Stockholms Observatorium
Institutionen för Astronomi

Tentamen i Kosmologi

Tisdagen den 18 december, kl 9-14


Maxpoängen är 24, gränsen för G är 12p, gränsen för VG är 18p.
Lycka till! / Edvard

1) Galaxy rotation curves are observed to be close to flat (see Fig. 1).
   (a) Show how one can compute the density profile using this information (you may assume \( \rho(r) \propto r^\alpha \)). What is the mass of the spiral galaxy NGC1097 within a radius of \( \sim 30 \text{ kpc} \) (corresponding to an angle of \( \sim 6 \text{ arcminutes} \))? (3p)
   (b) This tells us something important about the matter content of galaxies. Discuss this. (3p)

2) (a) According to recent observations (e.g., the Cosmic Background Radiation and Type Ia supernovae magnitudes), approximately how large is the fraction of the total energy density of the Universe in dark matter \( \rho_M \), radiation \( \rho_R \) and the cosmological constant \( \rho_\Lambda \)? (2p)
   (b) How does these different energy densities scale with redshift? (2p)
   (c) Compute at what approximate redshift interval each of these energy components dominate the Universe. (2p)

3) Study Fig. 2 showing the geometry of a gravitational lens system. The deflection angle is given by \( \alpha = \frac{4GM}{c^2} \).
   (a) Derive the lens equation, i.e., an expression for \( \beta \), the angle one would see the source at in absence of lensing in terms of the lens mass \( M \), \( \theta \) and the distances involved. (2p)
   (b) Calculate the Einstein angle, \( \theta_E \), corresponding to the case when the observer, lens and source are perfectly aligned (i.e., \( \beta = 0 \)). (2p)
(c) Study Fig. 3. The radius of the ring is \( \sim 0.47 \) arcseconds. The angular diameter distance to the lens is \( \sim 1.5 \) Gpc, the distance to the source is \( \sim 1.7 \) Gpc and the distance between the lens and source is \( \sim 600 \) Mpc. Estimate the mass of the lensing galaxy. \( \text{(2p)} \)

4) What is inflation? Why do we think we need it? Approximately when did it occur? \( \text{(3p)} \)

5) Einstein introduced the cosmological constant in order to obtain static solutions for the field equations since the Universe was believed to be static and eternal at the time. Show that in a static Universe, we must have \( \rho_\Lambda = \rho_m/2 \). Is it an open, flat or closed Universe? \( \text{(3p)} \)

![Figure 1: Rotation curve made of radio observations of the Doppler shifts in CO line emission and the HI (21 cm) line in the spiral galaxy NGC1097. Courtesy of Y. Sofue, Institute of Astronomy, Tokyo.](image)
Figure 2: The geometry of gravitational lensing. The direction of the source in the absence of lensing is specified by the angle $\beta$, the deflection angle is $\alpha$ and the observed direction is $\theta$. The distance to the source is $D_s$, the distance to the lens is $D_d$, and the distance between the lens and the source is $D_{ds}$. The ray passes the lens at a transverse distance $\xi$.

Figure 3: The Hubble Space Telescope picture of the distant galaxy 1938+666 which has been imaged into an Einstein ring by an intervening galaxy. The intervening galaxy shows up as the bright spot in the centre of the ring. Credit: L.J. King, U. Manchester.