Astrophysical Gasdynamics Notes:
2. More about shocks

November 26, 2007

1 Introduction

Shocks are treated in Chapter 7 of Clarke & Carswell. This document contains some further material about fluid shocks and discontinuities.

2 Shock jump conditions

There are various ways to write the shock jump conditions. The most common way (not given in the book) is to express it in terms of the shock's Mach number $\mathcal{M}$. This $\mathcal{M}$ corresponds to the $\mathcal{M}_1$ in the book, that is, it is given by u_1/c_1 where u_1 is the pre-shock velocity, measured in the shock frame.
The Rankine-Hugoniot conditions can then be manipulated into a form

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)\mathcal{M}^2}{2+(\gamma-1)\mathcal{M}^2} = \frac{u_1}{u_2} \tag{1}$$

$$\frac{p_2}{p_1} = \frac{2\gamma\mathcal{M}^2-(\gamma-1)}{\gamma+1} \tag{2}$$

referred to as the shock jump conditions. Since the sole parameter determining the magnitude of the jump is $\mathcal{M}$, this number is often used to express the strength of the shock, for example as in "a Mach 10 shock".

3 Refraction across shocks

The velocity component perpendicular to the shock is reduced across the shock front. The velocity components parallel to a shock do not change across the shock. This means that a shock in the x-direction changes the flow direction if there are y and z components to the velocity vector. See Fig. 1.

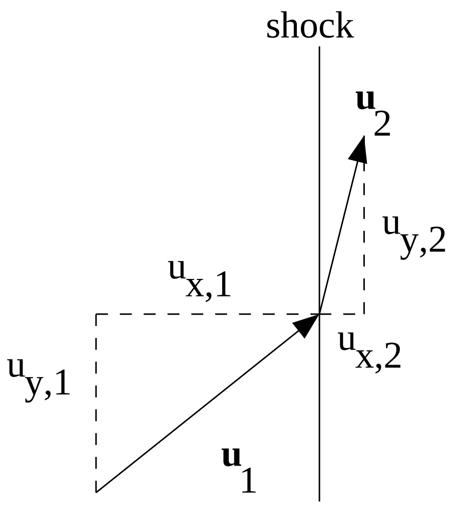

Figure 1: A shock refracting the velocity vector.

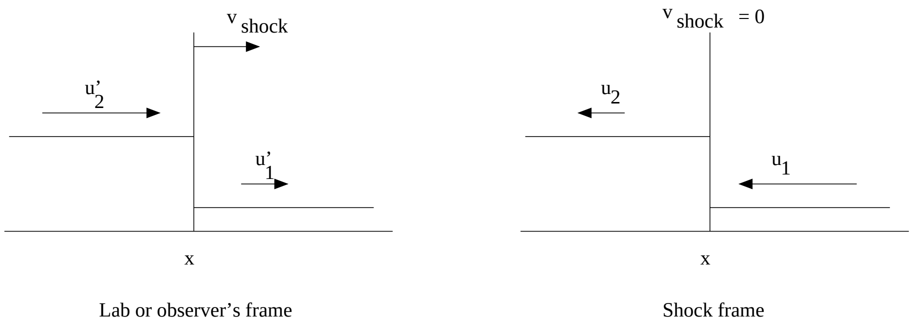

Figure 2: The relation between the observer's and shock frames of reference.

4 Frames of reference

The shock jump conditions were derived for the shock frame, i.e. the frame of reference in which the shock does not move. This gives the easiest shock jump conditions. However, one often needs to calculate results in another reference frame, for example that of a star. This is then called the observer's or lab frame. In this frame the shock will be moving. Let's call the shock velocity in the lab frame $v_{\rm sh}$, the pre- and post-shock velocities in the lab frame u_1' and u_2', and the pre- and post-shock velocities in the shock frame u_1 and u_2. Then obviously, $u_{1,2} = u_{1,2}' - v_{\rm sh}$. See Fig. 2.
Since the shock jump conditions can be conveniently written in terms of the pre-shock Mach number in the shock frame $\mathcal{M}$, it is also good to realize that $\mathcal{M} = (u_1' - v_{\rm sh})/c_1$.

5 Contact discontinuities

Looking back at the relations that let to the Rankine-Hugoniot conditions (eqs. (7.3), (7.6), (7.8) in Calarke & Carswell) one can see that they allow another, seemingly trivial solution:

$$u_1 = u_2 = 0 \tag{3}$$

$$p_1 = p_2 \tag{4}$$

and *no* condition on the densities. This solution does correspond to a physical phenomenon, one called a ***contact discontinuity***: a surface without pressure or velocity differences, but *with* a density jump. A contact discontinuity never forms spontaneously, but always originates from an initial discontinuity. Since the pressure is the same on either side, but the density is not, contact discontinuities separate areas of different entropy. Because of this they are also referred to as entropy waves.

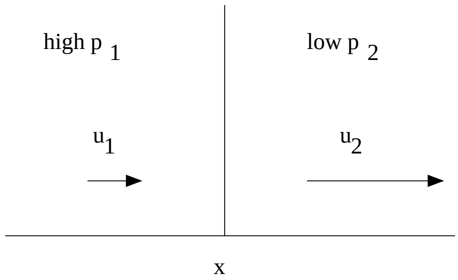

Figure 3: The impossible expansion shock.

6 Expansion waves

Shock waves can be said to be compression waves: material gets compressed in a shock. Contact discontinuities are entropy waves. The third kind of wave is the so-called ***expansion wave***, which is basically the reverse of a shock wave: material streams in with a low velocity and high pressure, and leaves with a high velocity and low pressure. Mathematically, expansion waves could also be discontinuities. However, this would mean that in such an expansion shock internal energy would be converted into bulk kinetic energy, or equivalently, that the entropy is lowered (see Fig. 3). This goes against the 2nd law of thermodynamics.

So expansion waves are not discontinuities, but smooth transition waves, where both ρ and p change to conserve entropy. Expansion waves are also known as rarefaction waves, as the density is lowered in them.

Expansion waves for example occur when you pull out a piston, and the gas has to adjust to the new larger volume (Fig. 4).

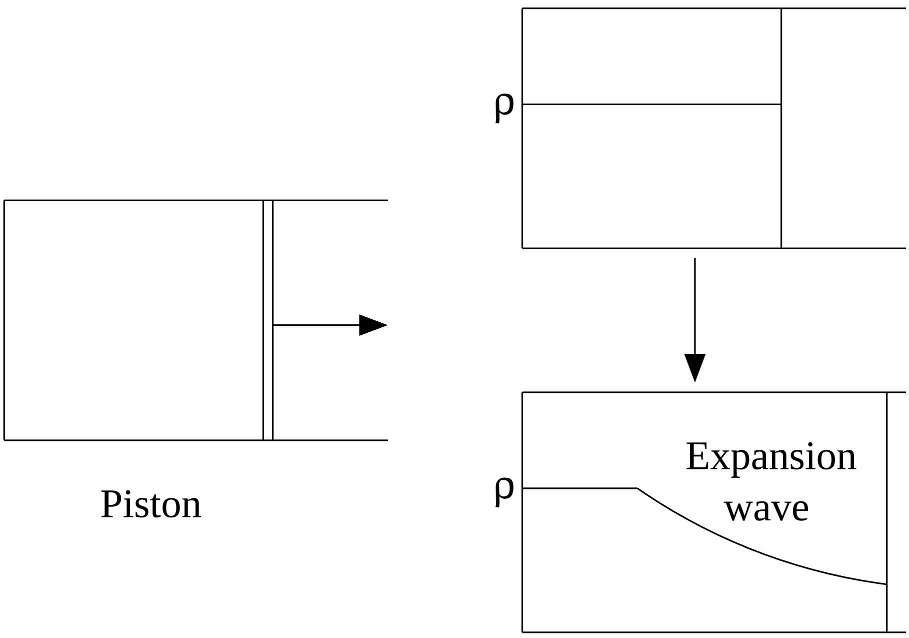

Figure 4: An example of an expansion wave.