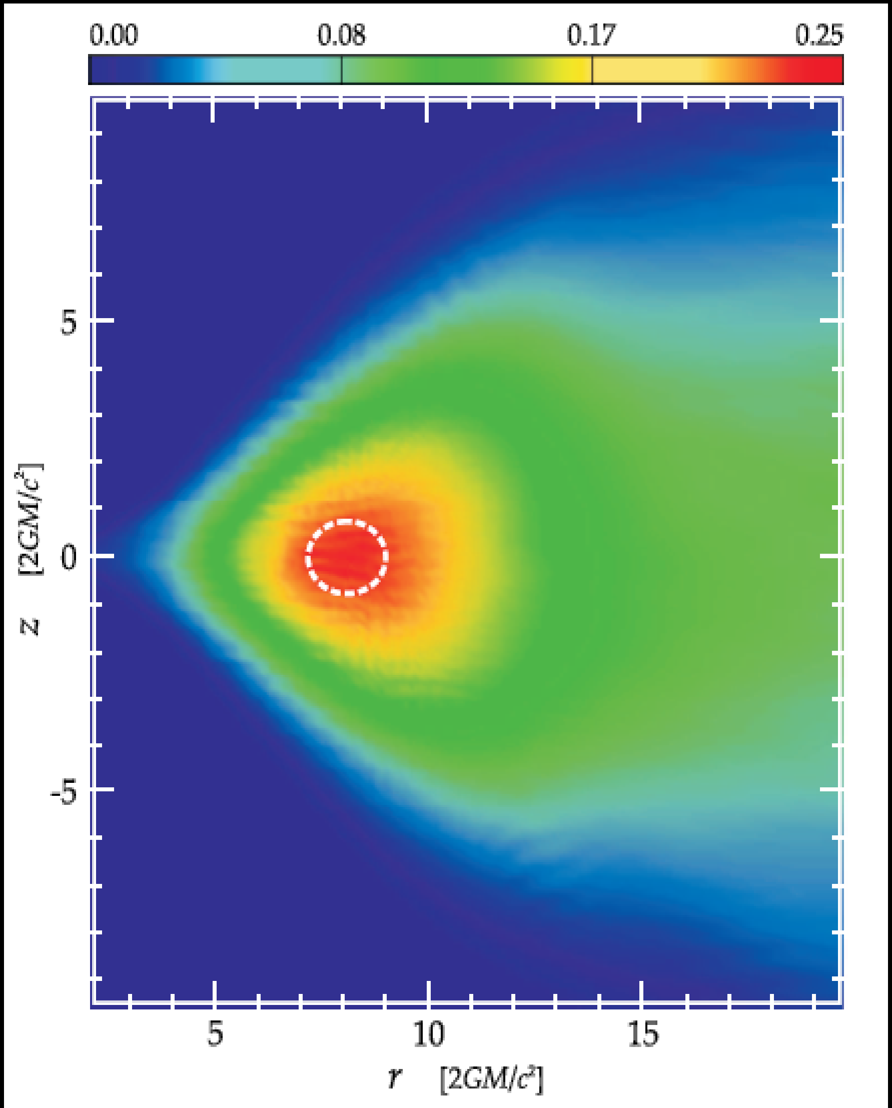
1. Papalouzou & Pringle (PPI), 1984
- constant angular momentum
- unstable under global non-axisym perturbations

2. PPI suppressed by accretion (Blaes, 1987)

3. Magneto-rotational (MRI), (Balbus & Hawley, 1998)
- weak field
- accretion tori are not stable!

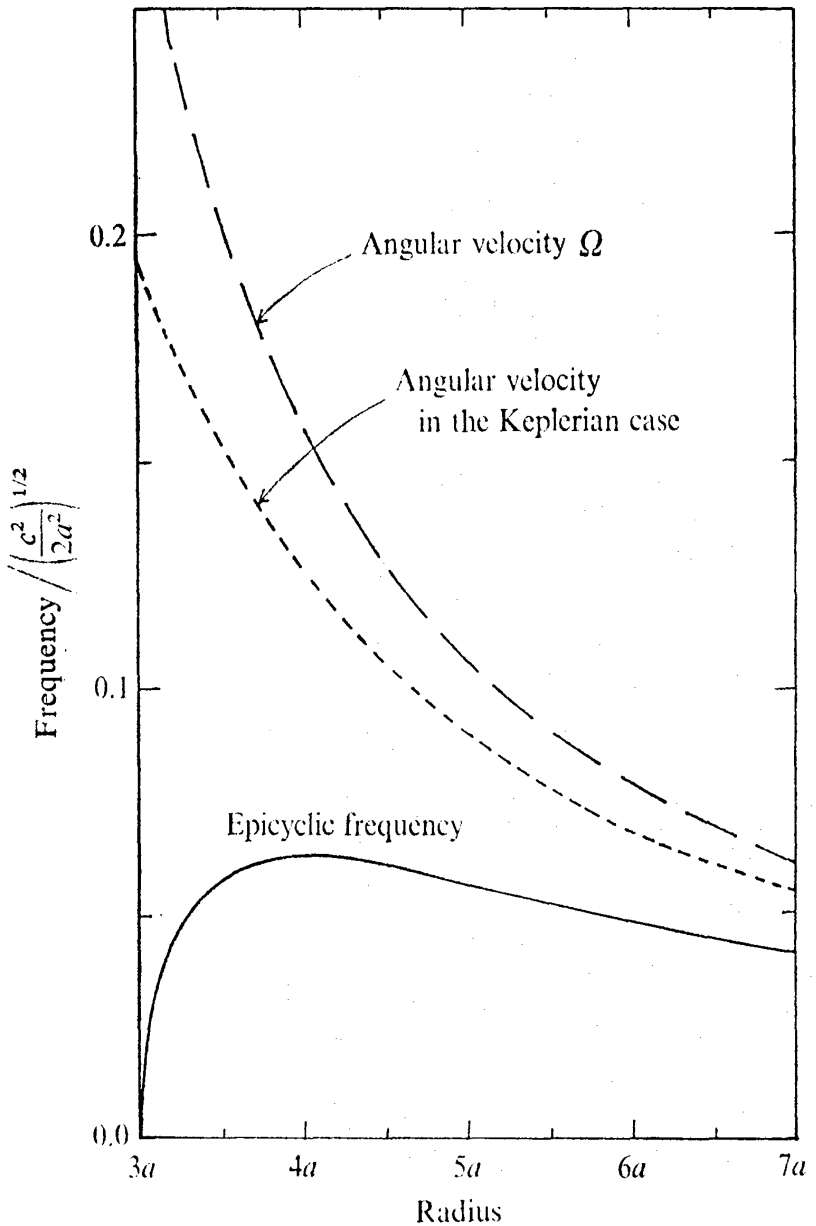
4. Runaway
- Roche surface

5. MHD simulations



(Machida et al., 2006)

Modelling the observed X-ray variability



The radial dependence of the epicyclic frequency κ is shown

p-modes: (Kato & Fukue, 1980)

- accretion disc supports discrete oscillatory modes

- epicyclic frequency decreases to zero as the ISCO is approached

- cavity that traps acoustic modes

g-modes, r-modes:

- disc oscillation modes
- propagating
- at low enough frequencies: standing
- damped

From the free particle orbit...

...to the torus

slender torus

- free particle – geodesic orbit
- perfect barotropic fluid
- non self-gravitating
- constant specific angular momentum
- azimuthal & time dependent perturbation

non-slender torus

- expansion to second order with respect to the torus thickness

$$\beta^2 \equiv \frac{2nc_{s0}^2}{A_0^2\Omega_0^2 r_0^2}$$

Epicyclic Frequencies

The effective potential

$$U = (-g^{tt} + 2l_0 g^{t\phi} - l_0^2 g^{\phi\phi})^{-1/2}$$

that has its minimum at the torus pressure maximum r_0.
A small perturbation of a test particle orbiting on a geodesic line $r = r_0$ with $l = l_0$ results in the radial and vertical epicyclic oscillation.

$$\omega_{r0}^2 = \frac{1}{2}\left(\frac{\mathcal{E}^2}{A^2 g_{rr}}\frac{\partial^2 U}{\partial r^2}\right)_0 \quad \text{and} \quad \omega_{\theta 0}^2 = \frac{1}{2}\left(\frac{\mathcal{E}^2}{A^2 g_{\theta\theta}}\frac{\partial^2 U}{\partial \theta^2}\right)_0$$

$$\omega_{r0}^2 = \Omega_0^2\left(1 - \frac{6}{r_0} + \frac{8a}{r_0^{3/2}} - \frac{3a^2}{r_0^2}\right) \quad \text{and}$$

$$\omega_{\theta 0}^2 = \Omega_0^2\left(1 - \frac{4a}{r_0^{3/2}} + \frac{3a^2}{r_0^2}\right)$$

Papaloizou-Pringle Equation

The general relativistic form of the Papaloizou-Pringle equation in terms of W,

$$\frac{1}{(-g)^{1/2}}\left\{\partial_\mu\left[(-g)^{1/2}g^{\mu\nu}f^n\partial_\nu W\right]\right\}-\left(m^2g^{\phi\phi}-2m\omega g^{t\phi}+\omega^2g^{tt}\right)f^nW$$
$$=-\frac{2n\bar{A}(m\bar{\Omega}-\bar{\omega})^2}{\beta^2r_0^2}f^{n-1}W$$

with

$$W=-\frac{\delta p}{A\rho(\omega-m\Omega)}$$

Here $\{\mu,\nu\}\in\{r,\theta\}$, $\bar{A}\equiv A^2/A_0^2$, $\bar{\Omega}\equiv\Omega/\Omega_0$, $\bar{\omega}\equiv\omega/\Omega_0$ and g denotes the determinant of the metric.

We rewrite the equation a eigenvalue problem

$$\hat{L}W=-2n\bar{A}\left(m\bar{\Omega}-\bar{\omega}\right)^2W$$

Expansion

We expand all relevant variables $\bar{\omega}$, $\bar{\Omega}$, $\bar{A}$, f, $\hat{L}$, W into a power series in β by writing

$$Q = Q^{(0)} + Q^{(1)}\beta + Q^{(2)}\beta^2 + \ldots \qquad \text{where} \qquad Q \in \{\bar{\omega}, \bar{\Omega}, \bar{A}, f, \hat{L}, W\}$$

Expansion to Second Order

The perturbation equation expanded to second-order in β reads

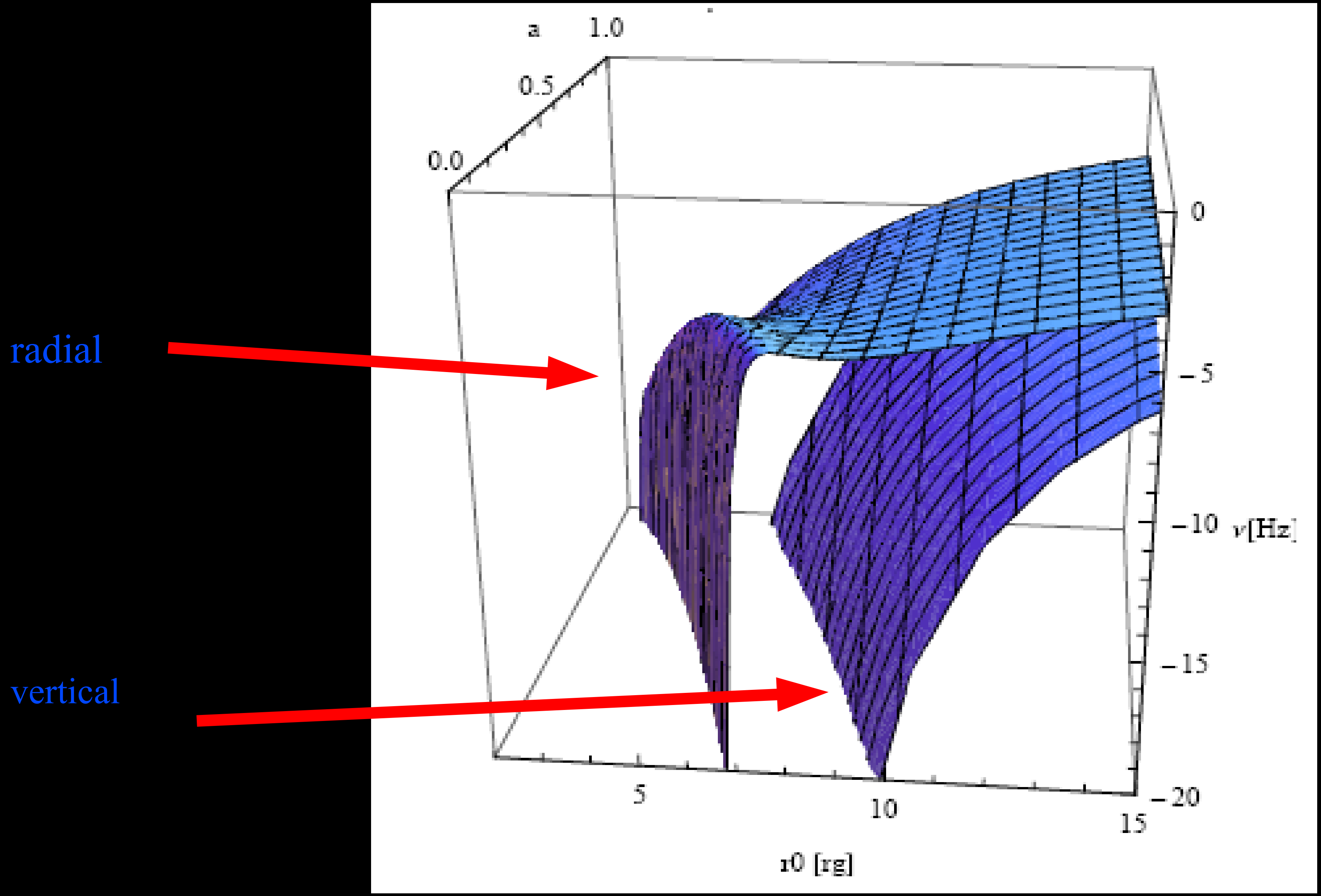
$$\begin{aligned}
\hat{L}^{(1)} W_j^{(1)} + \hat{L}^{(2)} W_j^{(0)} &= 2n \Big\{ \Big(2m\bar{\sigma}^{(0)2} \bar{\Omega}^{(1)} - \bar{\sigma}_j^{(0)2} \bar{A}^{(1)} \Big) W_j^{(1)} \\
&\quad + \Big[2m\bar{\sigma}_j^{(0)2} \bar{A}^{(1)} \bar{\Omega}^{(1)} - \bar{\sigma}_j^{(0)2} \bar{A}^{(2)} \\
&\quad + \Big(2m\bar{\sigma}_j^{(0)2} \bar{\Omega}^{(2)} - m^2 \bar{\Omega}^{(1)2} \Big) \\
&\quad - 2\bar{\sigma}_j^{(0)} \bar{\omega}_j^{(2)} \Big] W_j^{(0)} \Big\}
\end{aligned}$$

Second Order Correction

Analogous to the first-order case we obtain the formula for the second-order correction,

$$
\begin{aligned}
\bar{\omega}_i^{(2)} &= -\frac{1}{4n\bar{\sigma}_i^{(0)}}\Big[\Big\langle W_i^{(0)}\big|\hat{L}^{(2)} + 2nm^2\bar{\Omega}^{(1)2} - 4nm\bar{\sigma}_i^{(0)}\bar{\Omega}^{(2)} \\
&\quad +2n\bar{\sigma}_i^{(0)2}\bar{A}^{(2)} - 4nm\bar{\sigma}_i^{(0)}\bar{A}^{(1)}\bar{\Omega}^{(1)}\big|W_i^{(0)}\Big\rangle \\
&\quad +\sum_{i\neq j} b_{ij}\left\langle W_i^{(0)}\big|\hat{L}^{(1)} + (2n\bar{\sigma}_i^{(0)2}\bar{A}^{(1)} - 4nm\bar{\sigma}_i^{(0)2}\bar{\Omega}^{(1)})\big|W_j^{(0)}\right\rangle\Big]
\end{aligned}
$$

Radial & Vertical Correction



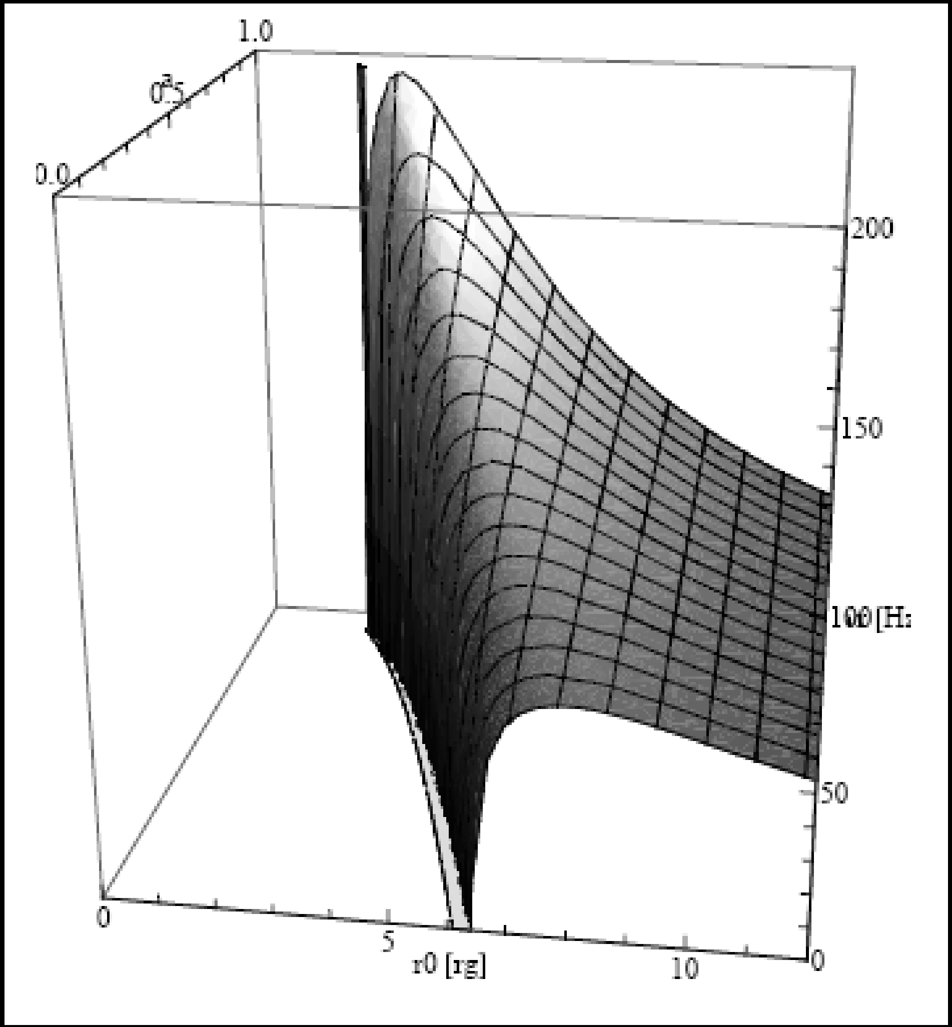
Epicyclic frequencies of non-slender tori

$$\bar{\omega}_r = \pm\bar{\omega}_r^{(0)} + m \mp \frac{\beta^2}{4n\bar{\sigma}_1^{(0)}}\left(P_1 + P_2 + P_3 + P_4\right) + \mathcal{O}(\beta^3)$$

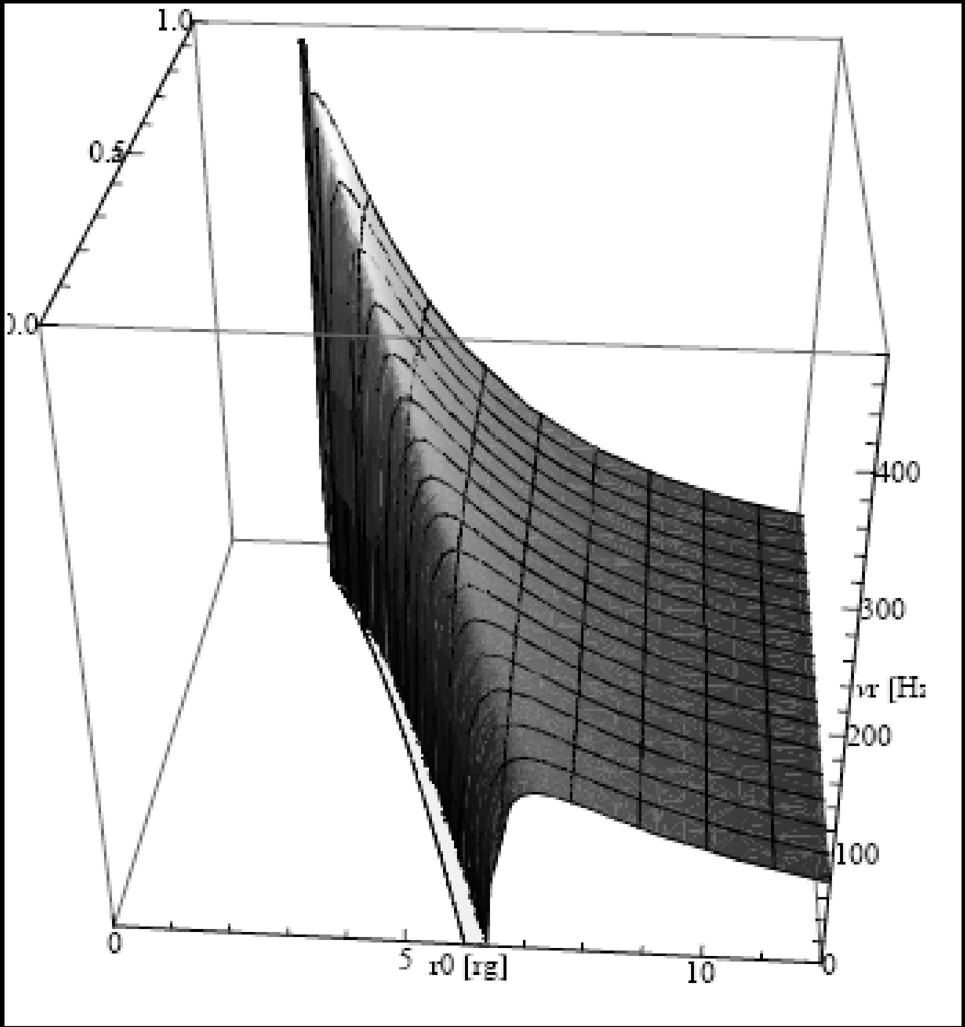
$$\bar{\omega}_\theta = \pm\bar{\omega}_\theta^{(0)} + m \mp \frac{\beta^2}{4n\bar{\sigma}_2^{(0)}}\left(P_5 + P_6\right) + \mathcal{O}(\beta^3)$$

Frequency profiles

$\beta = 0.1$



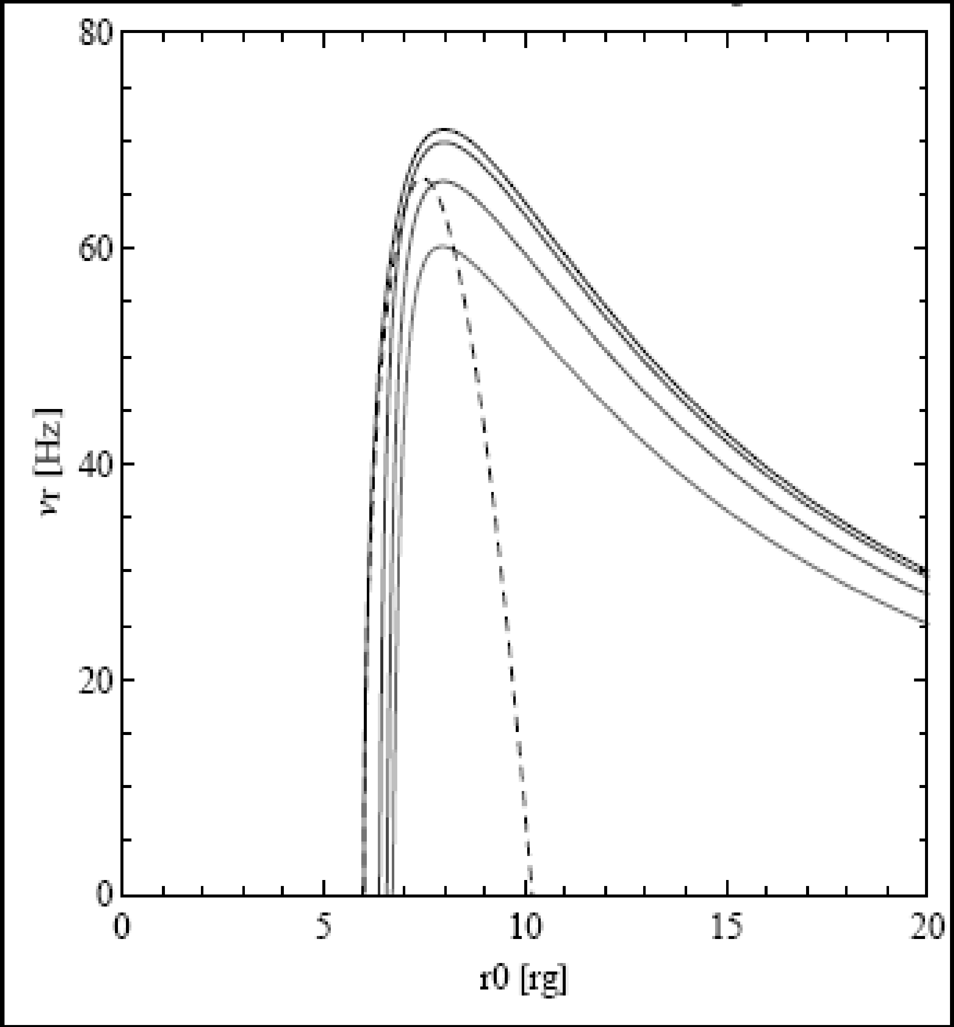
radial



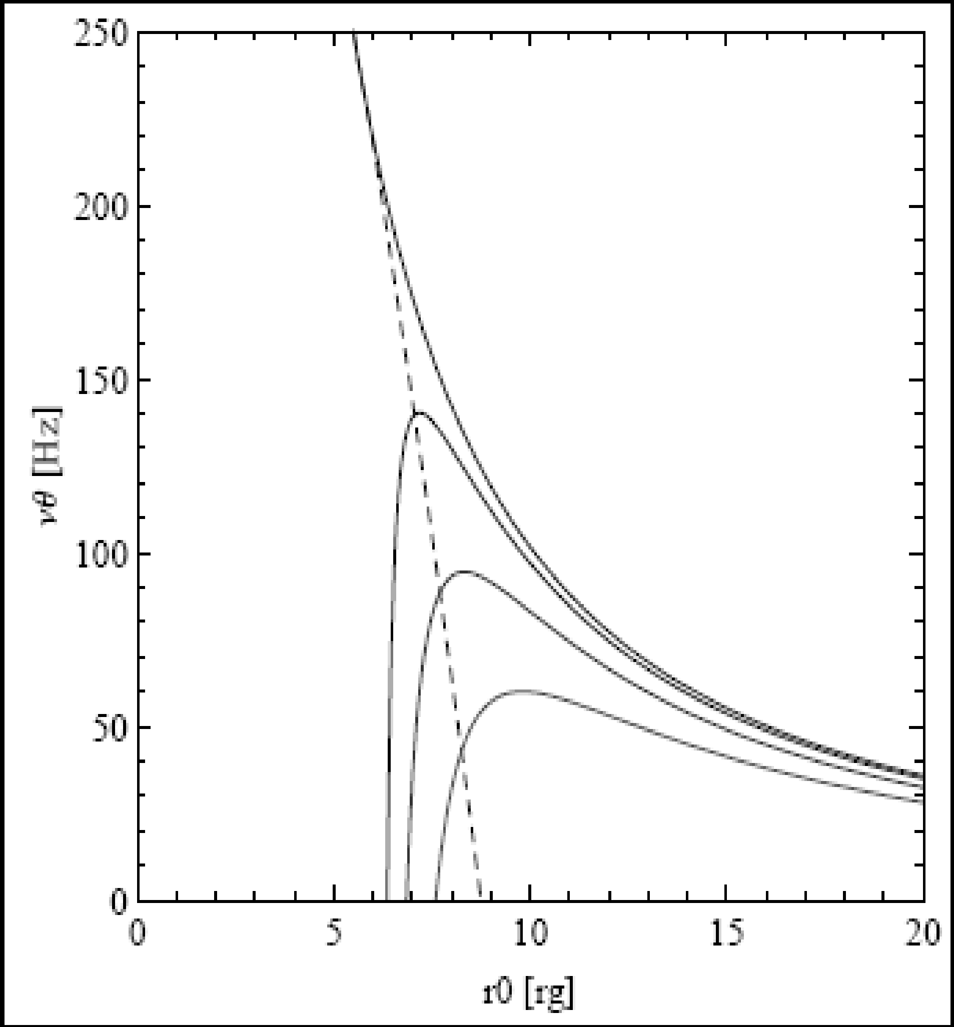
vertical

Schwarzschild black hole

a = 0



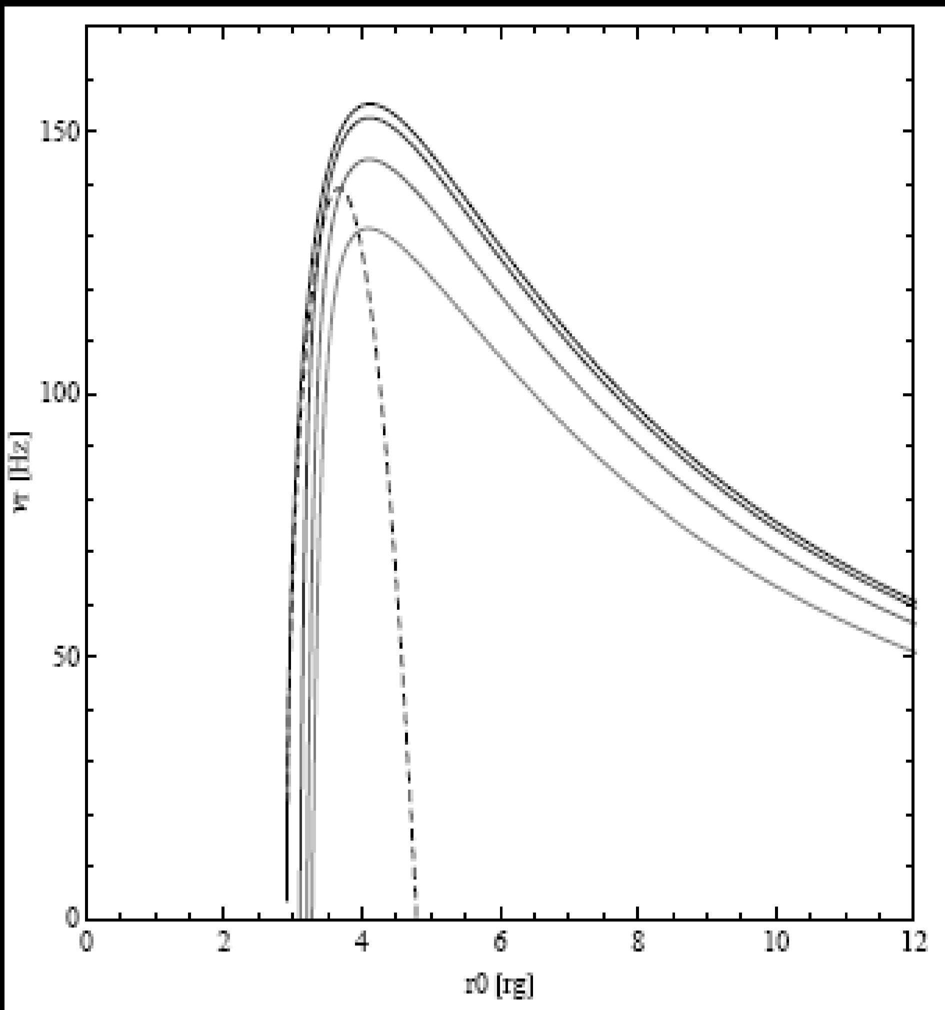
radial



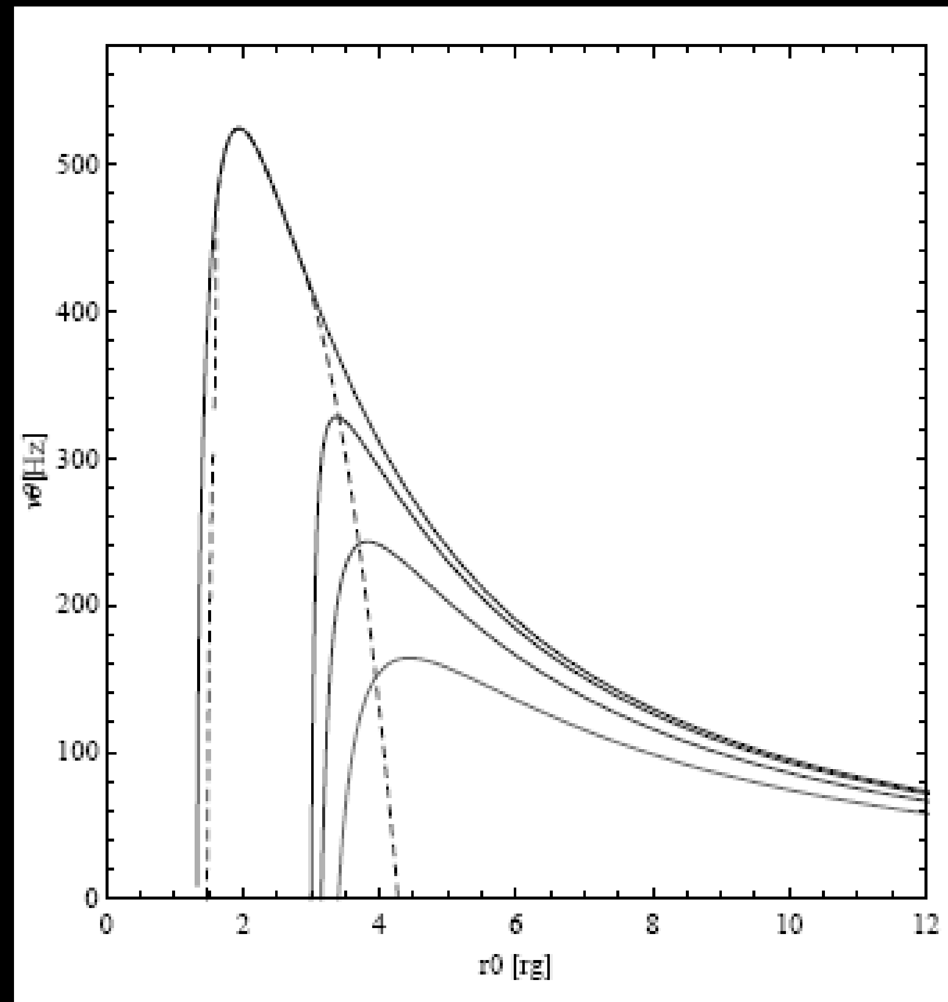
vertical

Kerr black hole

a = 0.8

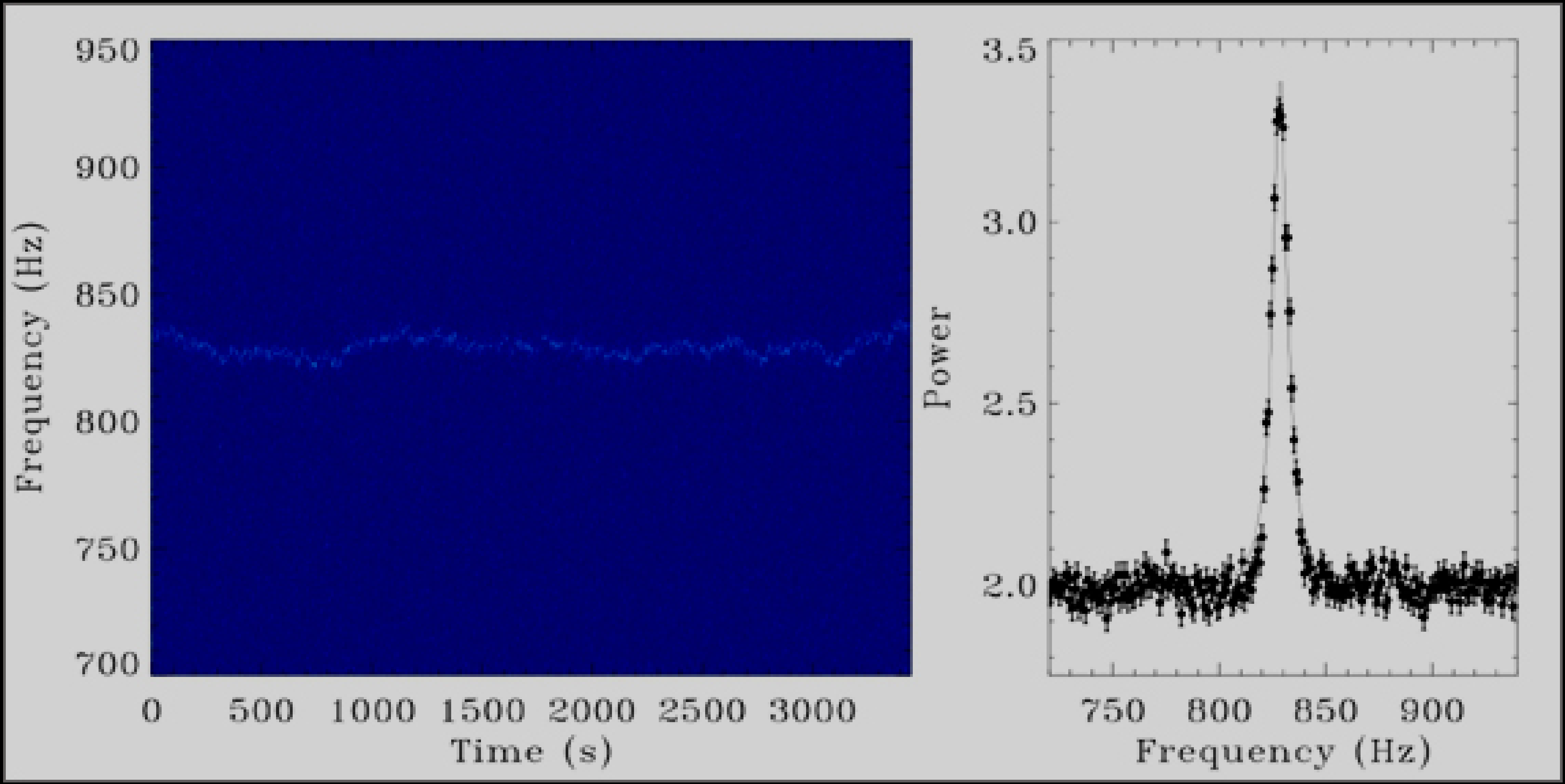


radial



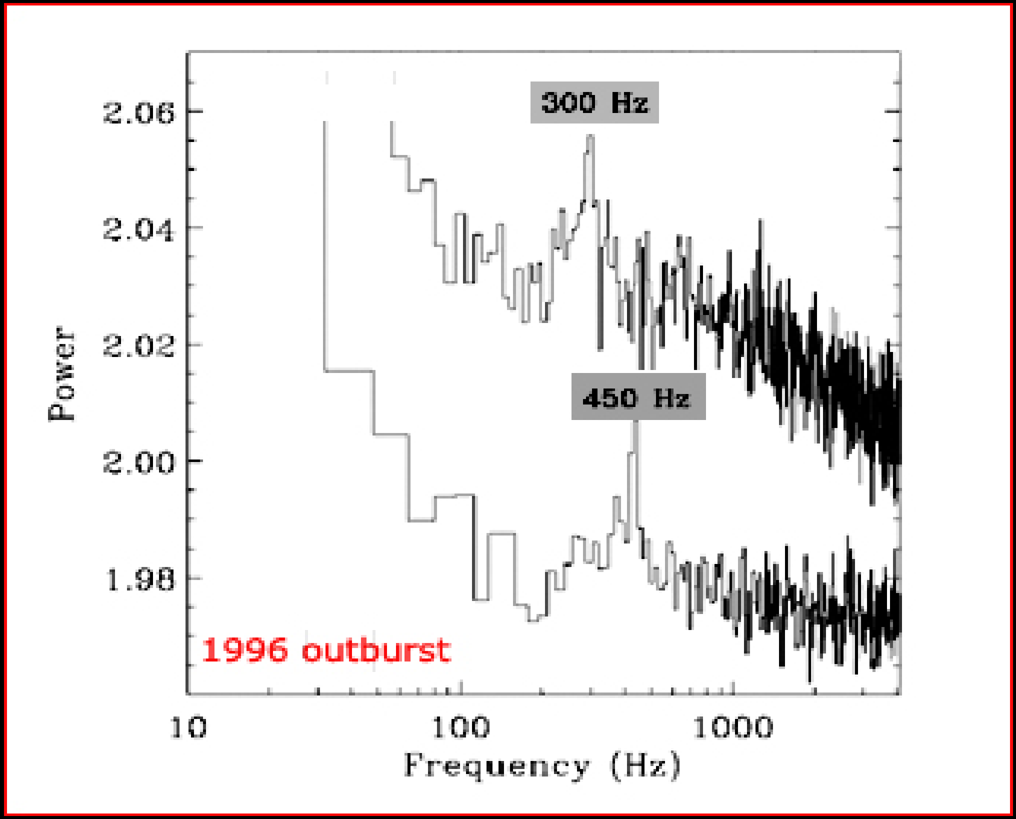
vertical

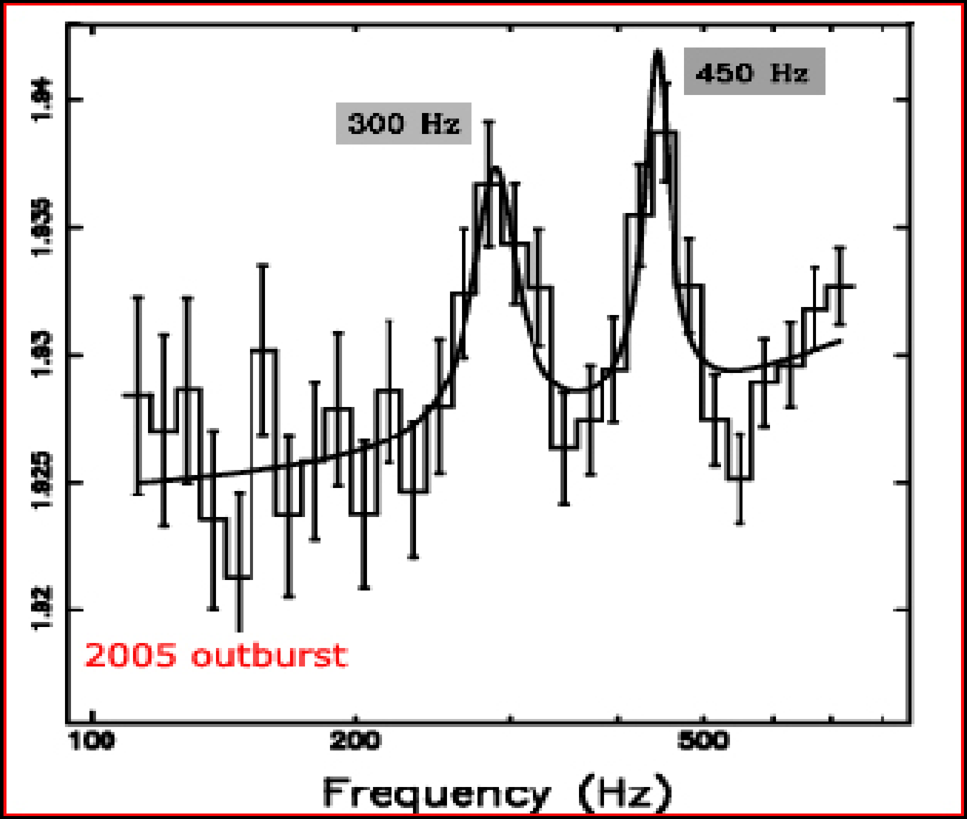
Observations



1. narrow peaks => real oscillations of accretion disc

Observations

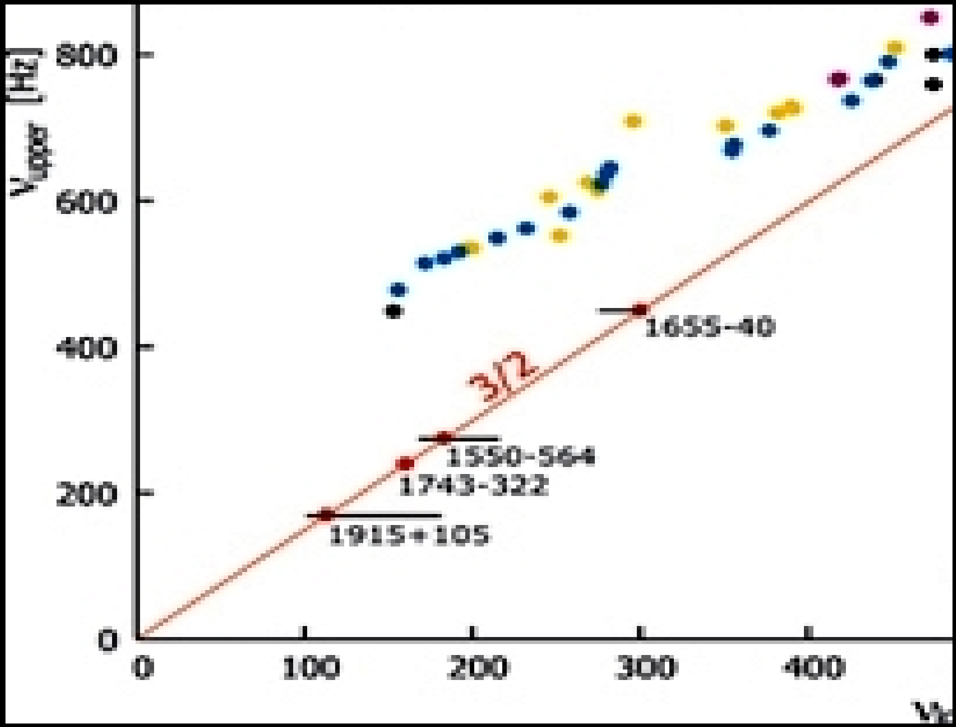




2. twin peaks: the same pair of high frequency QPOs reappears after years

=> QPOs are memorised i.e., determined by the BH

Observations



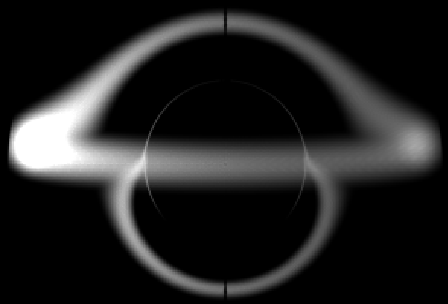
3. the two characteristic QPO frequencies of BHs are dependent on the amplitude and show a 3 : 2 ratio

Model

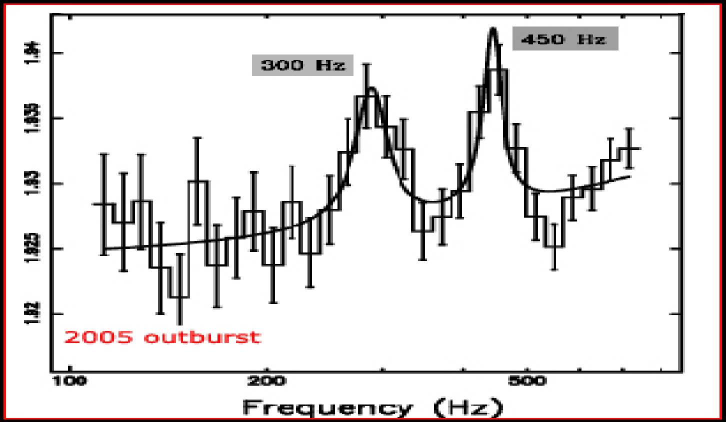
Resonance Model (Abramowicz & Kluzniak, 2001)

QPO twin peaks may be caused by a
non-linear resonance
between two (epicyclic) modes of oscillation
in the accretion flow

The 2 oscillation modes of a thin fluid torus in
resonance are composed of radial & vertical
eigenfrequencies



HF twin peak QPOs

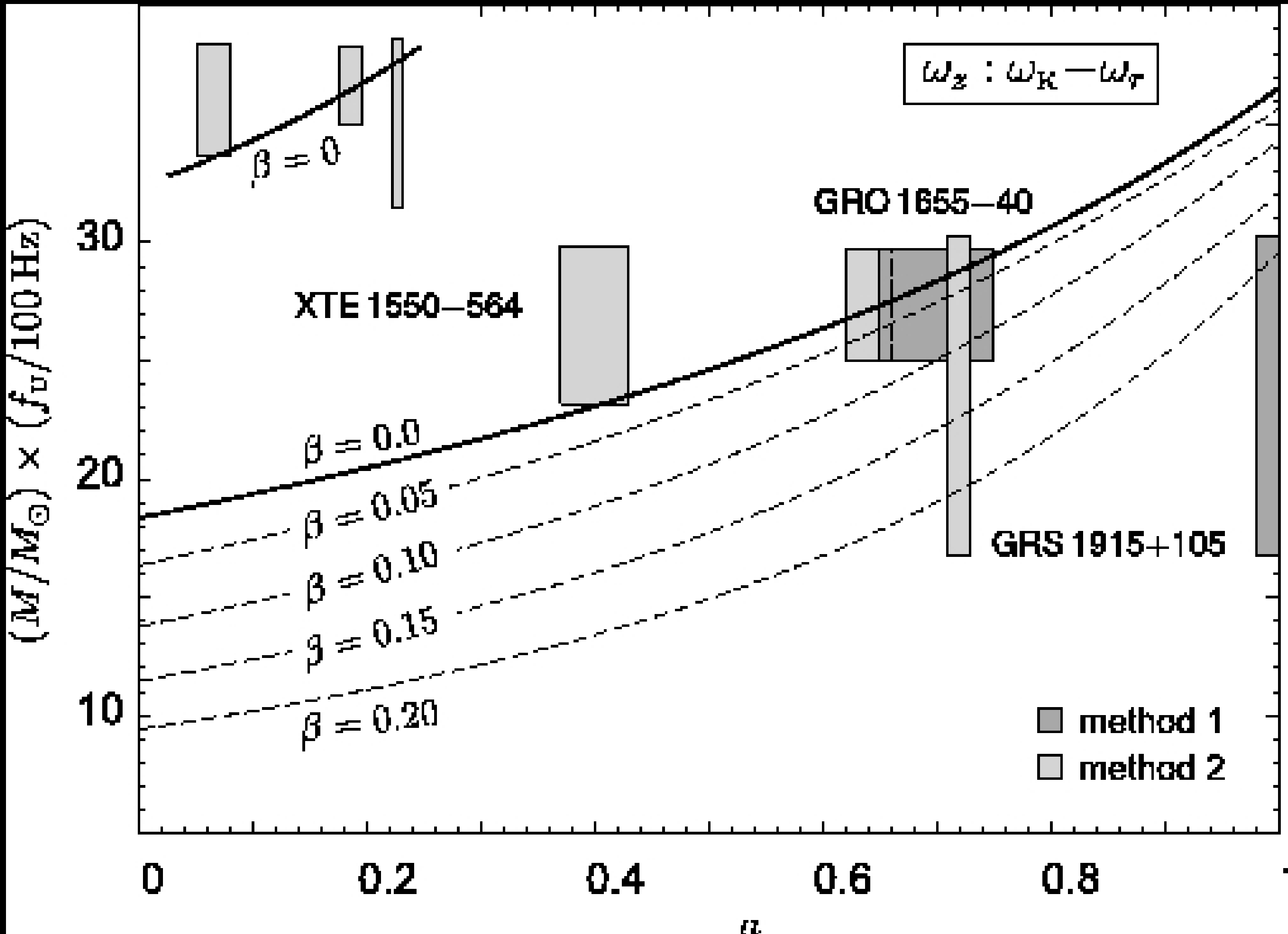


$$\nu_{upper} / \nu_{lower} = 3/2$$

$$\nu_U = \frac{\omega_\theta^0}{2\pi}, \quad \nu_L = \frac{\Omega_K - \omega_r^0}{2\pi}, \quad \frac{\nu_U}{\nu_L} = \frac{3}{2}.$$

$$\nu_U = \left(\frac{M}{M_\odot}\right)^{-1} \left[\frac{c^3\sqrt{x^2 - 4ax^{1/2} + 3a^2}}{2\pi G x (x^{3/2} + a)}\right]$$

Discussion

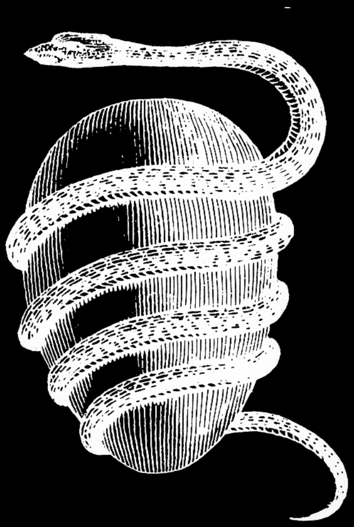


$\omega_z : \omega_K - \omega_r$
$\beta = 0$
GRO 1655−40
XTE 1550−564
$\beta = 0.0$
$\beta = 0.05$
$\beta = 0.10$
$\beta = 0.15$
$\beta = 0.20$
GRS 1915+105
method 1
method 2
$(M/M_\odot) \times (f_U/100\,\mathrm{Hz})$
10
20
30
0
0.2
0.4
0.6
0.8
1
a

Conclusions

for non-slender tori in Kerr metric:

- epicyclic frequencies are lowered
- resonance model:
  resonance radii and BH spins may occur at lower or higher positions (depending on the considered combination of modes!)



end

Funäsdalen, 27.03.2008