1) The Robertson-Walker metric is given by

\[ ds^2 = -c^2 dt^2 + a(t)^2 \left[ dr^2 + S_\kappa(r)^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right], \]

with

\[ S_\kappa(r) = r, R \sin(r/R), R \sinh(r/R). \]

for \( \kappa = 0, 1, -1 \) respectively.

(a) What does it describe? (2p)

(b) What is the area, \( A(t) \), of a sphere at \( r = r_c \)? What is the corresponding radius, \( d_p(t) \) (in terms of the proper distance)? (2p)

2) (a) According to recent observations (e.g., the Cosmic Microwave Background and Type Ia supernovae magnitudes), approximately how large is the fraction of the total energy density of the Universe in matter (\( \rho_m,0 \)), radiation (\( \rho_r,0 \)) and the cosmological constant (\( \rho_\Lambda,0 \))? (2p)

(b) How does these different energy densities scale with redshift? (2p)

(c) Compute at what approximate redshift interval each of these energy components dominate the Universe. (2p)

3) The Royal Swedish Academy of Sciences has decided to award the Nobel Prize in Physics for 2006 jointly to John Mather and George Smoot “for their discovery of the blackbody form and anisotropy of the cosmic microwave background radiation”. What can we learn about the universe from these two discoveries? (4p)
4) Galaxy rotation curves are observed to be close to flat (see Fig. 1).

(a) Show how one can compute the density profile using this information (you may assume $\rho(r) \propto r^\alpha$ and $v_{\text{rot}} = \text{constant}$). What is the mass of the spiral galaxy NGC1097 within a radius of $\sim 30$ kpc (corresponding to an angle of $\sim 6$ arcminutes)? (2p)

(b) This tells us something important about the matter content of galaxies. Discuss this. (2p)

(c) Show why the cosmological constant is not important for the dynamics in galaxies. (2p)

5) Consider Einstein’s static universe, in which the attractive force of the matter density is exactly balanced by the repulsive force of the cosmological constant, $\Lambda = 4\pi G\rho$. Suppose that some of the matter is converted into radiation (by stars, for instance). Will the universe start to expand or contract? Explain your answer. (This is Problem 4.2 in Ryden.) (4p)

Figure 1: Rotation curve made of radio observations of the Doppler shifts in CO line emission and the HI (21 cm) line in the spiral galaxy NGC1097. Courtesy of Y. Sofue, Institute of Astronomy, Tokyo.