Orbital evolution of the accreting millisecond pulsar SAX J1808.4–3658

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Abstract. We report here on the orbital evolution of the accreting millisecond pulsar SAX J1808.4–3658. In particular, we find for this source the first estimate of the orbital period derivative in an accreting millisecond pulsar, \(\dot{P}_{\text{orb}} = (3.40 \pm 0.12) \times 10^{-12} \text{s/s}\), and a refined estimate of the orbital period, \(P_{\text{orb}} = 7249.156499 \pm (1.2 \times 10^{-5}) \text{s}\). This derivative is positive and is more than one order of magnitude higher than what is expected from secular evolution driven by angular momentum losses caused by gravitational radiation under the hypothesis of conservative mass transfer. In the hypothesis that the measured derivative of the orbital period reflects the secular evolution of the system, we propose a simple explanation of this puzzling result assuming that during X-ray quiescence the source is ejecting matter (and angular momentum) from the inner Lagrangian point.

The proposed orbital evolution of the system suggests a degenerate or fully convective companion star and indicates that this kind of sources are capable to efficiently ablate the companion star, and therefore are black widows visible in X-rays during transient mass accretion episodes.

Keywords: stars: neutron – stars: magnetic fields – pulsars: general – pulsars: individual: SAX J1808.4–3658– X-ray: binaries

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INTRODUCTION

SAX J1808.4–3658 is the first one discovered among the eight known accreting millisecond pulsars (hereafter AMSPs), all of them transient X-ray sources, and is still the richest laboratory for timing studies of these sources. Although timing analysis have been now performed on most of the sources of this sample with interesting results (see Di Salvo et al. 2007 for a review and references therein), SAX J1808.4–3658 is the only known AMSP for which more than one outburst has been observed by the RXTE/PCA with high time resolution. Indeed SAX J1808.4–3658 shows outbursts approximately every two years (see [1] for a review).

Papitto et al. [2] performed a temporal analysis of the outbursts of SAX J1808.4–3658 that occurred in 1998, 2000, and 2002, which resulted in improved orbital parameters of the system. The large uncertainty caused by the relatively limited temporal baseline made it impossible to derive an estimate of the orbital period derivative. In this paper we use all the four outbursts of SAX J1808.4–3658 observed by RXTE/PCA, that means
a temporal baseline of more than 7 years, to derive an orbital period derivative, the first reported to date for an accreting millisecond pulsar. The value we find with high statistical significance is \((3.40 \pm 0.12) \times 10^{-12}\) s/s. However, the \(\dot{P}_{\text{orb}}\) we measure is an order of magnitude higher than what is expected from GR. In § 3 we propose a simple explanation of this result assuming that it reflects the secular evolution of the system. In particular, we hypothesize that during quiescence SAX J1808.4–3658 experiences a highly non-conservative mass outflow, in which a great quantity of mass is lost from the system with a relatively low specific angular momentum.

Indeed, SAX J1808.4–3658 and the other known AMSPs are all transient systems (accreting just for a very short fraction of the time), with small values of the mass function (implying small minimum mass for the secondary) and short orbital periods (less than a few hours). As predicted by King et al. [3], these pulsars may be part of the population of the so-called black widows which are millisecond pulsars in binary systems believed to evaporate their companion star. Although in some of these systems radio eclipses have been observed, clearly demonstrating the presence of matter around the system, a direct proof of severe mass losses from these system has never been found to date. The orbital evolution of SAX J1808.4–3658 indicates that this X-ray transient millisecond pulsar indeed may expel mass from the system for most of the time with just short episodes of accretion observed as X-ray outbursts. We therefore propose that SAX J1808.4–3658 (and perhaps most AMSPs) is indeed black widow still eating the companion star.

**TIMING ANALYSIS**

In this paper we analyse RXTE public archive data of SAX J1808.4–3658 taken during the April 1998 (Obs. ID P30411), the February 2000 (Obs. ID P40035), the October 2002 (Obs. ID P70080), and the June 2005 (Obs. ID P91056 and Obs. ID P91418) outbursts, respectively. In particular, we analysed data from the PCA [4], which is composed of a set of five xenon proportional counters operating in the 2 – 60 keV energy range with a total effective area of 6000 cm\(^2\). For the timing analysis, we used event mode data with 64 energy channels and a 122 \(\mu\)s temporal resolution; the arrival times of all the events were referred to the solar system barycentre by using JPL DE-405 ephemerides along with spacecraft ephemerides. This task was performed with the faxbary tool, considering as the best estimate for the source coordinates the radio counterpart position, that has a 90% confidence radius of 0.4 arcsec, which is compatible with that of the optical counterpart [5, 6].

For each of the outbursts we derived a precise orbital solution using standard techniques (see e.g. [7, 8], and references therein). In particular, we firstly corrected the arrival times of all the events with the orbital solution given by Papitto et al. [2]. Then we looked for differential corrections to the adopted orbital parameters as described in the following. We epoch-folded time intervals each with a duration of about 720 s (1/10 of the orbital period) at the spin period of 2.4939175936 ms, and fitted each pulse profile obtained in this way with a sinusoid in order to derive the pulse arrival times or pulse phases. Note that the determination of the pulse phases is insensitive to the exact value of the spin period chosen to fold the light curves providing that this value is not very far
from the true spin period of the pulsar.

We then looked for differential corrections to the adopted orbital parameters, which can be done by fitting the pulse phases as a function of time for each outburst. In general, any residual orbital modulation is superposed to a long-term variation of the phases, e.g. caused by a variation of the spin. However, as noted by Burderi et al. [9] for the 2002 outburst, SAX J1808.4–3658 shows a very complex behaviour of the pulse phases with time, with phase shifts probably caused by variations of the pulse shape that are difficult to model and to interpret. To avoid any fitting of this complex long-term variation of the phases, we preferred to restrict the fitting of the differential corrections to the orbital parameters to intervals in which the long-term variation and/or shifts of phases is negligible. We therefore considered consecutive intervals with a duration of at least 4 orbital periods (depending on the statistics), and fitted the phases of each of these intervals with the formula for the differential corrections to the orbital parameters (see e.g. [10] and eq. (3) in [8]). No significant corrections were found on the adopted values of the orbital period, $P_{\text{orb}}$, and the projected semi-major axis of the neutron star (NS) orbit, $a_1 \sin i/c$.

On the other hand we found that the times of passage of the NS at the ascending node at the beginning of each outburst, $T^*_N$, were significantly different from their predicted values, $T^*_0 + N P_{\text{orb}}$, where $T^*_0$ is the adopted time of ascending node passage at the beginning of the 1998 outburst and the integer $N$ is the exact number of orbital cycles elapsed between two different ascending node passages, i.e. $N$ is the integer part of $(T^*_N - T^*_0)/P_{\text{orb}}$ under the assumption that $|T^*_N - T^*_0| < P_{\text{orb}}$ that we have also verified a posteriori. We therefore fixed the values of $P_{\text{orb}}$ and $a_1 \sin i/c$ and derived the differential corrections, $\Delta T^*$, to the time of passage at the ascending node, obtaining a cluster of points for each outburst, which are plotted as a function of time in the inset of Fig. 1. Since each of the points corresponding to an outburst may be considered like an independent estimate of the same quantity, we computed the weighted mean of the points corresponding to each outburst, obtaining the four points shown in Fig. 1. These points show a clear parabolic trend that we fitted to the formula:

$$\Delta T^* = \delta T^*_0 + \delta P_{\text{orb}} \times N + (1/2)\dot{P}_{\text{orb}} P_{\text{orb}} \times N^2$$  \hfill (1)

In this way we found the best fit values $T^*_0 + \delta T^*_0 = 50914.8784320(7) \text{ MJD}$, $P_{\text{orb}} + \delta P_{\text{orb}} = 7249.15650(1) \text{ s}$, and $\dot{P}_{\text{orb}} = 3.4(1) \times 10^{-12} \text{ s/s}$ at $t = T_0 = 50914.8099 \text{ MJD}$, which is the beginning of the 1998 outburst (numbers in parentheses are the uncertainties in the last significant digit at 90% c.l.). The best fit $\chi^2$ is 2.2 for 1 d.o.f., which corresponds to a probability of 13% of obtaining a $\chi^2$ larger than the $\chi^2$ we found. Our result is therefore acceptable at more than 5% c.l., which is the standard confidence level to accept a model based on $\chi^2$ (see e.g. [11]). As a further check we fit with the same formula the points shown in the inset of Fig. 1, obtaining, as expected, the same results.

Note that the orbital period of SAX J1808.4–3658 is now known with a precision of $1/10^9$, that is one order of magnitude improvement with respect to the previous estimate by Papitto et al. [2] and two orders of magnitude with respect to Chakrabarty & Morgan [12]. We also find a highly statistically significant derivative of the orbital period that is at least an order of magnitude higher than what is predicted by a conservative
mass transfer driven by GR. This is the first measure of the orbital period derivative for an accreting millisecond pulsar. In the next section we discuss a possible explanation of this result.

**DISCUSSION AND CONCLUSIONS**

In order to compare the measured orbital period derivative with what is expected from the secular evolution of the system driven by GR, we have calculated the orbital period derivative and the mass transferred by the secondary star under the following assumptions. i) Angular momentum losses are due to GR and are given by: 
\[ \frac{\dot{J}}{J} = -\frac{32G^3 M_1 M_2 (M_1 + M_2)}{(5c^5a^3)} \]
where \( G \) is the Gravitational constant, \( M_1 \) and \( M_2 \) the masses of the primary and of the secondary, respectively, \( c \) the speed of light, \( a \) the orbital separation, and where \( J = M_1 M_2 G a / (M_1 + M_2) \)^{1/2} is the binary angular momentum (see e.g. [13, 14]). ii) For the secondary we have adopted a mass-radius relation \( R_2 \propto M_2^n \). iii) We have imposed that the radius of the secondary follows the evolution of the secondary Roche Lobe radius: 
\[ \frac{\dot{R}_2}{R_2} = \frac{\dot{R}_L}{R_L} \]
where for the radius of the secondary Roche Lobe we have adopted the Paczynski [15] approximation: 
\[ R_{L2} = 2/3^{4/3} \left[ q / (1 + q) \right]^{1/3} a \] that is valid for small mass ratios, \( q = M_2 / M_1 \leq 0.8 \).
In these hypotheses we derived a simple expression for the orbital period derivative and the mass transfer rate in the extreme cases of totally conservative and totally non-conservative mass transfer (see e.g. [3, 14, 16, 17]):

\[ \dot{P}_{orb} = -1.38 \times 10^{-12} \left[ \frac{n-1/3}{n+5/3-2g} \right] m_1^{5/3} q (1 + q)^{-1/3} P_2 h^{-5/3} \ s/s \]  
(2)

\[ \dot{M} = 4.0 \times 10^{-9} \left[ \frac{1}{n+5/3-2g} \right] m_1^{8/3} q^2 (1 + q)^{-1/3} P_2 h^{-8/3} \ M_\odot/yr \]  
(3)
where \( m_1 \) and \( m \) are, respectively, the mass of the primary and the total mass in units of solar masses, \( m_{2,0.1} \) is the mass of the secondary in units of 0.1 \( M_\odot \), \( P_{2h} \) is the orbital period in units 2 h, and \( g = q \) for totally conservative mass transfer, while \( g = (\alpha + q/3)/(1 + q) \) for totally non-conservative mass transfer, where \( \alpha = 1P_{\text{orb}}(M_1 + M_2)^2/(2\pi^2 M_1^2) \) is the specific angular momentum of the ejected matter, \( l \), in units of the specific angular momentum of the secondary.

Comparing the measured orbital period derivative with Eq. 2 (assuming that the orbital period derivative we measure reflects the secular evolution of the system), we note that, in order to have \( P_{\text{orb}} > 0 \) we have to assume an index \( n < 1/3 \). In the case of SAX J1808.4–3658 the secondary mass is \( m_2 \leq 0.14 \) at 95\% c.l. [12], and therefore the mass ratio \( q \leq 0.1 \) implies that for the totally conservative mass transfer case (\( g = q \)), the \( P_{\text{orb}} \) expected from GR must be of the order of \( 10^{-13} \) s/s, not compatible with what we measure for SAX J1808.4–3658. In other words, if we assume a conservative mass transfer for the system (that means that the mass transferred during outbursts is completely accreted by the NS, and during quiescence no or negligible mass is lost from the system), then it is impossible to explain the observed orbital period derivative with an evolution driven by GR.

On the other hand, if we assume that during X-ray quiescence the companion star is still overflowing its Roche Lobe but the transferred mass is not accreted onto the NS and is instead ejected from the system, we find a good agreement between the measured and expected orbital period derivative assuming that the matter leaves the system with the specific angular momentum at the inner Lagrangian point, \( \alpha = 1[(1 - 2/3\pi^2) q^{1/3} (1 + q)^{2/3}]^2 \). Adopting the measured value \( P_{\text{orb}} = 3.4 \times 10^{-12} \) s/s and the other parameters of SAX J1808.4–3658, Eq. 2 translates into a relation between \( m_1, m_2 \) and the mass-radius index \( n \); this is plotted in Fig. 2 (left panel) for different values of \( n \) going from 0 to \( n = -1/3 \). The constraint on \( m_1 \) vs. \( m_2 \) imposed by the mass function of the system is also plotted (the shadowed region in the figure) and indicates that the most probable value for \( n \) is \(-1/3 \), which in turn indicates a degenerate or, most probably, a fully convective companion star. In fact, in a system with orbital period less than 3 h, where the mass of the Roche Lobe filling companion is below \( 0.2 - 0.3 \) \( M_\odot \), the companion star becomes fully convective with a mass-radius hydrostatic equilibrium equation \( R \propto M^{-1/3} \) (e.g. [14, 18]). Also, for reasonable minimum, average and maximum values of the NS mass, 1.1, 1.56, and 2.2 \( M_\odot \), respectively, we obtain the following values for the secondary mass: 0.053, 0.088, and 0.137 \( M_\odot \), and the following values for the inclination of the system: 44°, 32°, and 26°, respectively.

Assuming therefore \( n = -1/3 \) we have plotted in Fig. 2 (right panel) the corresponding non-conservative mass transfer rate as a function of \( m_1 \). We find that for \( m_1 = 1.5 \) the mass transfer rate must be of the order of \( 10^{-9} \) \( M_\odot/\text{yr} \), much higher than what is expected in a conservative GR driven mass transfer case. Note that in Burderi et al. [9], the proposed interpretation of the complex observed behaviour of the pulse phase evolution during the 2002 outburst implied a mass accretion rate of \( 1.8 \times 10^{-9} \) \( M_\odot/\text{yr} \) at the beginning of the outburst; we note that this value is in agreement with the value of \( M \) necessary to explain the observed orbital period derivative in the hypothesis of a strongly non-conservative mass transfer during quiescence. Actually, during the X-ray outbursts, the mass transfer is conservative since the transferred matter is accreted onto the NS.
FIGURE 2. Left: Companion star mass vs. NS mass in the hypothesis of totally non conservative mass transfer (with matter leaving the system with the specific angular momentum at the inner Lagrangian point) and assuming the $P_{\text{orb}}$ measured for SAX J1808.4–3658. Different curves correspond to different values of the mass-radius index $n$ of the secondary. Horizontal lines indicate the limits for the secondary star mass corresponding to reasonable limits for the NS mass and to $n = -1/3$. Right: Mass rate outflowing the secondary Roche Lobe in the hypothesis of totally non conservative mass transfer (as above) and assuming $n = -1/3$.

However, the accretion phase duty cycle, about 40 days / 2 years = 5%, is so small that the totally non-conservative scenario proposed above is substantially unchanged.

The question to answer is why the accretion is inhibited during X-ray quiescence while the companion star is transferring mass at a high rate? We propose that the answer has to be found in the radiation pressure of the magneto-dipole rotator emission, with a mechanism that is similar to what is proposed to explain the behaviour of the so-called black widow pulsars (see e.g. [3, 16]; see also [19, 20]). Indeed, the possibility that the magneto-dipole emission is active in SAX J1808.4–3658 during X-ray quiescence has been used by Burderi et al. [21] to explain the optical counterpart of the source, which is observed to be over-luminous during quiescence. In conclusion, we propose that SAX J1808.4–3658 during quiescence behaves exactly as a black widow pulsar, efficiently ablatting its companion star. When (or if) the pressure of the outflowing matter becomes sufficiently high to temporarily overcome the radiation pressure of the magneto-dipole rotator, the source experiences a transient mass accretion episode, resulting in an X-ray outburst. We propose therefore that we are witnessing the behaviour of a hidden black widow eating its companion during X-ray outbursts and ablating it during quiescence.

REFERENCES