Roche Lobe Overflow **Longair 14.6**

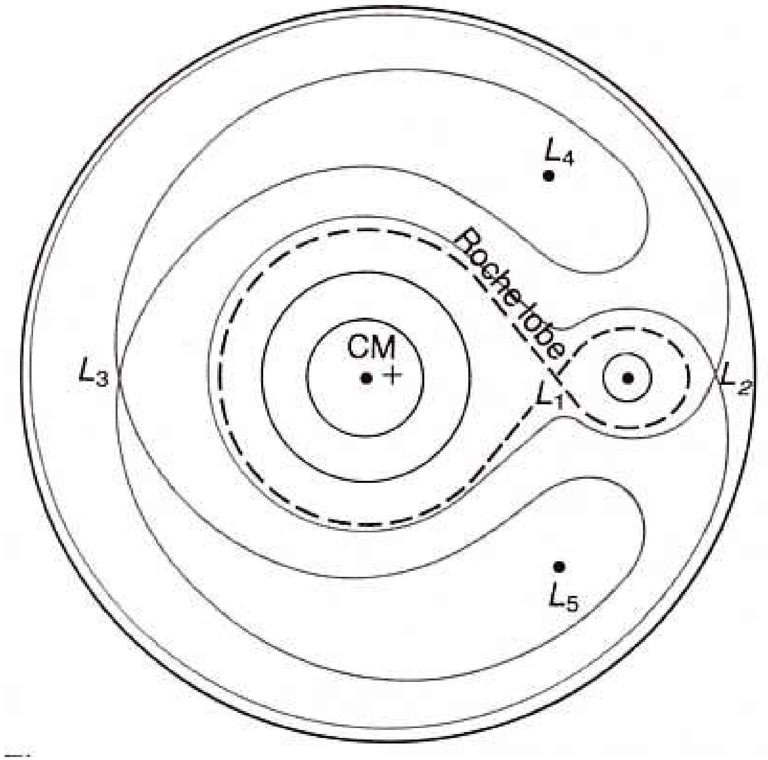
Transform Eulers equation to rotating frame

$$\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v} = -\nabla \boldsymbol{\Phi} - 2 \Omega \, \boldsymbol{x} \, \boldsymbol{v} - \frac{1}{\rho} \nabla p$$

$$\boldsymbol{\Phi} = \frac{-GM_1}{\boldsymbol{r} - r_1} - \frac{GM_2}{\boldsymbol{r} - r_2} - \frac{1}{2} (\Omega \, \boldsymbol{x} \, \boldsymbol{r})^2$$

Φ = Roche potential

L_1 = inner Lagrangian point



Magnetospheric accretion **(Longair 16.4.2)**

Assume a dipolar magnetic field

$$B = B_s \left(\frac{R_s}{r} \right)^3$$

Assume spherical accretion

Magnetic pressure

$$p_{mag} = \frac{B^2}{8\pi} = \frac{B_s^2}{8\pi} \left(\frac{R_s}{r} \right)^6$$

$$v \approx v_{ff} = \left(\frac{2GM}{r} \right)^{1/2}$$

Ram pressure

$$p_{ram} = \rho v^2 \approx \rho \left(\frac{2GM}{r} \right)$$

$$\dot{M} = 4\pi r^2 v \rho$$

$$p_{ram} = \frac{\dot{M}}{4\pi r^2 v} v^2 = \frac{\dot{M} v}{4\pi r^2}$$

At magnetosphere $p_{ram} = p_{mag} \quad \Rightarrow \quad \boxed{r_M = \frac{B_s^{4/7} R_s^{12/7}}{2^{3/7} \dot{m}^{2/7} (GM)^{1/7}}}$

Inside of r_M the magnetic field dominates the dynamics.

Flow along field lines

Ex. neutron star accreting at Eddington luminosity

$$\dot{m} = \frac{L}{\epsilon c^2} = \frac{1.3 \times 10^{38}}{0.1 c^2} = 1.4 \times 10^{18} g$$

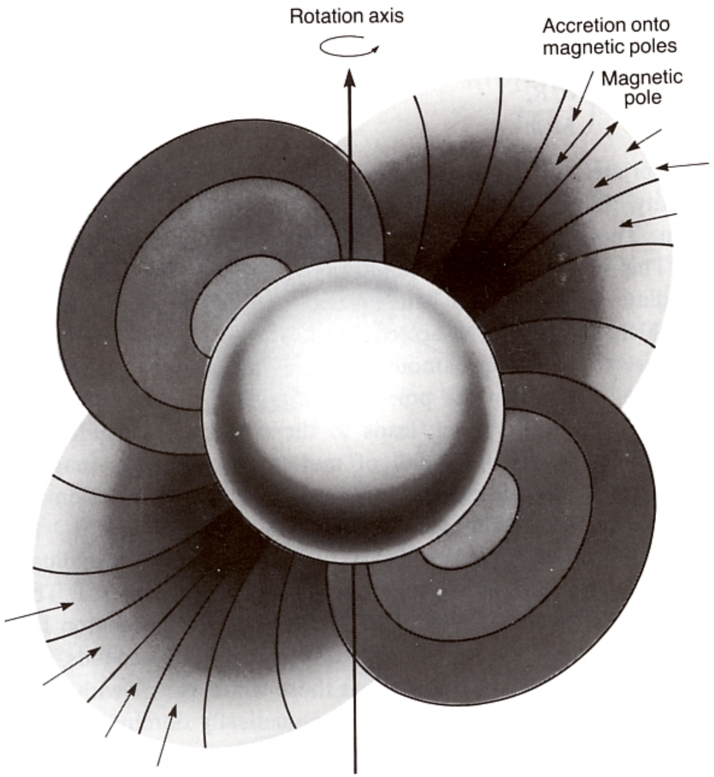
$$B_s = 10^{12} G$$

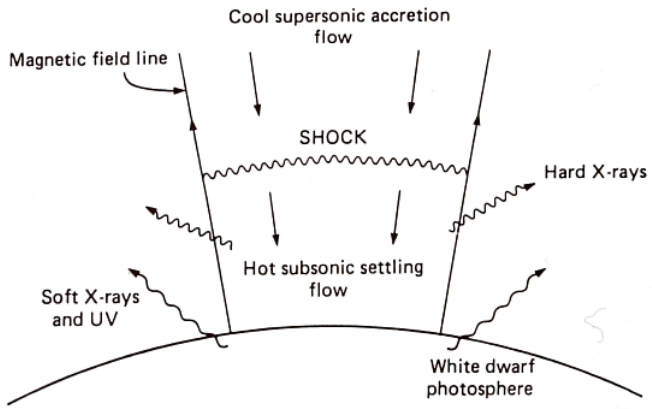
$$R_s = 10\, km$$

$$r_M = 1000\, km$$

Accretion mainly on the poles of the neutron star

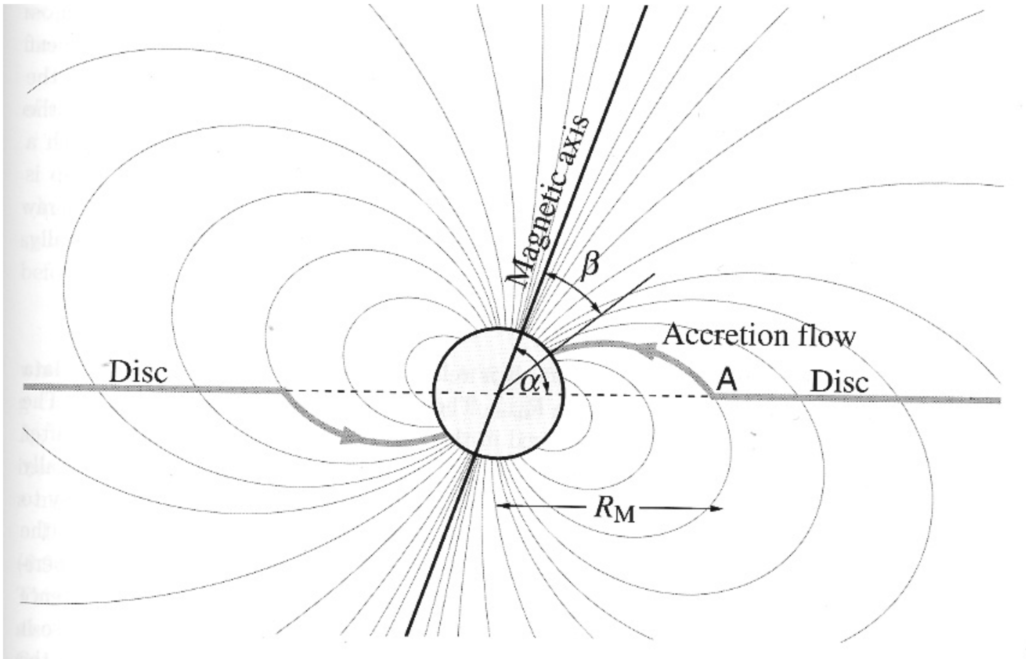
Accretion column





Disk accretion with magnetic field

Magnetospheric radius similar to the spherical case



Spin-up by accretion

Magnetic field of neutron star co-rotates with the star out to light cylinder.

Suppose $r_M < r_{l.c.}$

$$r_{lc} = \frac{c}{\Omega} = \frac{P\,c}{2\pi} = 4.8 \times 10^4\,P \quad km \quad \gg \quad r_M$$

If rotational frequency of neutron star << Keplerian frequency of the disk at the magnetosphere ⇨ torque on neutron star by disk ⇨ spin-up of neutron star

Keplerian ang. vel.at r_M

$$\Omega_K = \left(\frac{GM_s}{r_M^3}\right)^{1/2}$$

Angular momentum flux at r_M

$$\dot{L} = \dot{m}\,v\,r = \dot{m}\,\Omega_K(r_M)\,r_M^2$$

Change in angular momentum of star

$$I\,\dot{\Omega} = \dot{m}\,\Omega_K(r_M)\,r_M^2$$

$$I = \frac{2}{5} M_s R_s^2$$

Spin-up II

$$L = \frac{G \dot{m} M}{R_s}$$

$$r_M = \frac{B_s^{4/7} R_s^{12/7}}{2^{3/7} \dot{m}^{2/7} (GM)^{1/7}}$$

$$\frac{\dot{P}}{P} = \frac{\dot{\Omega}}{2\pi} P = \frac{1}{(2\pi I)} 2^{-3/14} B_s^{2/7} R_s^{12/7} (GM_s)^{-3/7} L^{6/7} P$$

* See below

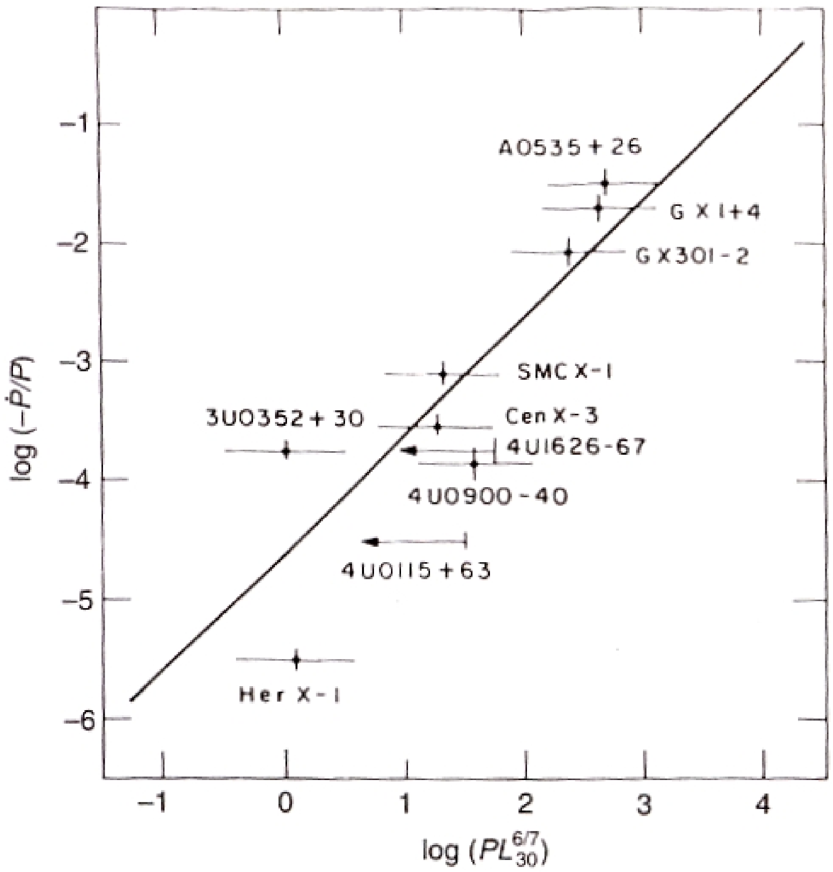
$M_s = 1\ M_O$, $R_s = 10$ km, $B_s = 10^{12}$ G

$$\log\left(\frac{-\dot{P}}{P}\right) = -4.4 + \log P + \frac{6}{7} \log L_{37}$$

* Error in Longair 16.45 $M^{-2/7}$ should be $M^{-3/7}$

Spin-up III

$$\log\left(\frac{-\dot{P}}{P}\right) = -4.4 + \log P + \frac{6}{7}\log L_{37}$$

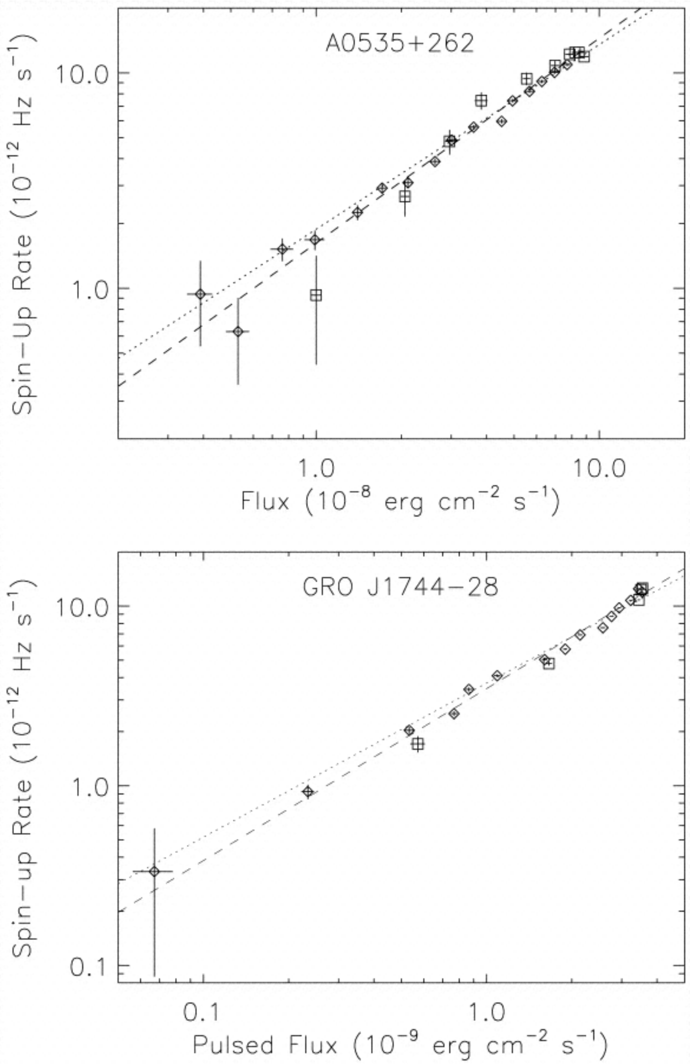


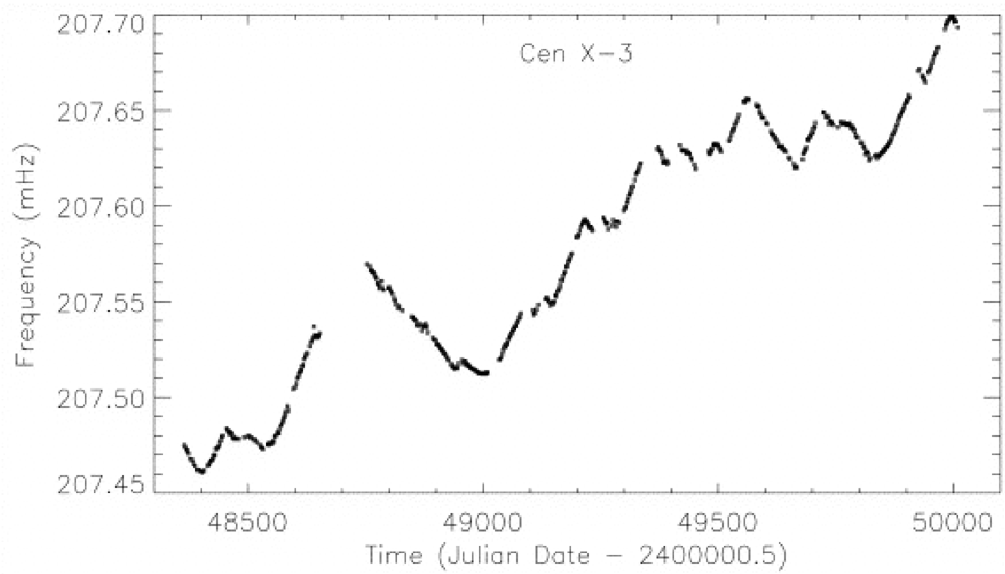
Support for disk accretion picture

Spin-up IV

$$\frac{\dot{P}}{P} = \frac{\dot{\Omega}}{2\pi} P = \frac{1}{(2\pi I)} 2^{-3/14} B_s^{2/7} R_s^{12/7} (GM_s)^{-3/7} L^{6/7} P$$

$$\log\left(\frac{-\dot{P}}{P}\right) = -4.4 + \log P + \frac{6}{7} \log L_{37}$$





Well behaved!

Not so well behaved!

Minimum spin-up period

Spin-up until co-rotation speed of n-star same as Keplerian at r_M

$$\Omega_{NS} = \frac{2\pi}{P_{NS}} = \Omega_K = \left(\frac{GM}{r_M^3}\right)^{1/2}$$

$$P_{NS} = 2\pi \left(\frac{r_M^3}{GM}\right)^{1/2}$$

$$r_M = \frac{B_s^{4/7} R_s^{12/7}}{2^{3/7}\dot{m}^{2/7}(GM)^{1/7}}$$

$$P_{min} = 2^{5/14}\pi\,\dot{m}^{-3/7}(GM)^{-5/7} R_s^{18/7} B_s^{6/7}$$

$$L = L_{Edd} = 1.3\times 10^{38}\,erg/s \qquad \dot{m} = 10^{18}\,g/s$$

$$P_{min} = 2.4\times 10^{-3}\left(\frac{\dot{m}}{10^{18}\,g/s}\right)^{-3/7}\left(\frac{B_s}{10^{9}\,G}\right)^{6/7} s$$

Maximum spin-down period for pulsars

For pulsars we have

$$B_s \approx 3 \times 10^{19} (P \dot{P})^{1/2} \quad G$$

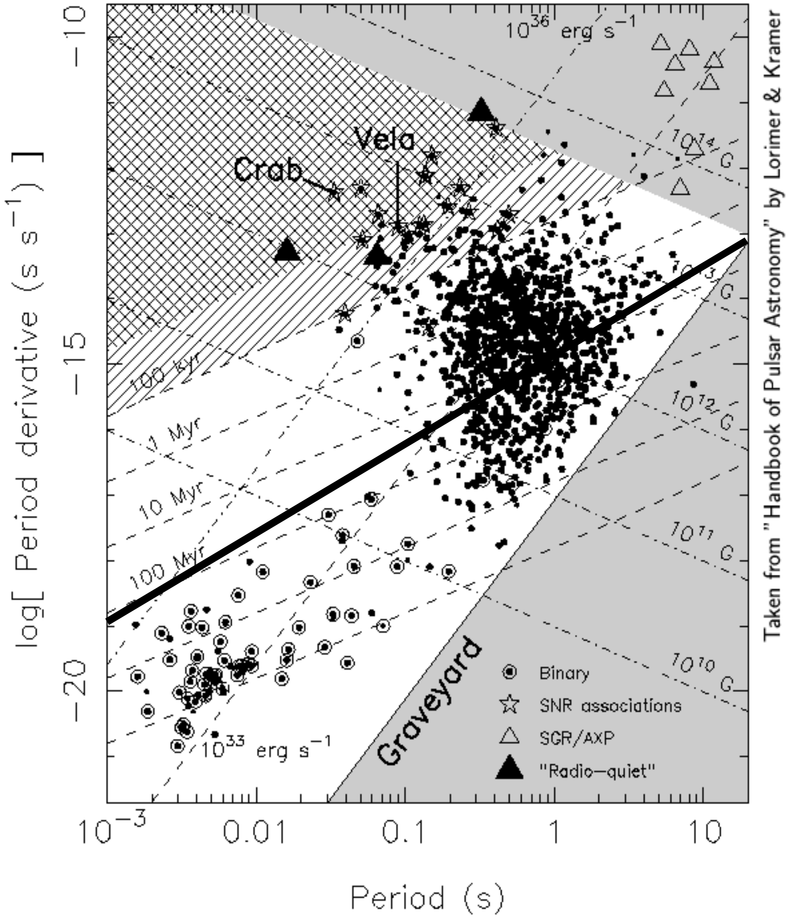
$$P_{min} = 2.4 \times 10^{-3} \left(\frac{\dot{m}}{10^{18} g/s} \right)^{-3/7} \left(\frac{B_s}{10^9 G} \right)^{6/7} \quad s$$

$$\dot{P} < 1.5 \times 10^{-15} P^{4/3} \quad s/s$$

Maximum spin-down rate for a pulsar if it has been spun-up
by disk accretion in a binary system before it became a radio pulsar

Pulsar spin-down II

$$\dot{P} < 1.5 \times 10^{-15} P^{4/3} \quad s/s$$



X-ray binaries

Companion star	Compact star		
	White dwarf	Neutron star	Black hole
Early-type, massive	None known	Massive X-ray binaries (e.g X-ray pulsars)	Cygnus X-1 LMC X-3
Late-type, low-mass	Cataclysmic variables (e.g. dwarf novae)	Low-mass X-ray binaries (e.g. galactic bulge sources)	A0620-00 plus at least ten others
Intermediate mass, or crossing Hertzsprung gap	Supersoft X-ray binaries	Progenitors of Cyg X-2 and some radio pulsars	Microquasars

X-ray binaries

Table 6.2. *Modes of accretion in compact binary systems*

Companion star	Type: mode	Compact star: White dwarf: Unmagnetized	Compact star: White dwarf: Magnetized	Compact star: Neutron star: Unmagnetized	Compact star: Neutron star: Magnetized	Compact star: Black hole
Early type, massive	O, B supergiant: wind, Roche lobe disc(?)	None known	None known	Unpulsed, massive X-ray binaries	Rapidly pulsing massive X-ray binaries [C]	Cygnus X-1 LMC X-3
	Be star: wind, eccentric, disc(?)	None known	None known	Unpulsed X-ray binaries	Slowly pulsing X-ray binaries [C]	None known
Late-type, low-mass ($\leq 2M_{\odot}$)	*Roche lobe*, disc (except AM Hers and some IPs)	Classical novae dwarf novae novalike variables	Intermediate polars [C], disc? AM Hers [C] only, no disc	Galactic bulge sources, bursters globular cluster sources	Her X-2 plus two others [C], pulsed	A0620-00 plus at least ten other soft X-ray transients
Intermediate mass, or crossing Hertzsprung gap	Roche lobe, disc	Supersoft X-ray binaries	Magnetized supersoft X-ray binaries [C], progenitor of AE Aqr and some AM Her systems [C]	Progenitor of Cyg X-2, transient	Progenitors of some radio pulsar binaries: unobserved, = quiescent transients?	Microquasars, transient

[C] denotes the presence of accretion columns.

Cataclysmic variables = dwarf novae **Longair 16.5.1**

Low mass star (0.1 – 1 $M_{\odot}$) + accreting white dwarf.

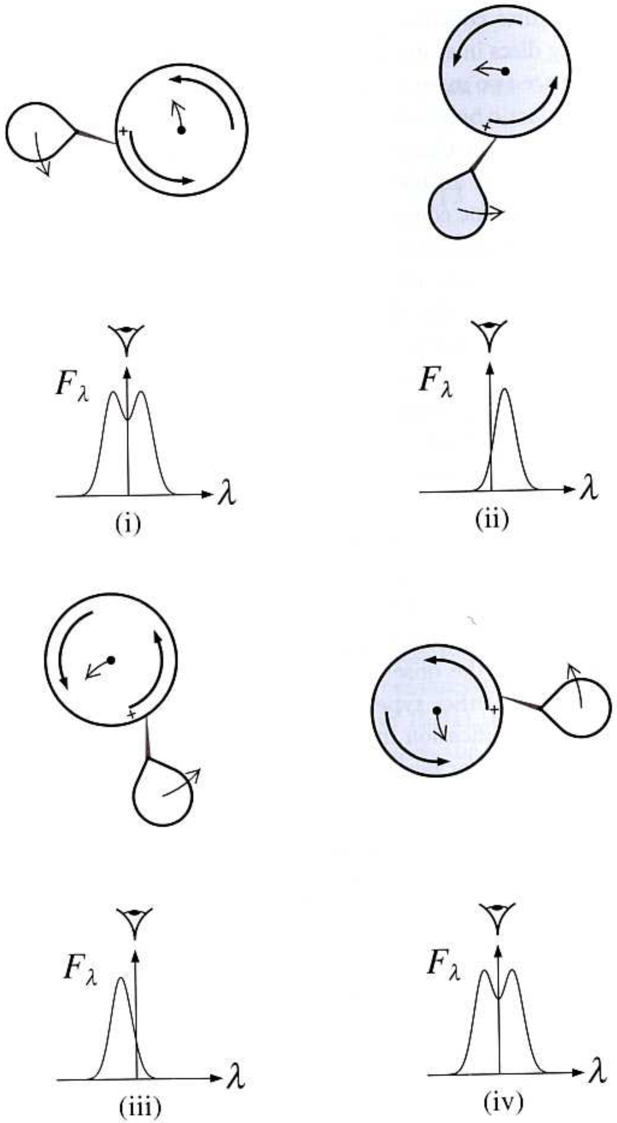
Roche lobe overflow

Luminosity $< 0.01\ L_{Edd}$ accretion onto WD does not swamp disk emission as in LMXBs.

Best test of accretion disk structure

Emission lines often double peaked likely to come from disk

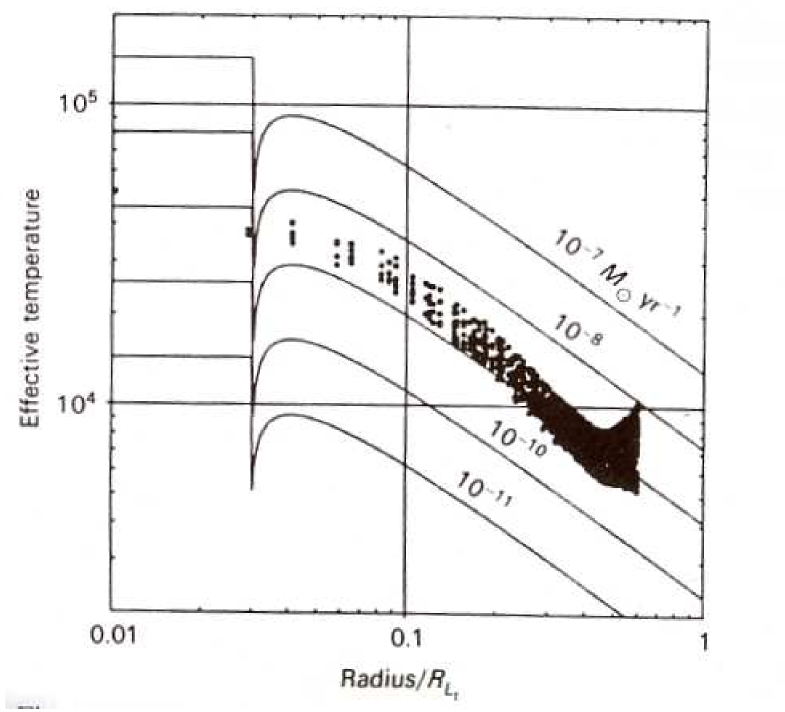
Disk emission



Eclipse mapping

gives temperature as function of radius

$$T = \left(\frac{3\,G\,M_s\,\dot{m}}{8\,\pi\,\sigma\,r^3} \right)^{1/4}$$



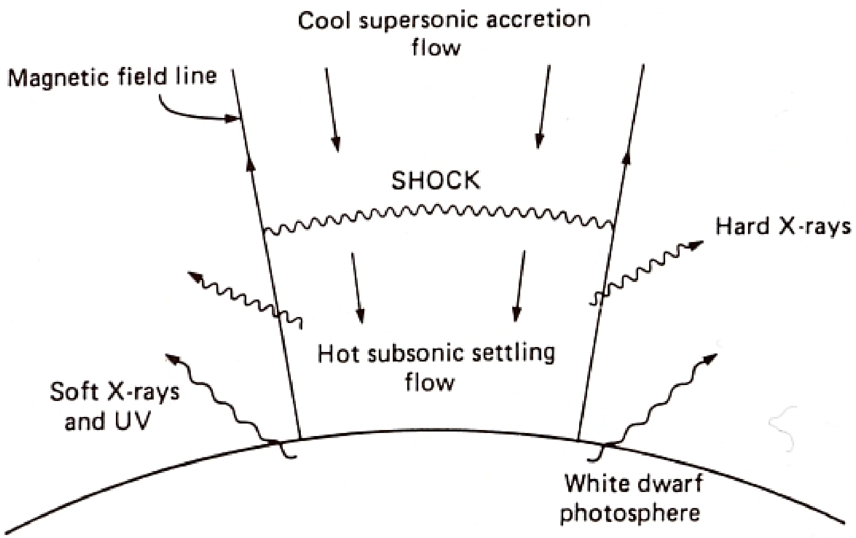
$T_{max} \sim 5x10^4$ K $\quad dM/dt \sim 10^{-9}\ M_O/yr$

Emission from accretion column

Disk has T ~ 10^4-10^5 K $\Rightarrow$ UV – EUV radiation

X-rays with E ~ 1-10 keV observed

Accretion onto poles $\Rightarrow$ shock $\Rightarrow$ thermalization



$$\frac{3}{2}k\left(T_p+T_e\right)=\frac{G M_s m_p}{R_s}$$

Assume $T_p \approx T_e \Rightarrow$

$$T_e=\frac{G M_s m_p}{3k R_s}$$

R_s ~ 10^4 km, M ~ 1 M_O $\Rightarrow$ kTe ~ 50 keV

Low mass X-ray binaries

NS + low mass companion with $M \sim M_o$

Most found in Galactic bulge and globular clusters. Old population

Edge on: Disk obscures the X-ray source weak in X-rays.
Show eclipses

Face on: Bright in X-rays, but no eclipses

Soft X-ray spectra

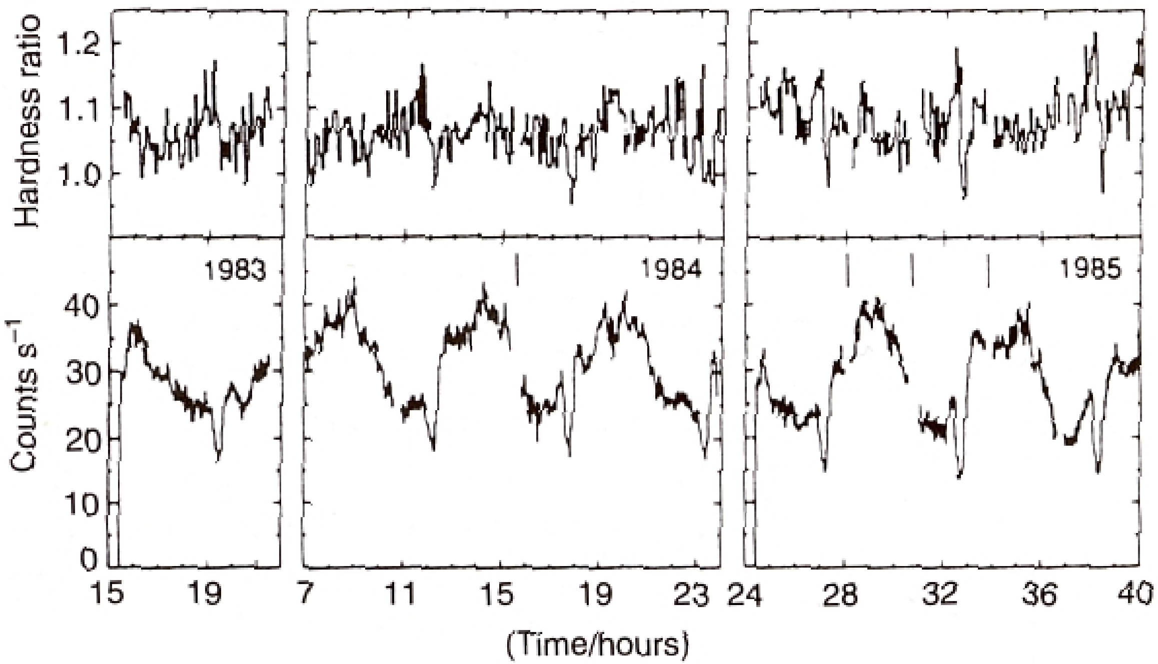
Some X-ray pulsars. Many show quasi-periodic oscillations = QPOs

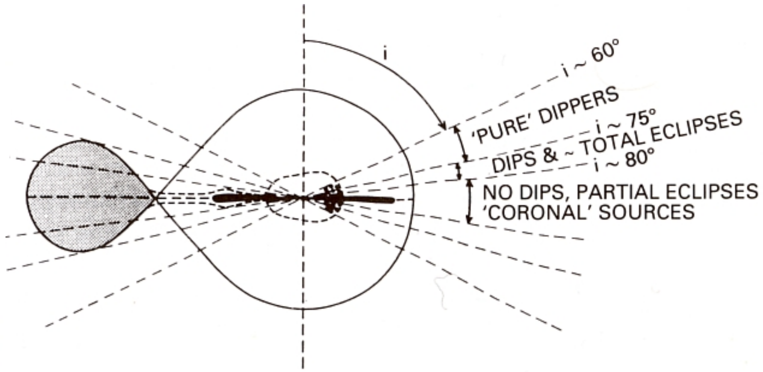
Beat frequency of rotation of neutron star and inner part of
disk extending to the surface.

Requires weak magnetic field, $B < 10^{10}$ G

Low mass X-ray binaries

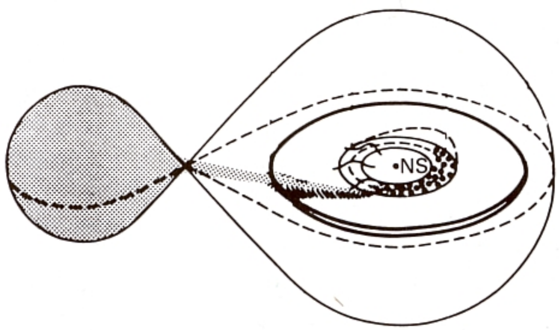
X-ray emission also during eclipses. Spectrum gets harder (higher energies) during eclipses





(a)

Hot corona scattering the X-ray emission



(b)

disk instabilities, flares...

Cyclotron emission

Resonant scattering absorption

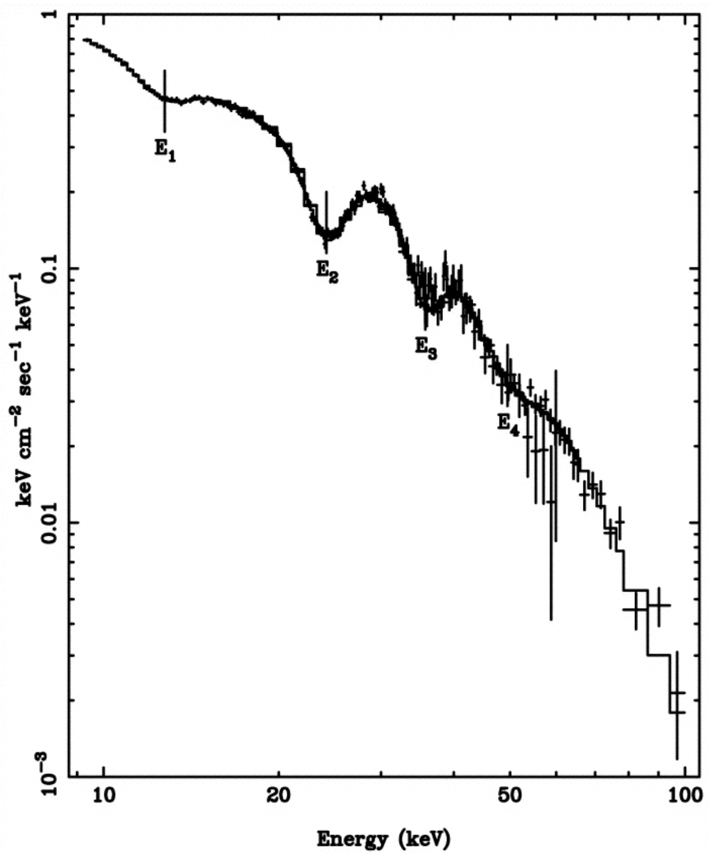
14 X-ray binaries with cyclotron absorption lines with energies 12 – 50 keV

Cyclotron frequency

$$\Omega = \frac{e\,B}{c\,m_e}$$

grav. redshift

$$E = 11.6\, B_{12}\ \ /(1+z)\quad keV$$



In X0115+63 four harmonics

E_n = 12.79 n keV

$B \sim 10^{12} - 10^{13}$ G

X-ray bursters

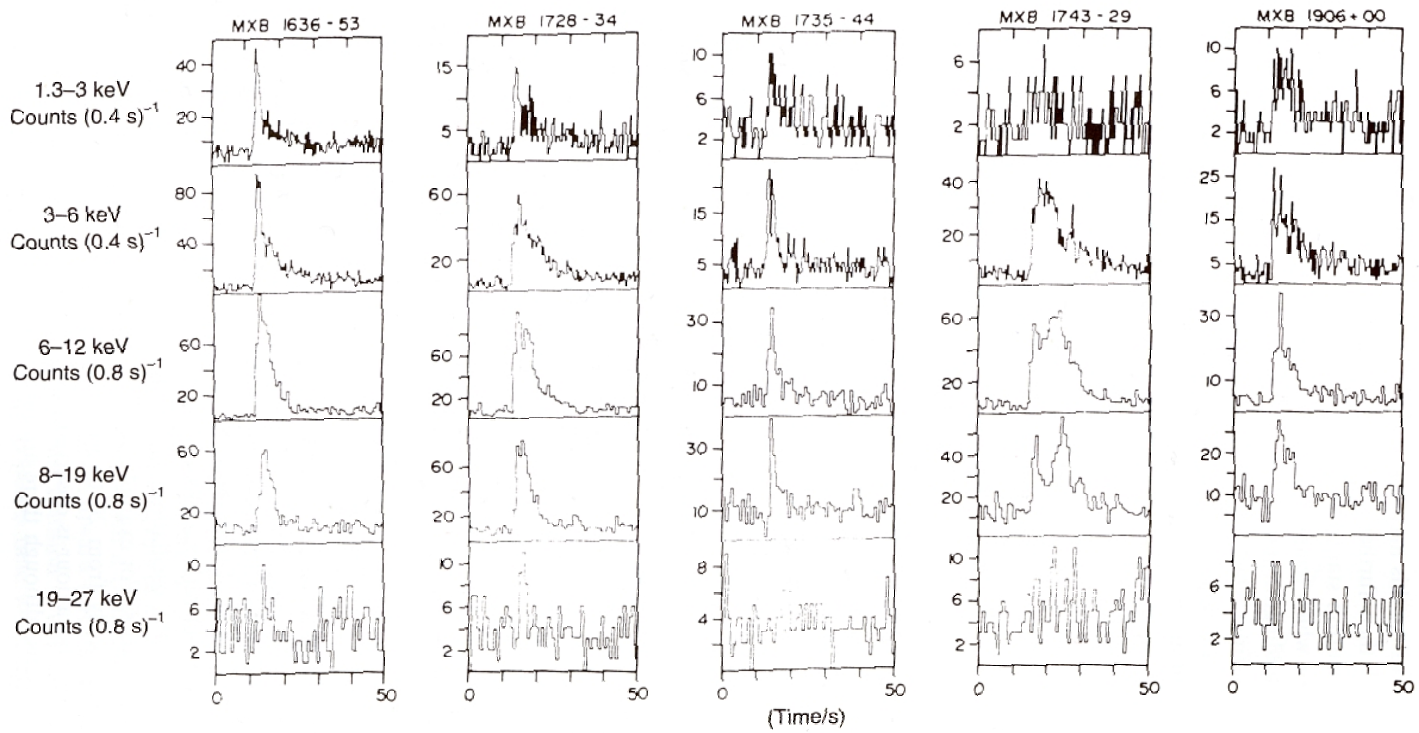
Type I

Rise time ~ second   decay time 10 s - minutes

Persistent luminosity + burst with intervals hours – days

L(burst) ~ 10 x L(persitent)

Δt_b



X-ray bursters II

Burst: thermonuclear runaway in a LMXB

1. Accretion of H.
2. CNO burning $\varepsilon \propto T^{17}$ or He burning + degenerate EOS at 10^7 K ⇒
   Thermonuclear runaway ⇒ matter expelled

Same mechanism as in accreting WDs (novae).

Accretion energy = $\Delta t_{\text{between bursts}} \times L_{\text{persistent}} = \varepsilon Mc^2$

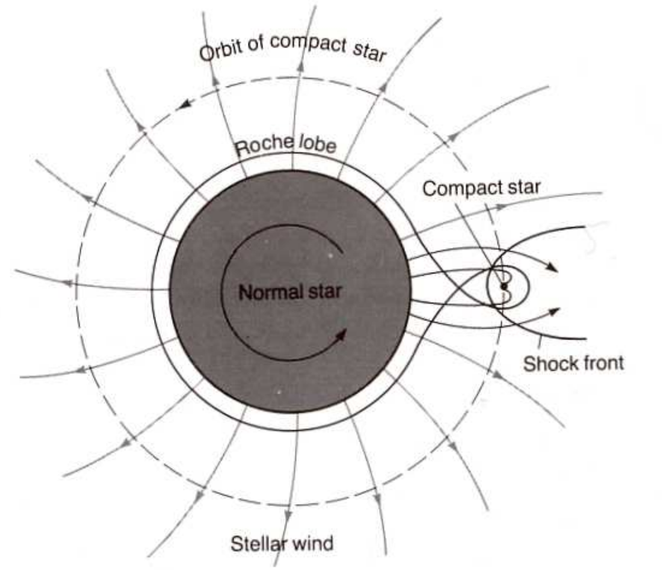
Burst energy = $\Delta t_{\text{burst}} \times L_{\text{burst}}$

E(burst)/E(persistent) ~ 10^{-2}

He H

$$\frac{E_{nuc}}{E_{acc}} = \frac{\epsilon_{nuc} \Delta M c^2}{\epsilon_{acc} \Delta M c^2} = \frac{0.002 - 0.007}{0.1} \approx 0.02 - 0.1$$

High Mass X-ray Binaries



Massive normal O-B star + compact object

Ex Cyg X-1 (BH), Cen X-3 (NS)

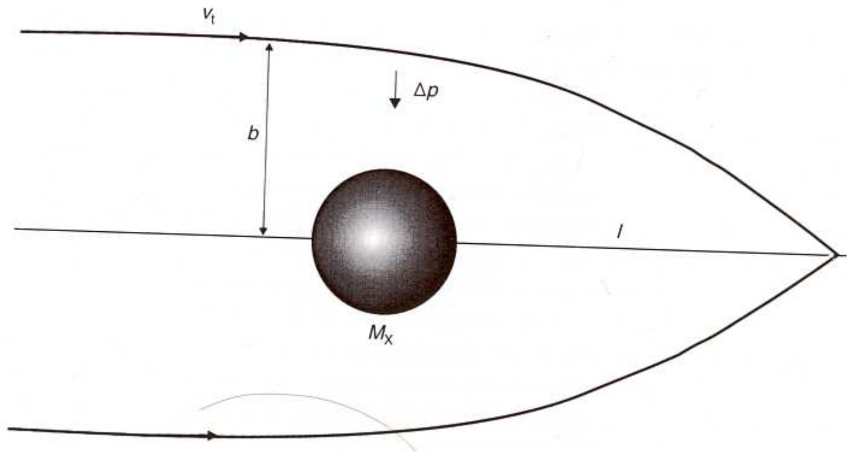
Stellar winds $v_w \sim$ 1000-3000 km/s
$dM/dt \sim 10^{-6} – 10^{-5}\ M_O/yr$

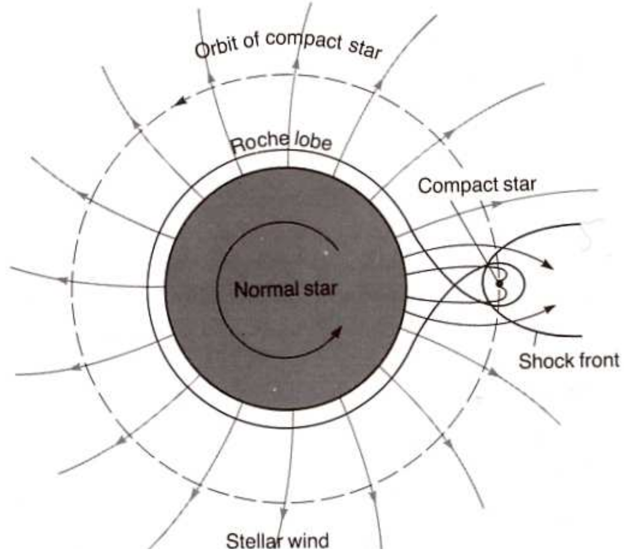
Young systems, confined to galactic plane

23/26 X-ray pulsars HMXB
3 BH candidates

Hard component in spectrum

Wind accretion





$$v_t^2 = v_w^2 + v_{orbit}^2$$

$$F = \frac{dp}{dt} = G\,m\,\frac{M_X}{b^2}$$

Perpendicular momentun $\Delta p = \frac{dp}{dt}\Delta t \approx \frac{dp}{dt} 2\frac{b}{v_t} = \frac{2\,G\,m\,M_X}{b\,v_t}$

Distance behind X-ray source $$l \approx b \frac{v_t}{v_p}$$

$$v_p = \frac{\Delta p}{m} = \frac{2GM_X}{b v_t}$$

$$l \approx b \frac{v_t}{v_p} = \frac{b^2 v_t^2}{2GM_X}$$

For capture by X-ray source $$\frac{GM_x}{l} > \frac{v_t^2}{2}$$

$$b < \frac{2GM_X}{v_t^2}$$

Capture radius $$R_c = \frac{2GM_X}{v_w^2 + v_{orbit}^2}$$

Fraction of wind captured

$$\dot{m} = \frac{\pi R_c^2}{4\pi R_p^2} \dot{M}_p = \frac{R_c^2}{4 R_p^2} \dot{M}_p$$

$$L_X = \epsilon \dot{m} c^2 = \epsilon \frac{R_c^2}{4 R_p^2} \dot{M}_p = \frac{\epsilon \dot{M}_p c^2}{4} \left(\frac{2 G M_X}{R_p \left(v_w^2 + v_{orbit}^2 \right)} \right)^2$$

$$v_w \gg v_{orbit} \qquad \boxed{L_X \approx \epsilon \dot{M}_p c^2 \left(\frac{G M_X}{R_p} \right)^2 v_w^{-4}}$$

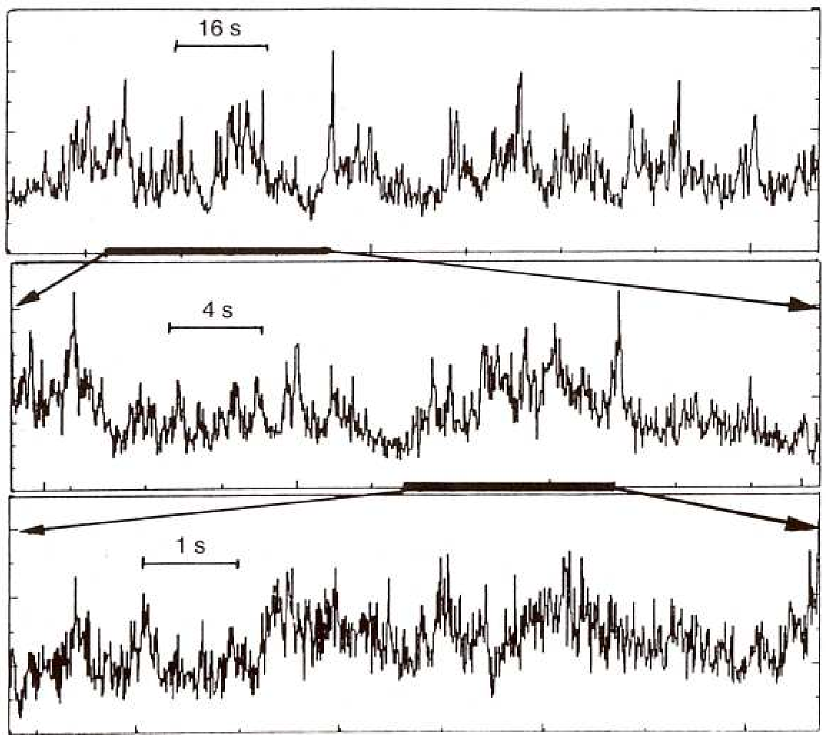
Black hole binaries Longair 16.2.3, 16.7

Signatures: 1. Mass of compact object > 3 M_O

2. 'Flickering'

$$T_{min} \approx \frac{r_g}{c} = 10^{-5} \frac{M}{M_O} \quad s$$

3. Luminosity in persitstent state lower than in NS sources



Black hole binaries II

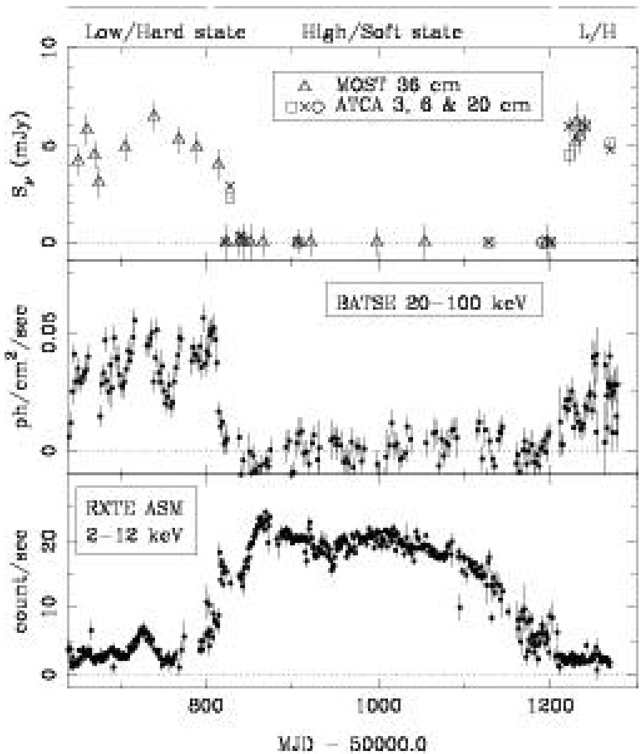
Large fraction X-ray transients: rise within a few days, decay on 10-100 days, recurrent on time scales of month – years. Always with low mass companion.

Ex A 0620-003

Companion K4V star, P = 0.32 days, M_{comp}=0.55-1.22, M_{BH}=9.4-15.9

Tranistions between high/soft and low/hard states

All low mass systems with BHs X-ray transients

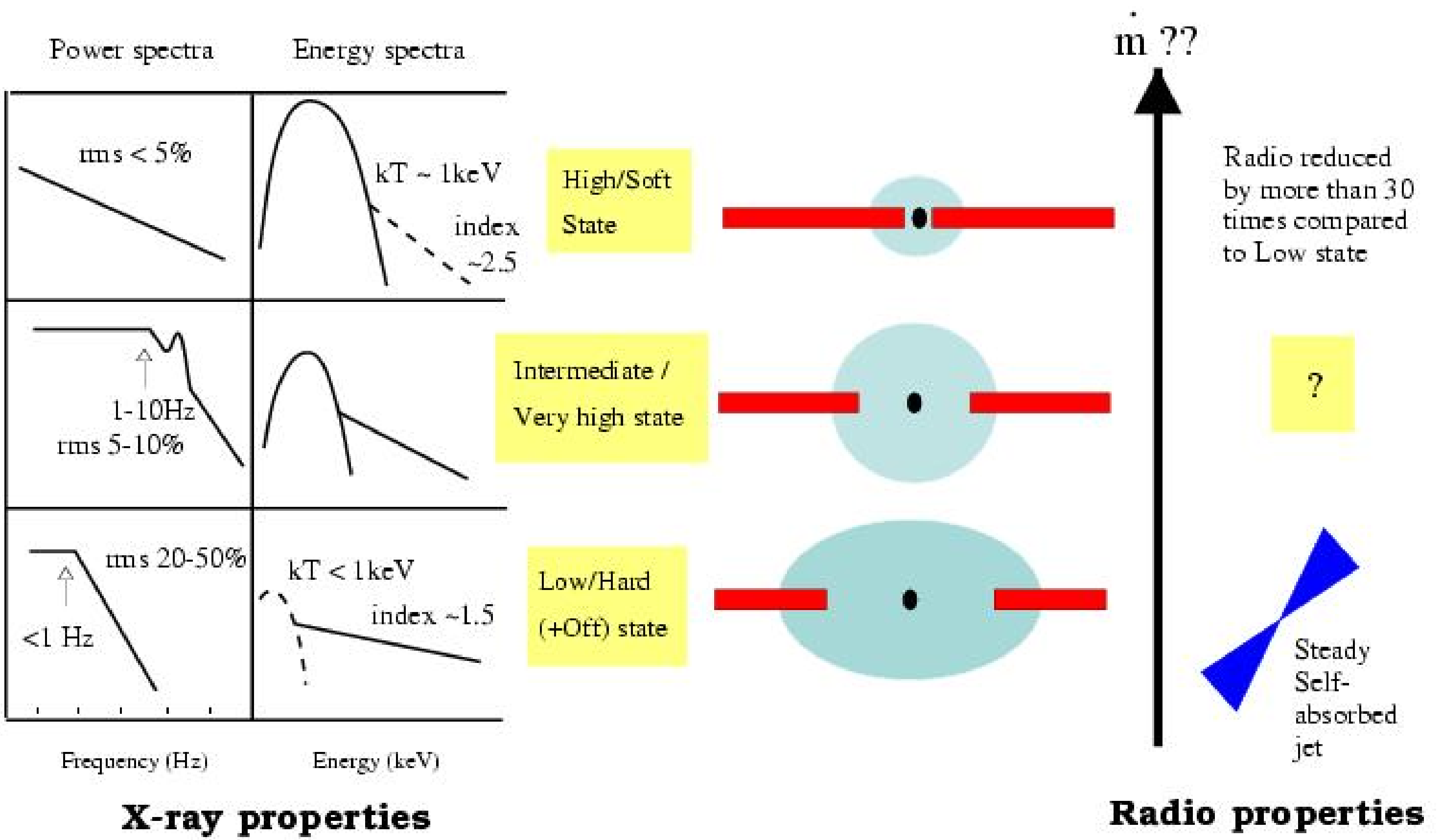


radio

Hard X

Soft X

High/soft to low/hard to



Fender 2001