Off-axis response of Compton and photoelectric polarimeters with a large field of view

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X-ray polarisation in astrophysics
-a window about to open?

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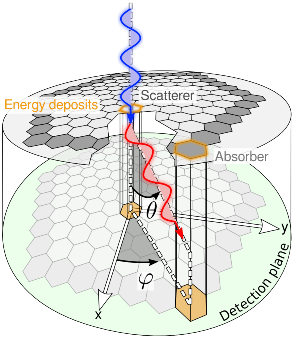
COST European Cooperation in Science and Technology
Polarisation as a tool to study the Solar System and beyond
Action MP1104

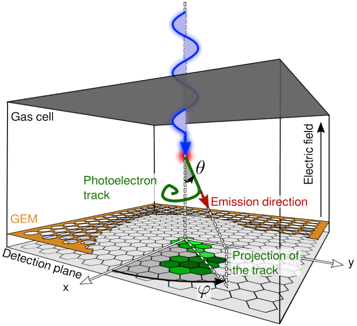
Outline

- An analytical approach to calculate the modulation curve
- Difference between on-axis and off-axis photons
- A selection of the results
- Comparison with measurements
- Conclusions

How we build the modulation curve

- The modulation curve $\mathcal{M}$ is just the number of counts produced in the direction φ





- Common instrumental designs are not sensitive to the angle ϑ
  - ➥ For example, Compton telescopes are an exception

How can this be expressed mathematically?

- The number of events produced in a certain direction is the differential cross section of the process:

$$\text{Photoelectric (K-shell)}: \frac{d\sigma_{Ph}}{d\Omega} = r_0^2 \alpha^4 Z^5 \left(\frac{m_e c^2}{E}\right)^{\frac{7}{2}} \frac{4\sqrt{2}\sin^2\theta\,\cos^2\phi}{(1-\beta\cos\theta)^4}$$

$$\text{Compton (Klein-Nishina)}: \frac{d\sigma_{Cm}}{d\Omega} = \frac{1}{2} r_0^2 \frac{E'^2}{E^2}\left[\frac{E}{E'} + \frac{E'}{E} - 2\sin^2\theta\,\cos^2\phi\right]$$

- Usually, the instruments are not sensitive to θ and then we have to integrate over this angle:

$$\mathcal{M}(\varphi) \propto \int_{\text{Min}}^{\text{Max}} \frac{d\sigma}{d\Omega} d\theta$$

- We have to sum the number of events in the meridian slide φ

For completely polarized photons and a photoelectric polarimeter:

$$\mathcal{M}(\varphi) \propto \int_{\text{Min}}^{\text{Max}} \frac{\sin^2\theta\ \cos^2\phi}{(1-\beta\cos\theta)^4} d\theta$$

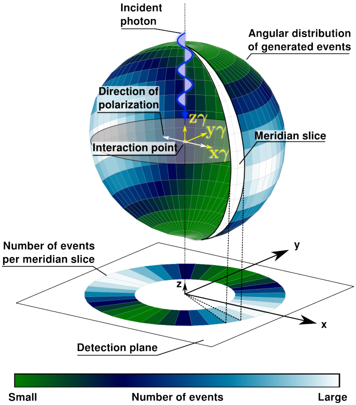
- On axis, this integral is trivial:

$$\mathcal{M}(\varphi) \propto \cos^2\phi \underbrace{\left[\int_{\text{Min}}^{\text{Max}} \frac{\sin^2\theta}{(1-\beta\cos\theta)^4} d\theta\right]}_{\text{constant}=A}$$

In the following, Min=0 and Max=π.

- Taking into account the error in the measurements of the event direction:

$$\mathcal{M}(\varphi) = A\cos^2(\varphi-\varphi_0) + B$$

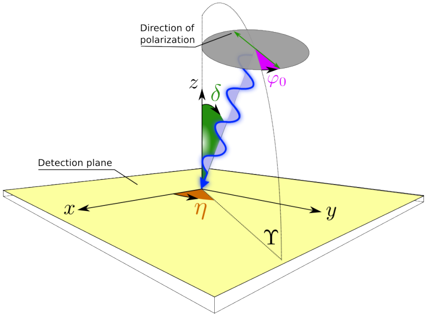


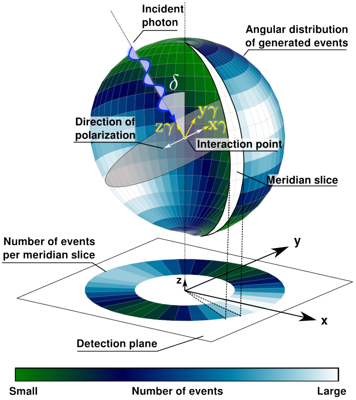
[Photoelectric, 100% polarized]

- The azimuthal and meridian directions are not decoupled as seen by in instrument
- The angular dependency of the events must be transformed in the instrument frame of reference

I used three rotations, representing

δ: inclination
η: azimuthal direction of the photons
φ_0: polarization angle





[Photoelectric, 100% polarized]

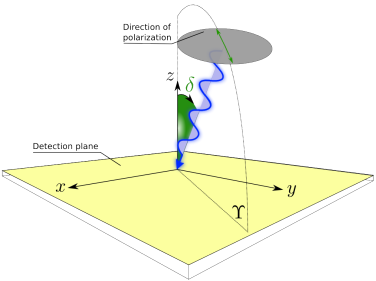
- There is no conceptual difference between photoelectric and Compton polarimeters
- Some algebra is required to arrive at the modulation curve
- In a simple configuration ($\eta = 0$ and $\varphi_0{=}0$), with a 1st order approximation in energy, for photoelectric polarimeters:

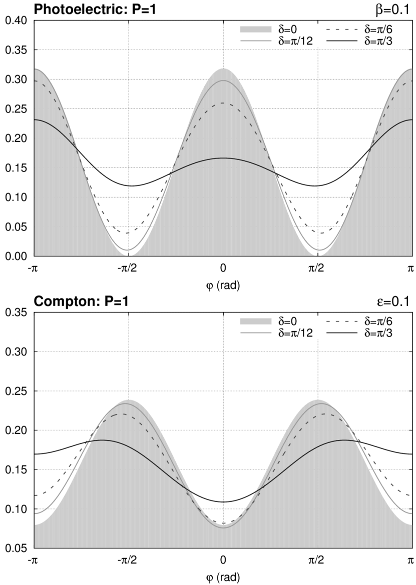
$$\begin{aligned}\mathcal{M}_{\rm Ph} = f\mathcal{N}_{\rm tot}\Bigg\{&\mathcal{P}\left[-\frac{9\beta\cos^2\delta\sin\delta}{8}\cos^3\varphi + \frac{\cos^2\delta}{\pi}\cos^2\varphi + \frac{3\beta\left(3\cos^2\delta - 1\right)\sin\delta}{8}\cos\varphi + \frac{\sin^2\delta}{2\pi}\right] + && \propto\text{Pol. component}\\ &+(1-\mathcal{P})\left[\frac{9\beta\sin^3\delta}{16}\cos^3\varphi - \frac{\sin^2\delta}{2\pi}\cos^2\varphi + \frac{3\beta(3\cos^2\delta - 4)\sin\delta}{16}\cos\varphi + \frac{3-\cos^2\delta}{4\pi}\right]\Bigg\} + && \propto\text{Unp. component}\\ &+\mathcal{N}_{\rm tot}\frac{1-f}{2\pi} && \propto\text{Instr. sensitivity}\end{aligned}$$

f: f-factor, conceptually similar to the modulation factor
►►► In the following we will assume an ideal instrument with $f = 1$

$\mathcal{P}$: polarization degree
$\mathcal{N}_{\rm tot}$: number of collected events
β: velocity of the photoelectron in c units, basically the energy of the photon

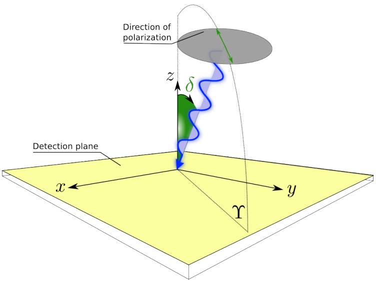
Increasing the inclination angle δ for completely polarized photons

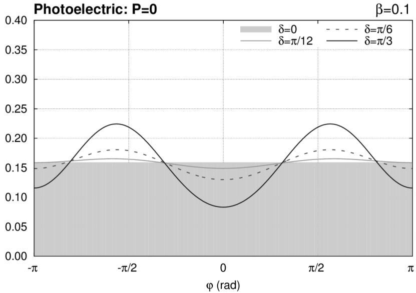


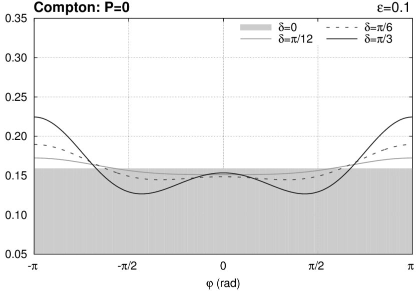


- Deviation from the cosine square behavior
  - ➥ Asymmetry of the two peaks
  - ➥ The modulation curve must be resolved over 360°
- Decreasing amplitude of the modulation with δ
- Increasing effects with the energy

Increasing the inclination angle δ for completely unpolarized photons

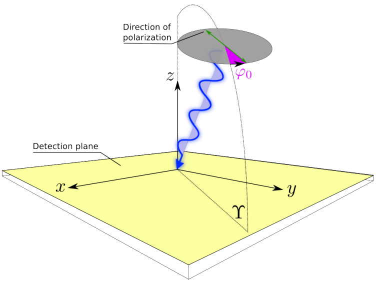




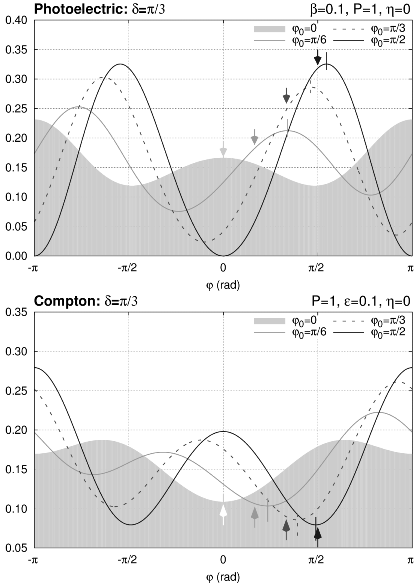


- A modulation is present also for off-axis unpolarized photons
  - ➥ In the low energy limit, this is exactly a cosine square
  - ➥ The amplitude increases with the inclination

Changing the angle of polarization φ_0



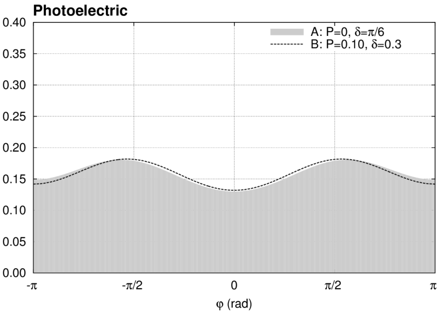
- The shape of the modulation curve depends on the angle of polarization
- The peak of the modulation does not correspond exactly to the polarization angle, even for 100% polarized photons



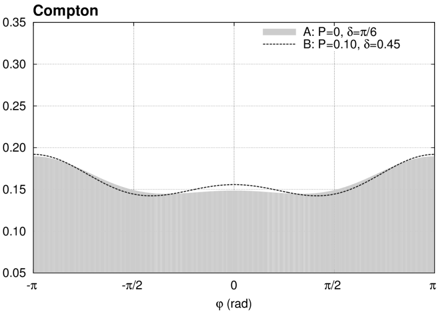
Modulation curve degeneracy

Modulation curve degeneracy

Comparison of 10% polarized and unpolarized photons with different inclination:



$\Delta\delta = 13^\circ$



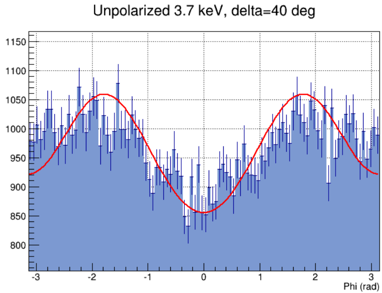
$\Delta\delta = 4^\circ$

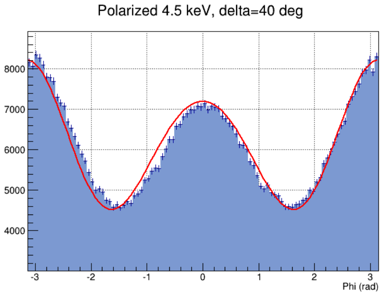
A similar result holds true if we consider different energies.

▶▶▶ The incident direction and energy of the photons have to be known to some degree

Comparison with real measurements

Photoelectric polarimeter (Gas Pixel Detector):





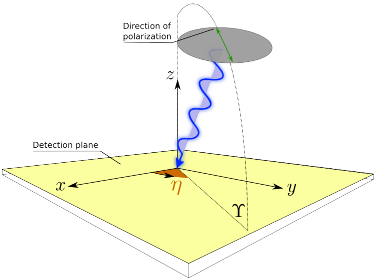
Red curves are not fit to data, but theory predictions without any free parameter.

- The modulation curve depends not only on the polarization, but also on the incident direction and energy in a complex but predictable way
  - ➥ The source position and the photon energy have to be known
- This is true for both usual “2-dimensional” photoelectric and Compton polarimeters
  - ➥ The only way of avoiding this is 3-dimensional polarimeters, like Compton telescopes, with a complete reconstruction of the event direction.

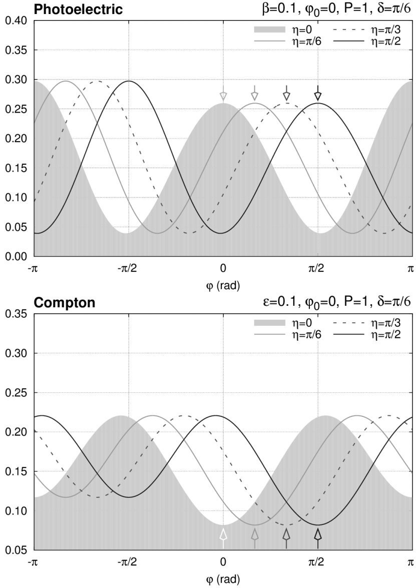
[see Muleri 2014, The Astrophysical Journal, 782:28 for more details]

Backup slides

Changing the azimuthal direction of the photons η



- The modulation curve just shift of η
  - ➥ This is true for the modulation produced by both polarized and unpolarized photons



- Both the shape and the phase of the modulation changes
  - ➥ Evolution from the behavior for unpolarized to polarized photons
  - ➥ The phase of the modulation is not correlated at all with the phase of the modulation