Exam 2006 - Aswers

① Ⓐ To get the same flow pattern, you need the same Reynolds number

$$\mathcal{R} = \frac{LV}{\nu}$$

So, if L is reduced 100 times, V needs to be increased 100 times.
So, the necessary velocity is 90 000 km/h. At room temperature this is a supersonic velocity ($c_s = 344$ m/s = 1230 km/h), so the experiment will fail since a shock wave will form.

Possible solutions:

① increase c_s by increasing the temperature (probably inpractical).

② reduce ν by either increasing the density ($\nu = \eta/\rho$) or reducing the temperature ($\eta \propto \sqrt{T}$), or by using another medium.
Probably also here a factor 100 is difficult to achieve

Ⓑ $\mathcal{R} = \frac{LV}{\nu}$

$\nu_{air} = 0.145 \ cm^2 s^{-1}$ (from book)

$\rightarrow \mathcal{R} = 2.9 \times 10^6$

$\mathcal{R} \gtrsim 3000 \rightarrow$ turbulence

The dissipation length is approximately

$$\frac{l_d}{L} \sim \mathcal{R}^{-3/4} \xrightarrow[L=5\,m]{} l_d \sim 72\ \mu m$$

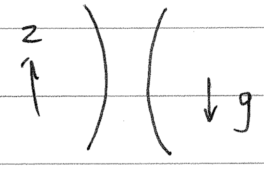
(c) If the collision time is longer, the particles can travel further between collisions. So, the fact that there are temperature and velocity differences in a flow is communicated to a larger region (within a given time), so the thermal conduction and viscosity effects are larger

(2) See exercise (11)

(3) (A) See Sect. 9.2 in Clarke & Carswell

(B) continuity eq: $\rho u A = \dot{m}$ (constant)

momentum eq: $u \cdot D u = -\frac{1}{\rho}\vec{\nabla} p - g$



$$-\frac{1}{\rho}\vec{\nabla} p = -\frac{1}{\rho}\vec{\nabla}\rho \frac{dp}{d\rho} = -\frac{c_s^2}{\rho}\vec{\nabla}\rho$$

for z: $\displaystyle -\frac{c_s^2}{\rho}\frac{d\rho}{dz}$ and $\displaystyle u \cdot \vec{\nabla} u = u\frac{du}{dz}$

Also $\rho u A = \dot{m} \rightarrow \ln\rho + \ln u + \ln A = \ln\dot{m}$

$$\frac{d\ln\rho}{dz} + \frac{d\ln u}{dz} + \frac{d\ln A}{dz} = 0$$

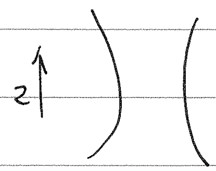
$$u \cdot \frac{du}{dz} = c_s^2 \left(\frac{1}{u}\frac{du}{dz} + \frac{1}{A}\frac{dA}{dz} \right) - g$$

$$(u^2 - c_s^2)\frac{1}{u}\frac{du}{dz} = \frac{c_s^2}{A}\frac{dA}{dz} - g$$

$$(M^2 - 1)\frac{1}{u}\frac{du}{dz} = \frac{1}{A}\frac{dA}{dz} - \frac{g}{c_s^2} \rightarrow (1 - M^2)\frac{1}{u}\frac{du}{dz} = -\frac{1}{A}\frac{dA}{dz} + \frac{g}{c_s^2}$$

ⓒ The sonic point has $M = 1$, so here

$$-\frac{1}{A}\frac{dA}{dz} + \frac{g}{c_s^2} = 0 \Rightarrow \frac{d \ln A}{dz} = \frac{g}{c_s^2} > 0$$



$\frac{d \ln A}{dz} > 0 \rightarrow$ above the narrowes point.

④ Ⓐ $\rho_0 u_0^2 + p_0 = \rho_1 u_1^2 + p_1$
in the frame of the shock

$$u_0 = -V_b$$

$$u_1 = \frac{\gamma - 1}{\gamma + 1} u_0 = -\frac{\gamma - 1}{\gamma + 1} V_b \qquad \rho_1 = \frac{\gamma + 1}{\gamma - 1}$$

$$p_0 \simeq 0$$

$$p_1 = \rho_0 u_0^2 - \rho_1 u_1^2 = \rho_0 V_b^2 - \left(\frac{\gamma - 1}{\gamma + 1}\right)\rho_0 \left(\frac{\gamma + 1}{\gamma - 1}\right)^2 V_b^2$$

$$= \frac{2}{\gamma + 1}\rho_0 V_b^2$$

Ⓑ $M_1^2 = \frac{u_1'^2}{c_{s,1}^2}$ $\qquad u_1' = u_1 + V_b$

$$= -\frac{\gamma - 1}{\gamma + 1} V_b + V_b = \frac{2}{\gamma + 1} V_b$$

$$c_{s,1}^2 = \frac{\gamma p_1}{\rho_1} = \gamma \cdot \frac{2}{\gamma+1}\rho_0 V_b^2 \cdot \frac{\gamma-1}{\gamma+1}\frac{1}{\rho_0} = \frac{2\gamma(\gamma-1)\,V_b^2}{(\gamma+1)^2}$$

$$\Rightarrow \mathcal{M}_1^2 = \frac{4}{(\gamma+1)^2} V_b^2 \cdot \frac{(\gamma+1)^2}{2\gamma(\gamma-1)V_b^2} = \frac{2}{\gamma(\gamma-1)}$$

For $\gamma = 5/3$, $\mathcal{M}_1 = 1.34 \rightarrow$ post-shock gas moves supersonically in the frame of the ISM & the cloud.

(c) The cloud surface is a contact discontinuity. Its origin is the initial discontinuity between ISM and cloud.

(D) Just as for u_1 and p_1 we can find

$$p_4 = \frac{2}{\gamma+1} \rho_c V_c^2$$

$$u_4 = \frac{2}{\gamma+1} V_c$$

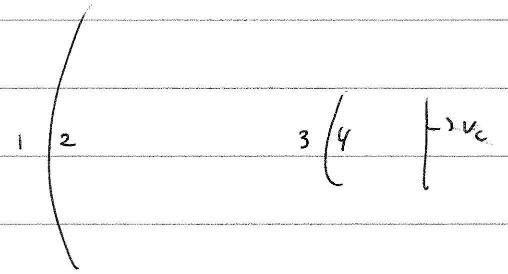
$$\mathcal{M}_1^* = \frac{u_1 - u_4}{c_{s,1}} = \frac{\frac{2}{\gamma+1}V_b - \frac{2}{\gamma+1}V_c}{c_{s,1}} = \frac{\frac{2}{\gamma+1}V_b}{c_{s,1}}(1 - V_c/V_b)$$

$$= \mathcal{M}_1 (1 - V_c/V_b) = \mathcal{M}_1(1-x)$$

$x > 1$: $V_c > V_b$, impossible

$x < 0$: $V_c < 0$, impossible

E



Between 1 and 2: shock jump conditions
(in frame of cloud $V_{sh}(1,2)=0$)
3 and 4: contact disc. ($p_3 = p_4$)

2 and 3: Bernoulli

For completeness:

$$2\to3 \quad \frac{1}{2}u_2^2 + \frac{\gamma P_2/\rho_2}{\gamma-1} = \frac{1}{2}u_3^2 + \frac{\gamma P_3/\rho_3}{\gamma-1} = \frac{\gamma P_3/\rho_3}{\gamma-1} \quad \text{(Bernoulli)}$$

$$u_2^2 = \mathcal{M}_2^{*2}\,\gamma P_2/\rho_2$$

$$\left(\frac{1}{2}\mathcal{M}_2^{*2}\gamma + \frac{\gamma}{\gamma-1}\right)P_2/\rho_2 = \gamma P_3/\rho_3$$

Between 2 and 3 entropy is constant (no shocks)

$$\to P/\rho \propto p^{(\gamma-1)/\gamma}$$

$$\left(\frac{1}{2}\mathcal{M}_2^{*2}\gamma + \frac{\gamma}{\gamma-1}\right)P_2^{(\gamma-1)/\gamma} = \gamma\frac{P_3^{(\gamma-1)/\gamma}}{\gamma-1}$$

$$1\to2 \quad P_2 = \frac{2\gamma\mathcal{M}_1^{*2} - (\gamma-1)}{(\gamma+1)}P_1 \quad \text{(shock jump)}$$

$$(1\to2) \text{ into } (2\to3): \left(\frac{1}{2}\mathcal{M}_2^{*2} + \frac{1}{\gamma-1}\right)\left(\frac{2\gamma\mathcal{M}_1^{*2} - (\gamma-1)}{(\gamma+1)}\right)^{(\gamma-1)/\gamma} P_1^{(\gamma-1)/\gamma} = P_3^{(\gamma-1)/\gamma}$$

$$\to \frac{P_3}{P_1} = \left(\frac{1}{2}(\gamma-1)\mathcal{M}_2^{*2} + 1\right)^{\gamma/(\gamma-1)}\left(\frac{2\gamma\mathcal{M}_1^{*2} - (\gamma-1)}{(\gamma+1)}\right)$$

$$\mathcal{M}_2^{*2} = \frac{2 + (\gamma-1)\mathcal{M}_1^{*2}}{2\gamma \mathcal{M}_1^2 - (\gamma-1)} \qquad \text{(e.g. Eq. (7.38) in C\&C)}$$

3→4: $p_4 = p_3 \;\rightarrow\; \frac{p_4}{p_1} = \frac{p_3}{p_1}$

$$\frac{p_4}{p_1} = \left(\frac{1}{2}(\gamma-1)\,\frac{2 + (\gamma-1)\mathcal{M}_1^{*2}}{2\gamma\mathcal{M}_1^2 - (\gamma-1)} + 1\right)^{\gamma/(\gamma-1)} \left(\frac{2\gamma\mathcal{M}_1^{*2} - (\gamma-1)}{(\gamma+1)}\right)$$

which can be rewritten in the form (6)

(F) $\frac{p_4}{p_1} = \frac{\frac{2}{\gamma+1}\rho_c v_c^2}{\frac{2}{\gamma+1}\rho_0 v_b^2} = \frac{\rho_c v_c^2}{\rho_0 v_b^2} \equiv F$ (→ given by $\mathcal{M}_1^*$)

$$\rightarrow v_c = F \frac{v_b}{\sqrt{\rho_c/\rho_0}}$$

$$\mathcal{M}_1^{*2} = \frac{2}{\gamma(\gamma-1)}(1-x)^2 \simeq 1.8(1-x)^2$$

$x \rightarrow 0 \quad \mathcal{M}_1^{*2} = 1.8$

$x \rightarrow 1 \quad \mathcal{M}_1^{*2} = 0$

$\rightarrow \frac{p_4}{p_1} = 3.15 - 0$

$x=1 \qquad x=0$

(G) K-H as material flows past the cloud. Since the shock speed is ~constant there is not much change for R-T.

(5) See exercise (14)