Introduction and Fundamental Observations

Prelude

Cosmology is the study of the universe taken as a whole ⇒ ruthless simplification necessary (e.g. homogeneity)!

Cosmology describes the past, the present, and predicts the future of the universe.

Cosmology deals with very large masses and length scales, thus, cosmologists use units of megaparsecs (Mpc) and solar masses (M⊙):

1 Mpc = 10^6 pc = 3.1 \times 10^{22} \text{ m} \\
(1 \text{ pc is the distance at which 1 AU subtends 1 arcsec.})

1 M⊙ = 2.0 \times 10^{30} \text{ kg}

Cosmology also deals with very small things. For instance, we measure energy in electron volts (eV) 1 eV = 1.6 \times 10^{-19} \text{ J} \quad (m_e c^2 = 511000 \text{ eV} = 0.511 \text{ MeV}, \quad \text{and } m_P c^2 = 938.3 \text{ MeV}).

Since cosmology is ultimately based on observations, we review some of the observations on which modern cosmology is based.

The night sky is dark.

The observation that the night sky is dark is called “Olbers’ paradox” (after the 19th century astronomer Heinrich Wilhelm Olbers).

Let \( n \) be the average number density of stars, and let \( L \) be the average stellar luminosity. The average flux received from a single star at distance \( r \) will then be:

\[
f(r) = \frac{L}{4\pi r^2}.
\]

Now consider a spherical shell of stars, of radius \( r \) and thickness \( dr \), centered on the Earth. The intensity (power per unit area per steridian) is

\[
dJ(r) = \frac{L}{4\pi r^2} \cdot nr^2 dr = \frac{nL}{4\pi} dr,
\]
and the total intensity received at Earth:

\[ J = \int_0^\infty dJ = \frac{nL}{4\pi} \int_0^\infty dr = \infty, \]

⇒ the sky should be infinitely bright.

Assumption # 1: Every star in the universe is visible. Nevertheless, if every line of sight ends at the surface of a star, the average surface brightness of the sky should be equal to the surface brightness of a typical star.

Assumption # 2: The number density and luminosity of stars are constant (or at least \( nL = \text{constant} \)).

Assumption # 3: The universe is infinitely large (\( r \to \infty \)).

Assumption # 4: The universe is infinitely old (\( t \to -\infty \)). (The speed of light is finite.)

Solution: The universe is not infinitely old.

On large scales, the universe is isotropic and homogeneous.

Saying the universe is isotropic means that there are no preferred directions in the universe. Saying the universe is homogeneous means that there are no preferred locations in the universe.

Note that homogeneity does not imply isotropy, and that isotropy about a single point does not imply homogeneity. The cosmological principle states, “There is nothing special about our location in the universe.” This greatly simplifies our calculations (and seems to be true)!

(The perfect cosmological principle states that all locations in time are pretty much the same, as well but we know that the universe is evolving.)

It isn’t until you get to 100 Mpc scales that the universe is isotropic and homogeneous. For instance, a sphere 1 meter in diameter, centered on your navel, will have an average density of \( \sim 100 \text{ kg m}^{-3} \). The average density of the universe is \( \rho_0 \sim 3 \times 10^{-27} \text{ kg m}^{-3} \).
Galaxies show a redshift proportional to their distance.

Consider a particular absorption (or emission) line with wavelength (as measured in a laboratory) $\lambda_{em}$. The wavelength we observe for the same line in a galaxy’s spectrum, $\lambda_{obs}$, will not, in general, be the same. We say that the galaxy has a redshift

$$z \equiv \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}}.$$

(If $z < 0$, we call this quantity a blueshift.)

If the redshift is due to the radial velocity $v$ of the distant galaxy, then the Doppler shift is

$$z \approx \frac{v}{c},$$

in the limit $v \ll c$. In 1925, Slipher had measured the shift in wavelength for the light from $\sim 40$ galaxies, of which nearly all were redshifted $\Rightarrow$ galaxies are preferentially moving away from us.

Measuring redshifts is easy; determining distances is hard. In 1929, Hubble had distances and $z$ for 20 galaxies. From a plot of redshift $z$ versus distance $r$, Hubble found the famous relation now called Hubble’s Law:

$$z = \frac{H_0}{c}r,$$

where $H_0$ is the Hubble constant. The Hubble constant has units of velocity over distance, or inverse time. Hubble’s Law can also be written in the form

$$v = H_0r,$$

in the limit $v \ll c$.

Hubble: $H_0 \sim 500 \text{ km s}^{-1} \text{ Mpc}^{-1}$
Hubble Space Telescope: $H_0 = 70 \pm 7 \text{ km s}^{-1} \text{ Mpc}^{-1}$
($H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$ where $h$ is the dimensionless Hubble constant.

What we see is exactly what you would expect if the universe were expanding uniformly in all directions (observers in any other galaxy would also see distant galaxies moving away from them).
Consider a pair of galaxies separated by a distance $r$, with a relative velocity $v = H_0 r$. If there are no forces acting to accelerate or decelerate the expansion, the time that has elapsed since the galaxies were in contact is

$$t_0 = \frac{r}{v} = \frac{r}{H_0 r} = H_0^{-1}.$$ 

The time $H_0^{-1}$ is referred to as the “Hubble time”. For $H_0 = 70 \text{ s}^{-1} \text{ Mpc}^{-1}$, the Hubble time is $H_0^{-1} = 14 \text{ Gyr}$. ⇒ The Big Bang scenario.

If the universe was expanding more rapidly in the past than it is now, the universe is younger than $H_0^{-1}$ (matter). If the universe was expanding more slowly in the past than it is now, it is older than $H_0^{-1}$ (cosmological constant).

Solution to Olbers’ paradox: The horizon size $d_{\text{hor}}$ is the distance traveled by light since the beginning of the universe,

$$d_{\text{hor}} \approx c/H_0 = 4300 \text{ Mpc}.$$ 

Since $nL \sim 2 \times 10^8 L_\odot \text{ Mpc}^{-3}$ (corresponding to a 40 W light bulb in a 1 AU radius sphere), integrating to $d_{\text{hor}}$ instead of infinity gives $J_{\text{gal}}/J_{\text{sun}} \sim 3 \times 10^{-10}$!

(Hubble’s Law can also be explained in a Steady State scenario, obeying the Perfect Cosmological Principle in which the global properties of the universe, such as the Hubble constant $H_0$ and the mean density $\rho_0$, remain constant with time. If $H_0$ is constant with time,

$$r(t) \propto \exp H_0 t.$$ 

In order to have a constant density of matter within a sphere that is growing in volume, matter must be continuously created, at a rate

$$\dot{M} = \rho_0 \dot{V} = \rho_0 3H_0 V.$$ 

The Steady State theory lost favor, finally, as the result of the discovery of the Cosmic Microwave Background.)

The universe contains different types of Stuff

The objects that surround us in our everyday life are made of protons, neutrons, and electrons.
Protons and neutrons are baryons (made up of three quarks) with masses $m_p c^2 = 938.3\text{ MeV}$ and $m_n c^2 = 939.6\text{ MeV}$. Electrons ($e^-$) are leptons (not made of quarks) with $m_e c^2 = 0.511\text{ MeV}$. Since the number of protons is equal to the number of electrons and protons outmass electrons by a factor of 1836 to 1, the component of the universe made up of ions, atoms, and molecules is generally referred to as baryonic matter.

The photon ($\gamma$) is massless and interact with electrons, protons, and neutrons.

Neutrinos ($\nu$, leptons) have no charge but there is some experimental evidence that neutrinos may have a small mass. Neutrinos interact via the weak nuclear force – a typical neutrino would have to pass through a few parsecs of solid lead before having a 50% chance of interacting with a lead atom.

Observational astronomers refer to any massive component too dim to be detected (stellar remnants such as white dwarfs, neutron stars, and black holes) as dark matter. A more stringent definition is any massive component of the universe which doesn’t emit, absorb, or scatter light at all. The usual way of detecting dark matter is by measuring its gravitational influence on luminous matter.

The universe is filled with a Cosmic Microwave Background

Penzias and Wilson in 1965 found an excess signal at a wavelength $\lambda = 7.4\text{ cm}$. After consulting with Bob Dicke and his research group at Princeton University, they realized they were observing a cosmic background of radiation (CMB).

The signal was consistent with a blackbody spectrum at a temperature of just 3.5 Kelvin. The COBE satellite (COsmic Background Explorer) show a blackbody spectrum with a temperature

$$T = 2.725 \pm 0.001 \text{ K}.$$
If the universe was initially very hot and dense, the CMB arises naturally. The universe would have been ionized, and thus opaque. Protons, neutrons, electrons, and photons frequently interact with each other, and hence attain thermal equilibrium ⇒ a blackbody function:

$$\varepsilon(f)df = \frac{8\pi h}{c^3} \frac{f^3 df}{\exp(hf/k_B T) - 1}.$$ 

As the universe expanded, it cooled and ions and electrons combined to form neutral atoms ⇒ the universe became transparent ⇒ blackbody photons started streaming freely through the universe ⇒ the CMB.

(In the Steady State theory, the existence of a 3K background requires a contrived explanation.)

**Summary**

The evidence cited so far – the dark night sky, the homogeneity & isotropy of the universe on large scales, the expansion of the universe, and the presence of the CMB – all can be explained within the framework of a Hot Big Bang model. The universe was initially very hot and very dense, and since then has been expanding and cooling.