1 Homework problem 1

For neutrino energies much less than $m_n c^2 \sim 1$ GeV the cross section for the nucleon scattering is

$$\sigma_\nu = \sigma_0 \left(\frac{E_\nu}{m_e c^2} \right)^2 \tag{1.1}$$

where

$$\sigma_0 = \frac{4 G_F^2 m_e^2}{\hbar^4} = 1.76 \times 10^{-44} \mathrm{cm}^2. \tag{1.2}$$

1. Show that the mean free path for scattering is

$$\lambda_\nu \approx 2 \times 10^5 \left(\frac{E_\nu}{10\ \mathrm{MeV}} \right)^{-2} \rho_{12}^{-1}\ \mathrm{cm} \tag{1.3}$$

   The typical neutrino energy is ~ 20 MeV, so the mean free path is only $\sim 0.5\ \rho_{12}^{-1}$ km.

2. Scattering is a diffusion process, and from the diffusion equation in spherical geometry one finds that the time for a neutrino to diffuse a radial distance R is

$$t_{diff} = \frac{R^2}{3 \lambda_\nu c} \tag{1.4}$$

   Diffusion can also be seen as a random walk in space. Show that after N steps the photon has diffused from its origin a distance $R = N^{1/2} \lambda_\nu$, and that the diffusion time scale is $t_{diff} \sim \Delta R^2 / (\lambda_\nu c)$.

   Do not worry about the numerical factor in Eq. (1.4) originating in the geometry.

3. Show that for a uniform density sphere of mass 1.4 $\mathrm{M}_\odot$ we get from Eq. (1.3)

$$t_{diff} = 3.9 \times 10^{-3}\ \rho_{12}^{1/3} \left(\frac{E_\nu}{10\ \mathrm{MeV}} \right)^2\ \mathrm{s}. \tag{1.5}$$

4. Estimate the neutrino energy as the Fermi energy of the collapsing core to obtain $E_\nu \approx E_F = 35 (\rho_{12})^{1/3}$ MeV, and therefore that

$$t_{diff} \approx 5 \times 10^{-2}\ \rho_{12}\ \mathrm{s} \tag{1.6}$$

5. Estimate now the diffusion time scale after the collapse when the protoneutron star radius is ~ 30 km.

6. Show that the binding energy for a uniform density of the neutron star is

$$E_b = \frac{3}{5}\frac{GM^2}{R} = 3.1 \times 10^{53} \left(\frac{M}{1.4\ \mathrm{M}_\odot}\right)^2 \left(\frac{R}{10\mathrm{km}}\right)^{-1} \text{ergs} \tag{1.7}$$

   Note that we here should use the radius of the cool neutron star $10 - 20$ km. A more accurate calculation, taking the non-uniform density distribution into account, gives a similar result.

7. Estimate the total electron-antineutrino luminosity detectable from the collapse.

Homework problem 2

1. Fill in the steps in the derivation of the fluid energy equation Eq. 8.13 in the lecture notes.

2. Discuss the case of isothermal (i.e. $\gamma = 1$) spherical accretion along the lines in Sect. 10 in the lecture notes.

3. For those who have some extra energy! Not compulsary! Discuss the spherical wind solution along the lines of Sect. 10 and Fig. 20.

Homework problem 3

1. Read the new section 11 of the lecture notes (gan be found on the home page). Show how one obtains the standard disk equations for the case of Kramer's opacity and gas pressure (Eqns. (11.8) in the lecture notes). Do not worry about the constants, only the dependence on the parameters, eg., $\Sigma \propto \alpha^{-4/5} \dot{M}_{16}^{7/10} m^{1/4} r_{10}^{-3/4} f^{14/5}$.

2. Calculate the total mass of the disk for this case.

Homework problem 4

Assume that a supernova expands with constant velocity 2×10^4 km s^{-1} into the stellar wind of the progenitor star. We assume that this is a Wolf-Rayet star with massloss rate 10^{-5} M$_\odot$ yr^{-1} and a wind velocity of 1000 km s^{-1}. The expansion causes a shocked region of hot stellar wind material.

1. Calculate the temperature, mass density and the thermal energy density behind the shock 10 days after the explosion.
2. Assume that the magnetic field energy density is a factor ϵ_B of the thermal energy behind the shock. Calculate the magnetic field behind the shock for $\epsilon_B = 0.1$.
3. What is the typical Lorentz factor of the electrons responsible for radio emission at 10 GHz? For X-rays with $E = 1$ keV?
4. For which frequency is the synchrotron cooling time scale (defined by $\tau = \gamma m_e c^2/dE/dt$ equal to the age of the supernova. What does this mean for the spectrum?
5. Assuming that the effective temperature of the radiation fom the supernova is 2×10^4 K and luminosity is 2×10^{42} erg s^{-1}, calculate the Compton cooling time scale and compare with the synchrotron time scale.
6. Estimate the frequency of the inverse Compton scattered photons from the photosphere upscattered by the radio emitting electrons.
7. Calculate the ratio of the total inverse Compton flux to the synchroton flux.
8. Scetch the spectrum, assuming that the electrons injected at the shock have a distribution $dn/dE \propto E^{-p}$ with $p = 2$.

1 M$_\odot$ $= 2 \times 10^{33} g$, 1 W $= 10^7$ erg

Homework problem 5

Estimate when radiative cooling becomes important for a supernova remnant with total energy 10^{51} ergs in a constant density gas of density 1 cm^{-3}. What is the temperature and shock velocity at this time? Use the cooling function given in the lecture notes.

Homework problem 6

Show that one from a determination of the Sunyaev-Zeldovich Compton parameter y and from X-ray observations can determine the absolute distance of a cluster of galaxies, and therefore the Hubble parameter. Simple scaling arguments is sufficient.