Recapitulation

Definition:
Cosmology is the study of the structure, evolution and origin of the universe taken as a whole.

Most important fact:
The universe is expanding. The expansion is described by the scale factor $a(t)$, or equivalently the Hubble parameter $H = \dot{a}/a$. $H_0$ is the Hubble constant.

Assuming the cosmological principle holds, i.e. that (on large scales) the universe is isotropic and homogeneous, the equations that describe how the scale factor evolves with time are the Friedmann equation

$$H(t)^2 = \frac{8\pi G}{3c^2} \epsilon(t) - \frac{\kappa c^2}{R_0^2 a(t)^2},$$

and the acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2} (\epsilon + 3P).$$

Here, $\kappa = 0, +1, -1$ for zero, positive and negative curvature.

Non-relativistic particles (matter): $\epsilon_m = \epsilon_{m,0}/a^3$.

Relativistic particles (radiation): $\epsilon_r = \epsilon_{r,0}/a^4$.

A cosmological constant: $\epsilon_\Lambda = \text{const}$.

The curvature term in the Friedmann equation is proportional to $a^{-2}$.

$\Rightarrow$ Different components dominate at different epochs.

Measuring Cosmological Parameters

By measuring distances and redshifts to far-away sources (e.g. standard candles), we can deduce the expansion history of the universe and thus compute the energy densities.

Cepheid observations (at $z \ll 1$) give the expansion rate today (i.e. $H_0$).

Type Ia supernovae observations (at $0 < z < 1.5$) give the expansion rate in
the past. At a given redshift, an accelerating universe yields fainter standard candles than a decelerating universe. Observations shows that the universe is currently accelerating and will continue to expand for all eternity.

**Benchmark model**

Define

\[ \Omega(t) \equiv \frac{\epsilon(t)}{\epsilon_c(t)}, \]

where the **critical density** is

\[ \epsilon_c(t) \equiv \frac{3c^2}{8\pi G}H(t)^2. \]

All current observations are consistent with the (Hot) Big Bang model – the universe was initially very hot and very dense, and since then has been expanding and cooling. The universe is (close to) flat and contains radiation (photons and neutrinos), matter (baryons and dark matter) and a cosmological constant (???).

\[ H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}. \]

\[ \Omega_r,0 \sim 10^{-4} \text{ dominant at } 3600 < z < \infty \Rightarrow a \propto t^{1/2} \]

\[ \Omega_m,0 \sim 0.3 \text{ dominant at } 1/3 < z < 3600 \Rightarrow a \propto t^{2/3} \]

\[ \Omega_\Lambda,0 \sim 0.7 \text{ dominant at } 0 < z < 1/3 \Rightarrow a \propto \exp Kt \]

The current age of the universe is 13.5 Gyr.

Note that the average matter density of the universe is very low, \( \rho_0 \sim 3 \cdot 10^{-27} \text{ kg m}^{-3} \). Only one per cent of this is in stars. 10% is in baryonic matter. Thus, most of the matter is dark matter and the next largest part is dark baryons.

**A brief history of the universe**

**Origin?**

Big Bang.
**Inflation** \((t \sim 10^{-36} \text{ s})\)

Our ever so successful Big Bang model has a few problems, namely the flatness problem, the horizon problem and the monopole problem. The solution to these are inflation; the very early universe underwent a period when the scale factor increased exponentially (or at least \(\dot{a} > 0\)) for at least a hundred e-foldings (that is, \(a\) increased by a factor of at least \(e^N\), where \(N = 100\)).

\((A\ \text{typical inflation model has } t_i \approx 10^{-36} \text{ s and } N \sim 100.\)

Apart from solving the aforementioned problems, inflations also provides the seeds for structure in the universe. On submicroscopic scales, there are always quantum fluctuations, i.e. on quantum scales, the universe is intrinsically inhomogeneous. Inflation takes the submicroscopic quantum fluctuations and blows them up to macroscopic scales. The energy fluctuations that result are the origin of all the inhomogeneities that we see today, including humans, cars, galaxies and clusters.

**Nucleosynthesis** \((t \sim 3 \text{ min})\)

The physics in the early universe is well understood \(\Rightarrow\) we can predict the abundances of (light) elements which are synthesized during the first \(\sim 3\) minutes. Most of the matter is in hydrogen and helium. Elements heavier than lithium is made in stars (up to Fe) and exploding stars (including us!).

We can constrain the baryon density of the universe by comparing the predictions of BBN computer codes to the observed density of especially deuterium but also helium and lithium \(\Rightarrow\)

\[
\Omega_{\text{bary},0} = 0.04 \pm 0.01 .
\]

**CMB** \((\text{originating from } t \sim 400 000 \text{ years})\)

The CMB is a perfect black body (thermal) spectrum with \(T = 2.7K\).

Subtracting the monopole, we see a dipole \(\frac{\delta T}{T} \sim 10^{-3}\) which is due to our peculiar motion.

Subtracting also this, we are left with \(\frac{\delta T}{T} \sim 10^{-5}\) on degree scales, representing fluctuations at time of emission. What we see is a snapshot of the Universe at the time of photon decoupling (approximately at the time of formation of neutral hydrogen or recombination) at \(z \sim 1100\). The CMB is redshifted to microwaves.

The perfect thermal spectrum is evidence for the hot big bang model.
The anisotropies give information on cosmological parameters.

**Structure formation**

Up to this point, we have regarded the universe as being homogeneous and isotropic.

The basic mechanism for growing structures is **gravitational instability**: “The rich get richer and the poor get poorer.”

We begin with a state of thermal eq. with some perturbations (generated by, e.g., inflation). These are then amplified by gravity (matter attracts matter) and forms today’s structure (voids, walls, clusters, galaxies). The growth depends on the amount and type of matter, e.g. HDM or CDM (rel. or non-rel. at decoupling).

Observations are consistent with initial fluctuations from inflation and the nonbaryonic matter being primarily CDM. Small structures form first, larger structures later.

**Outlook**

We have a standard model of cosmology consisting of an expanding Universe described by GR with a Big Bang and perhaps an early period of inflation.

Remaining questions:
- How did it all begin?
- What is the inflaton field?
- What is the DM?
- What is the dark energy/cosmological constant?