Notes for Cosmology course, fall 2005

Inflation and the Very Early Universe

Prelude

Galaxies, quasars, and supernovae tell us about the relatively recent universe. The CMB tell us about the universe at the time of photon decoupling ($z_{\rm dec} \approx 1100$, $t_{\rm dec} \approx 400\,000\,{\rm yr.}$)
The observed abundances of light elements tell us about the universe at the time of Big Bang Nucleosynthesis ($z_{\rm nuc} \approx 3 \times 10^8$, $t_{\rm nuc} \approx 3\,{\rm min}$).
We have a good understanding of the universe as far back as the time of neutron-proton freezeout, at $t \approx 1\,{\rm s}$.

Problems with the Hot Big Bang scenario:
The flatness problem: "The universe is (close to) spatially flat now, and was even flatter in the past."
The horizon problem: "The universe is nearly homogeneous and isotropic, and was even more so in the past."
The monopole problem: "The universe is not dominated by magnetic monopoles."

Solution: The first second!
The universe underwent a period of *inflation*; a period when the scale factor increased exponentially for at least a hundred e-foldings (that is, a increased by a factor of at least e^{100}).

The flatness problem

The Friedmann equation:

$$1 - \Omega(t) = -\frac{\kappa c^2}{R_0^2 a(t)^2 H(t)^2} \ .$$

If $\kappa = 0$, then (at all times) $\Omega(t) = 1$ $\Rightarrow$ a perfectly flat universe is perfectly flat at all times. At the present, we know

$$|1 - \Omega_0| < 0.2 \ (0.02) \ .$$

The density could equally well be $\Omega_0 = 10^{-9}$, or $\Omega_0 = 10^9$, without violating the laws of physics. Is there any underlying principle why the energy density should be close to the critical density, or is it a coincidence? Perhaps the initial conditions of the universe just happened to have the right values to make the universe close to being spatially flat? Not bloody likely:

We can write

$$1 - \Omega(t) = \frac{H_0^2(1-\Omega_0)}{H(t)^2 a(t)^2} ,$$

and

$$1 - \Omega(a) \approx \begin{cases} (1-\Omega_0)a^2 \propto t & \text{(radiation-dominated)} \\ (1-\Omega_0)a \propto t^{2/3} & \text{(dust-dominated)} \ . \end{cases} \tag{9}$$

The difference between Ω and 1, no matter how small it was at early times, invariably *increases* with time during the radiation-dominated and dust-dominated epochs of the universe's expansion. Since $|1 - \Omega_0| < 0.2$, we must have had

$$|1 - \Omega_{\rm rm}| < 2 \times 10^{-4} ,$$

and at the time of Big Bang Nucleosynthesis

$$|1 - \Omega_{\rm nuc}| < 3 \times 10^{-14} .$$

If we push our extrapolation to the Planck time at $t_P \approx 5 \times 10^{-44}$ s, we find

$$|1 - \Omega_P| < 10^{-60} ,$$

$\Rightarrow$ for Ω_0 to be within shouting distance of one today, it had to be fanatically close to one during the early stages of the universe.

The horizon problem

The horizon distance is given by

$$d_{\rm hor}(t) = ca(t) \int_0^t \frac{dt}{a(t)} .$$

The current horizon distance is $d_{\rm hor}(t_0) \approx 3ct_0 \approx 14\,000$ Mpc. Anything further away from us at the present moment is causally disconnected from us, e.g. two antipodal points on the surface of last scattering, 180° away from each other. They haven't had time to send messages to each other, and in particular they haven't had time to come into thermal equilibrium. Nevertheless, they have the same temperature to within one part in 10^5!

Even worse, the horizon size at the time of last scattering was

$$d_{\rm hor}(t_{\rm ls}) \approx 0.4\,{\rm Mpc}\ ,$$

i.e. points more than 0.4 Mpc away from each other were totally ignorant of each other. Two points that were 0.4 Mpc away from each other at the time of last scattering, will be separated by an angle

$$\delta\theta = \frac{0.4\,{\rm Mpc}}{13\,{\rm Mpc}} = 2^\circ\ .$$

Nevertheless, we find $\delta T/T \sim 10^{-5}$ on scales much larger than 2°. We can divide up the surface of last scattering into 20,000 patches, the center of each was out of touch with the other patches at the time of last scattering but that all have $\delta T/T \sim 10^{-5}$.

The monopole problem

Currently, it is customary to talk about the four fundamental forces of nature: gravitational, electromagnetic, weak, and strong. The strong and electroweak forces should be unified in a single Grand Unified Force at a particle energy of $E_{\rm GUT} \sim 10^{15}\,{\rm GeV}$, corresponding to a temperature of $T_{\rm GUT} \sim 10^{28}\,{\rm K}$. The universe had a temperature equal to the GUT temperature when its age was $t_{\rm GUT} \sim 10^{-36}\,{\rm s}$. *(The GUT energy, you will note, is still 4 orders of magnitude smaller than the Planck energy, $E_P \sim 10^{19}\,{\rm GeV}$. The Planck energy is the energy at which the ultimate unification of forces occurs. At the Planck energy, the gravitational, strong, and electroweak forces all unite to form a single force.)*

When the temperature dropped below the GUT temperature the universe underwent a **phase transition** that ought to result in the creation of magnetic monopoles (analogous to the flaws and bubbles that are created in ice as it freezes) with mass $m_M c^2 \sim E_{\rm GUT} \sim 10^{15}\,{\rm GeV}$. The typical distance between magnetic monopoles will be comparable to the horizon size at $t_{\rm GUT} \Rightarrow$ the number density

$$n_M(t_{\rm GUT}) \sim \frac{1}{(2ct_{\rm GUT})^3} \sim 10^{82}\,{\rm m}^{-3}\ ,$$

and energy density

$$\epsilon_M(t_{\rm GUT}) = m_M c^2 n_M(t_{\rm GUT}) \sim 10^{94}\,{\rm TeV\,m^{-3}}\ .$$

$\Rightarrow$ the universe should be dominated by magnetic monopoles at $t > 10^{-16}$ s.

The Inflation Solution

Inflation is defined as an epoch when the expansion was accelerating outward, i.e. $\ddot{a} > 0$. As an example, assume that the universe was temporarily dominated by a cosmological constant:

$$\frac{\dot{a}}{a} = H_i = \sqrt{\frac{\Lambda_i}{3}}\ .$$

and

$$a(t) \propto \exp\left(\sqrt{\frac{\Lambda_i}{3}}t\right)\ .$$

A period of exponential growth during the universe's early, radiation-dominated phase can resolve the flatness, horizon, and monopole problems.

If the exponential growth was switched on at a time t_i, and lasted until t_f, we may write the scale factor as

$$a(t) = \begin{cases} a_i(t/t_i)^{1/2} & t < t_i \\ a_i e^{H_i(t-t_i)} & t_i < t < t_f \\ a_i e^{H_i(t_f-t_i)}(t/t_f)^{1/2} & t > t_f \end{cases} \tag{7}$$

Between the time t_i and t_f, the scale factor increased by a factor

$$\frac{a(t_f)}{a(t_f)} \sim e^{H_i(t_f-t_i)} = e^N\ .$$

Let's assume a model for inflation, with $t_i \approx 10^{-36}$ s and $N \sim 100$ ($t_f \sim 2\times 10^{-34}$ s).

Resolving the flatness problem: During a radiation-dominated era,

$$|1-\Omega(t)|_{\rm rad} \propto t\ ,$$

the deviation of Ω from one *grows* with time. During an inflationary era (or any Λ-dominated era),

$$|1 - \Omega(t)|_{\rm inf} \propto e^{-2H_i t} \ ,$$

the deviation of Ω from one *decreases exponentially* with time. Suppose that prior to inflation, the universe was strongly curved, with

$$|1 - \Omega(t_i)| \sim 1 \ .$$

The deviation of Ω from one immediately after inflation would then be (for $N \sim 100$)

$$|1 - \Omega(t_f)| \sim e^{-2N} \sim e^{-200} \sim 10^{-87} \ .$$

Even if the universe wasn't particularly close to being flat prior to inflation, 100 e-foldings of inflation will flatten it like a pancake!

Resolving the horizon problem: The horizon distance is

$$d_{\rm hor}(t) = a(t)c \int_0^t \frac{dt}{a(t)} \ .$$

At the beginning of inflation we have (radiation-domination) $\Rightarrow$

$$d_{\rm hor}(t_i) = 2ct_i \ .$$

The horizon distance at the end of inflation will be

$$d_{\rm hor}(t_f) = a_i e^N c \left[\int_0^{t_i} \frac{dt}{a_i (t/t_i)^{1/2}} + \int_{t_i}^{t_f} \frac{dt}{a_i \exp[H_i(t - t_i)]} \right] \ .$$

If inflation goes on for many e-foldings,

$$d_{\rm hor}(t_f) \approx e^N c [2t_i + H_i^{-1}] \ .$$

For our model inflation

$$d_{\rm hor}(t_i) \approx 2ct_i \approx 6 \times 10^{-28}\,{\rm m} \ ,$$

and immediately after inflation

$$d_{\rm hor}(t_f) \approx e^N 3ct_i \approx 2 \times 10^{16}\,{\rm m} \approx 0.8\,{\rm pc} \ .$$

During the 10^{-34} s that inflation lasts, the horizon distance is increased from submicroscopic scales up to parsecs. After the end of inflation, the horizon reverts to growing at linear rate.

Immediately after inflation, the currently visible universe was crammed into a sphere of radius

$$d_p(t_f) \sim a(t_f)d(t_0) \sim 0.9\,\mathrm{m} \ .$$

Immediately prior to inflation, the currently visible universe was enclosed within a sphere of radius

$$d_p(t_i) \sim e^{-N} d_p(t_f) \sim 3 \times 10^{-44}\,\mathrm{m} \ .$$

Thus, the currently visible universe had plenty of time to come into thermal equilibrium before inflation started.

Resolving the monopole problem: If magnetic monopoles were created before, or during, the inflationary era, then the number density of monopoles is simply diluted to an extremely tiny value. During the time when the universe is expanding exponentially ($a \propto e^{H_i t}$), the number density of monopoles is decreasing exponentially ($n_M \propto e^{-3H_i t}$). If the number density of monopoles at $t_i \sim t_{\mathrm{GUT}} \sim 10^{-36}\,\mathrm{s}$ was $n_M(t_i) \sim 10^{82}\,\mathrm{m}^{-3}$, after inflation, the number density was $n_M(t_f) \sim e^{-300} n_M(t_i) \sim 15\,\mathrm{pc}^{-3}$. The number density today, is thus $n_M(t_0) \sim 10^{-61}\,\mathrm{Mpc}^{-3}$. The probability of finding even a single monopole in the visible universe is extremely small.

The Physics of Inflation

What triggers the start of the exponential expansion at $t_i \sim 10^{-36}$ s and what turns it off at $t_f \sim 10^{-34}$ s?

Why doesn't it dilute particles, such as photons, to undetectably low densities? After inflation, you would expect to find a single photon in a box 25 AU on a side, as compared to the 400 photons per cubic centimeter in the universe today.

Why doesn't it flatten out the local curvature due to fluctuations in the energy density? Inflation makes the universe TOO homogeneous and isotropic!

Assume that we have a scalar field ϕ with potential energy density $V(\phi)$. The field ϕ contributes to the total energy density ϵ of the universe

$$\epsilon_\phi = \frac{1}{2\bar{h}c^3}\dot{\phi}^2 + V(\phi) \ .$$

An example is if ϕ is the elevation above sea-level, i.e. a kinetic and a potential term.
The pressure is given by

$$P_\phi = \frac{1}{2\bar{h}c^3}\dot{\phi}^2 - V(\phi) \ .$$

Assuming that $\dot{\phi}$ is small, we have

$$\omega_\phi \sim -1 \ ,$$

$\Rightarrow$ equivalent to a cosmological constant $\Rightarrow$ exponential expansion.

To minimize the energy, the field settles into a minimum of the potential energy $V(\phi)$, if such a minimum exists $\Rightarrow$ inflation stops.

The energy which is lost by the scalar field goes into **reheating** the universe, i.e. the reason why the universe isn't very cold and dilute today is that the energy density which drove inflation was dumped back into the universe in the form of radiation which heated the universe back up to its pre-inflation temperature. (Protons, neutrons, electrons, and so forth are then made by pair production from the highly energetic photons.)

Inflation predicts that the density fluctuations immediately after inflation would be

$$\frac{\delta\epsilon}{\bar{\epsilon}} \sim e^{-100} \sim 10^{-43} \ .$$

This would leave the CMB (and present universe) too smoothe. On submicroscopic scales, there are always quantum fluctuations in any field, i.e. on quantum scales, the universe is intrinsically inhomogeneous. Inflation takes the submicroscopic quantum fluctuations in the field ϕ and blows them up to macroscopic scales. The energy fluctuations that result are the origin of all the inhomogeneities that we see today. The Coma cluster, a huge agglomeration of galaxies and dark matter, was once a tiny quantum fluctuation.

Summary

Our ever so succesful Big Bang model has a few problems, namely the flatness problem, the horizon problem and the monopole problem. The solution to these are inflation; the early universe underwent a period when the scale factor increased exponentially (or at least $\ddot{a} > 0$) for at least a hundred e-foldings (that is, a increased by a factor of at least e^{100}).