Stockholms Observatorium
Institutionen för Astronomi

Tentamen i Kosmologi

Tisdagen den 21 december, 2004, kl 9-14

Tillåtna hjälpmedel: Physics Handbook,
s. 210-211 i Raine/Thomas (medföljer), miniräknare.

Maxpoängen är 24, gränsen för G är 12p, gränsen för VG är 18p.
Lycka till! / Edvard

1) The Robertson-Walker metric is given by

\[ ds^2 = c^2 dt^2 - R(t)^2 \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right], \]

with \( k = 0, 1, -1. \)

(a) What does it describe? (2p)
(b) What is the area of a sphere at \( r = r_e \)? What is the corresponding radius? (2p)
(c) Show why it is possible to have only three discrete values of \( k \). (2p)

2) Study Fig. 1 showing the geometry of a gravitational lens system. The deflection angle is given by \( \alpha = \frac{4GM}{c^2} \), where \( M \) is the mass of the lens.

(a) Derive the lens equation, i.e., an expression for \( \beta \), the angle one would see the source at in absence of lensing in terms of the lens mass \( M \), \( \theta \) and the distances involved. (2p)
(b) Calculate the Einstein angle, \( \theta_E \), corresponding to the case when the observer, lens and source are perfectly aligned (i.e., \( \beta = 0 \). (2p)
(c) Study the attached IR colour composite of the galaxy cluster CL2244-02. The radius of the Einstein ring is \( \sim 12 \) arcseconds. The angular diameter distance to the lens is \( \sim 0.8 \) Gpc, the distance to the source is \( \sim 1.6 \) Gpc and the distance between the lens and source is \( \sim 1.1 \) Gpc. Estimate the mass of the lensing galaxy. (2p)
3) For a model with a general equation of state \( p = \omega \rho c^2 \), use the equation of local energy conservation to show that \( \rho \propto R^{-3(1+\omega)} \). How does non-relativistic matter, \( \rho_m \), evolve with \( R \)? (This is Problem 39 in Raine/Thomas.) (3p)

4) The mean free path of a photon in the present Universe depends on the density and state of ionization of the intergalactic medium. Estimate a lower bound on the mean free path by taking all the baryonic matter in the present Universe to be uniformly distributed and in fully ionized form. (This is Problem 55 in Raine/Thomas.) (3p)

5) What is the Cosmic Microwave Background (CMB)? What can we learn from it? (3p)

6) What is inflation? Why do we think we need it? Approximately when did it occur? (3p)

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Figure 1: The geometry of gravitational lensing. The direction of the source in the absence of lensing is specified by the angle \( \beta \), the deflection angle is \( \alpha \) and the observed direction is \( \theta \). The angular distance to the source is \( D_s \), the angular distance to the lens is \( D_d \), and the angular distance between the lens and the source is \( D_{ds} \). The ray passes the lens at a transverse distance \( \xi \).