Astrophysical Gasdynamics: Exercises II

November 23, 2007

8 Shock velocity and post-shock velocity

A shock whose strength is characterized by a Mach number $\mathcal{M}$=3 is moving into a medium of density $\rho_1 = 5/3$, velocity $u_1 = 10$, pressure $p_1 = 1$.
a) Find the shock velocity $v_{\rm sh}$ and the post-shock conditions ρ_2, u_2, p_2. Use $\gamma = 5/3$. Hint: think about the direction (sign of the velocity) as well.
b) If an observer at a distance D measures the shock speed as a change in angle $\Delta\theta$ in a time Δt, and the post-shock velocity u_2 through some other means (e.g. spectroscopy), and derives the distance D by equating the two velocities, how big a mistake does he make? (Assume that the distance D is large enough to approximate $\Delta\theta = \Delta x/D$ instead of $\arctan(\Delta x/D)$.
This last result is relevant for measurements of the distance to Planetary Nebulae. High resolution imaging with HST allows us to measure the angular expansion of the nebula over time. Spectroscopy allows us to measure the velocity of the gas. Equating the two velocities gives us a distance. But as seen above, if the nebular shell is a shock front, the two velocities may differ, and the distance found be wrong.

9 Rankine-Hugoniot Relations

Show that the shock jump conditions can also be written as

$$\mathbf{F}_2 - \mathbf{F}_1 = v_{\rm sh}(\mathbf{W}_2 - \mathbf{W}_1) \quad (1)$$

where $\mathbf{W} = (\rho, \rho u, e)^T$ is the (1D) state (of conserved variables), and $\mathbf{F} = (\rho u, \rho u^2 + p, (e+p)u)^T$ is the corresponding flux.
This is an example of the symmetry that can be found in the shock jump conditions, and is an expression that is used in computational fluid dynamics.

10 Piston Shock

Consider an infinitely long cylinder filled with a polytropic gas of density ρ_1 and pressure p_1. Into this cylinder a a piston is driven. The piston has a velocity U.
a) Argue that if $U^2 > \gamma p_1/\rho_1$ a shock wave will form.

b) Find the speed of this shock wave.
The answer is:

$$v_{\rm sh} = (\gamma+1)U/4 + \sqrt{(\gamma+1)^2U^2/16 + \gamma p_1/\rho_1} \tag{2}$$

Hint: Sketch the flow sitation in the pipe; there will be two regions, separated by the shock. You know the state of the unshocked gas, and you know the velocity of the shocked gas (U, why?). From this solve for the shock conditions. The solution will give you the shock velocity.

11 Double shocks

A shock whose strength is characterized by Mach number 4 travels into a medium of (number) density $n_1 = 1$ cm^{-3}, temperature $T_1 = 100$ K and velocity $u_1 = 0$ km s^{-1}. Assume the medium to consist of neutral hydrogen and take the adiabatic index to be $\gamma = 5/3$.

a) What is the shock velocity of this shock, and what are the post-shock density, velocity and temperature?

This shock is followed by a second shock, this one characterized by a Mach number of 2 (see Fig. 1).

b) Show that the shock velocity of this second shock is higher than that of the first shock, and also here derive the post-shock density, velocity and temperature.

It can be shown that a second shock is *always* faster than the first one, or in other words, the second shock will always catch up with the first one. Here you are asked to prove this for two *strong* shocks for which both Mach numbers can be taken to be much larger than the other terms in the shock jump conditions.

c) What are the shock jump conditions for a strong shock?

d) Show that for two subsequent strong shocks, the second one always moves faster than the first one.
This result is relevant in many astrophysical systems, for example those that have a time variable outflow or wind. Each new disturbance will tend to catch up with the previous ones.

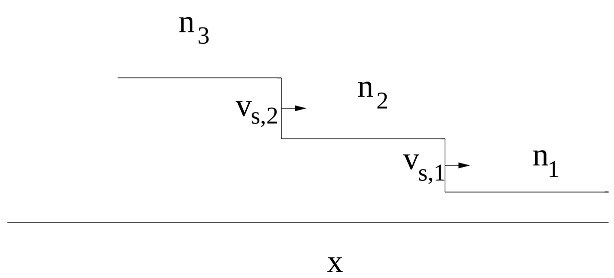

Figure 1: Two shocks travelling with speeds $v_{\rm sh,1}$ and $v_{\rm sh,2}$ through a medium with original density n_1.