Problem set 5

1. A neutron star in the X-ray source Cen X-3 with mass $1 \, M_\odot$ and radius 10 km orbits an O-star at a distance of $1.2 \times 10^{12}$ cm. The period is 2.08 days. The O-star has a mass loss rate $10^{-5} \, M_\odot \, \text{yr}^{-1}$ and wind velocity 2000 km s$^{-1}$. Estimate
   
   (a) the mass of the O star, assuming the orbit to be circular.
   
   (b) the accretion rate of the neutron star
   
   (c) the luminosity of the X-ray source. (Do NOT assume a given efficiency!)

2. The inner regions of an accreting disk has usually a high enough temperature that the opacity is dominated by electron scattering and the pressure by radiation pressure. Therefore, $\kappa = \kappa_T = \sigma_T/m_p = 0.4 \, \text{cm}^2\text{g}^{-1}$ is the Thompson opacity, and $p = \sigma T^4/3c$.

   (a) Show from the disk equations that the thickness of the disk is given by
   
   \[ H = \frac{3\dot{m}\sigma_T}{8\pi cm_p} [1 - \left(\frac{r_s}{r}\right)^{1/2}] . \]

   (b) By writing the accretion rate in terms of the total luminosity of the disk show that
   
   \[ \frac{H}{r_s} = \frac{3L}{L_{Edd}} [1 - \left(\frac{r_s}{r}\right)^{1/2}] , \]

   where $L_{Edd}$ is the Eddington luminosity. A disk radiating close to the Eddington luminosity can therefore in the inner parts no longer be considered as thin.

3. An active galaxy has a central black hole mass of $10^7 \, M_\odot$ and accretes in a disk at a rate of $0.1 \, M_\odot \, \text{yr}^{-1}$.

   (a) Assume that the black hole is non-rotating. What is the radius of the last stable orbit?

   (b) Suppose the disk extends to this radius and that no energy is released inside this radius. What is the total luminosity of the disk?

   (c) What fraction of the Eddington luminosity of the black hole is this?
(d) Estimate the maximum temperature of the disk, assuming it radiates like a black-body.

4. Consider the synchrotron emitting lobes of the radio galaxy Cyg A. These have a size of \( R \approx 50 \text{ kpc} = 1.5 \times 10^{23} \text{ cm} \). The total luminosity is \( L_\nu \approx 8 \times 10^{35} \text{ erg/s/Hz at 178 MHz} \). Calculate the minimum magnetic field and total energy.

5. A supernova remnant has a radius of 4.1 pc and an age 400 years. The density of the interstellar medium in this direction is 1 cm\(^{-3}\).

(a) Estimate the total energy in the explosion.

(b) Calculate the temperature behind the shock.

(c) Assume that the magnetic energy is 10 % of the thermal energy behind the shock. What is the magnetic field?

(d) Calculate the frequency where synchrotron losses cause a break in the spectrum.

6. A cluster of galaxies has an X-ray luminosity of \( 10^{43} \text{ erg s}^{-1} \) and a gas temperature of \( 10^8 \text{ K} \). The radius of the X-ray emitting region is 0.2 Mpc. Estimate the density and mass of the hot gas in the cluster, assuming that the density and temperature are roughly constant.

7. Consider a cluster of galaxies. The magnetic field is assumed to be \( 10^{-6} \text{ G} (10^{-10} \text{ T}) \). The non-thermal radio emission is assumed to be synchrotron emission. For many clusters there is also a non-thermal component observed with energy higher than the thermal X-rays. In some models this is assumed to be inverse Compton scattering of the Cosmic Microwave Background, with temperature 2.73 K, by the non-thermal electrons.

(a) Calculate the Lorentz factor needed to explain the emission at 50 keV.

(b) Approximately at what radio frequency do these electrons emit synchrotron radiation?

\[
1 \text{ pc} = 3.09 \times 10^{18} \text{ cm}, \ 1 \text{ erg} = 10^{-7} \text{ W}, \ \kappa_T = 0.4 \text{ cm}^2 \text{ g}^{-1}
\]