Notes for Cosmology course, fall 2005

Nucleosynthesis and the Early Universe

Prelude

The photons of the CMB are the oldest photons in the universe. Although the CMB is tremendously useful in determining what the universe was like at the time of last scattering, it also represents a frustrating barrier - we can't directly view the universe during its earliest stages.

We determine the properties of the early universe by extrapolating backward in time. For $t > 10^{-12}\,\mathrm{s}$, the physics of the universe is well understood and much simpler than the later universe. The models of the early universe can then be tested by, e.g. Big Bang Nucleosynthesis (BBN).

Temperatures and energies

The temperature T of the photons (and any component coupled to the photons) is

$$T = \frac{T_0}{a} \sim \frac{2.7\,\mathrm{K}}{a} \ .$$

At $a \ll a_{\mathrm{rm}} \sim 3 \times 10^{-4}$, the universe is radiation-dominated, with $a \propto t^{1/2}$ and

$$T(t) \approx 10^{10}\,\mathrm{K} \left(\frac{t}{1\,\mathrm{s}}\right)^{-1/2} \ .$$

The average energy per CMB photon is

$$E_{\mathrm{mean}}(t) \approx 3\,\mathrm{MeV} \left(\frac{t}{1\,\mathrm{s}}\right)^{-1/2} \ ,$$

i.e. comparable to the binding energy per baryon of an atomic nucleus at $t = 1$ s. Just as recombination follows once the energy per photon falls far enough below the ionization energy of $13.6\,\mathrm{eV}$, protons and neutrons will combine to form more complicated nuclei once the energy per photon falls far enough below the binding energy of a few MeV.

BBN takes place when the temperature is $T_{\rm nuc} \sim 10^9\,{\rm K}$ ($t \sim 200\,{\rm s}$). The energy density at that time was

$$\epsilon_{\rm nuc} = \alpha T_{\rm nuc}^4 \sim 10^{33}\,{\rm MeV\,m^{-3}} \ .$$

This corresponds to an equivalent mass density of $\epsilon_{\rm nuc}/c^2 \sim 2\,{\rm ton\,m^{-3}}$. The mass density of baryons was

$$\rho_{\rm bar}(t_{\rm nuc}) = \rho_{\rm bary,0}\left(\frac{T_{\rm nuc}}{T_0}\right)^3 \sim 0.007\,{\rm kg\,m^{-3}} \ .$$

The density and particle energies are reasonable $\Rightarrow$ the physics that goes into Big Bang Nucleosynthesis is well understood.

Five particle types are relevant to Big Bang Nucleosynthesis: protons, neutrons, electrons, neutrinos, and photons.

Buildup

BBN involves fusing together protons and neutrons to form stable atomic nuclei such as deuterium (D), helium-3 (^{3}He), helium-4 (^{4}He) and lithium-7 (^{7}Li) containing three protons and four neutrons.

Nucleosynthesis takes place because it is energetically favorable. When a neutron and proton are bound together to form deuterium, energy is released:

$$p + n \rightarrow D + 2.22\,{\rm MeV} \ ,$$

i.e. the binding energy of a deuterium nucleus is 1.11 MeV per baryon. The most tightly bound nucleus is iron, with 9 MeV per baryon. Three-fourths of the baryonic component is still in unbound protons (^{1}H). Why is not all baryonic matter iron, i.e. why is BBN so inefficient?

Assume the age of the universe is $t \approx 0.1\,{\rm s}$, so the temperature is $T \approx 3 \times 10^{10}\,{\rm K}$, and the mean energy per photon is $E_\gamma \sim 3kT \sim 10\,{\rm MeV}$. This energy is much greater than the rest energy of an electron-positron pair $\Rightarrow$

$$\gamma + \gamma \leftrightarrow e^- + e^+ \ .$$

Neutrons and protons are in equilibrium through the interactions

$$n + \nu_e \leftrightarrow p + e^-$$

and

$$n + e^+ \leftrightarrow p + \overline{\nu}_e \ .$$

Their number density is given by the Maxwell-Boltzmann distribution:

$$n_p = \left(\frac{m_p kT}{2\pi\hbar^2}\right)^{3/2} \exp\left(-\frac{m_p c^2}{kT}\right) \ ,$$

and the number density of neutrons is

$$n_n = \left(\frac{m_n kT}{2\pi\hbar^2}\right)^{3/2} \exp\left(-\frac{m_n c^2}{kT}\right) \ .$$

Thus, the ratio of neutrons to protons in statistical equilibrium is

$$\frac{n_n}{n_p} \approx \exp\left(-\frac{Q_n}{kT}\right) \ .$$

where $Q_n = m_n c^2 - m_p c^2 = 1.29\,\mathrm{MeV}$. At $kT \gg Q_n$, the number of neutrons and protons are nearly equal. At $kT < Q_n$, protons begin to be strongly favored.

However, neutrons and protons fall out of statistical equilibrium at about the time of neutrino decoupling at $kT_{\mathrm{freeze}} = 0.8\,\mathrm{MeV}$, $T_{\mathrm{freeze}} = 9 \times 10^9\,\mathrm{K}$, $t_{\mathrm{freeze}} = 1\,\mathrm{s}$ and the ratio of n_n/n_p is "frozen" at a fixed value,

$$\frac{n_n}{n_p} = \exp\left(-\frac{Q_n}{kT_{\mathrm{freeze}}}\right) = \exp\left(-\frac{1.29\,\mathrm{MeV}}{0.8\,\mathrm{MeV}}\right) = 0.2 \ .$$

The scarcity of neutrons explains why BBN leaves over 75% of the baryonic mass in the form of unfused protons.

The neutron-proton fusion reaction is

$$p + n \leftrightarrow D + \gamma \ .$$

This is a strong (em?) interaction. Proton-proton and neutron-neutron reactions are weak and negligible (proton-proton reactions also has the Coulomb barrier to overcome). BBN proceeds until every free neutron has been bonded into an atomic nucleus, with the leftover protons remaining solitary.

The helium fraction is

$$Y_d \equiv \frac{\rho(\,^4\mathrm{He})}{\rho_{\mathrm{bar}}} \; .$$

To compute the maximum permissible value of Y_d, suppose that *every* neutron is incorporated into a ^{4}He nucleus. Given $n_n/n_p = 1/5$, we have

$$Y_d(\mathrm{max}) = \frac{\rho_{\mathrm{He}}}{\rho_{\mathrm{bar}}} = \frac{n_{\mathrm{He}} m_{\mathrm{He}}}{n_{\mathrm{bar}} m_{\mathrm{bar}}} \sim \frac{n_n 2 m_p}{(n_n + n_p) m_p} \sim \frac{1}{3} \; .$$

If the observed value of Y_d is greater than this, we have to worry. Fortunately, $Y_d = 0.23$.

Assume now that $t = 1\,\mathrm{s}$. Proton-neutron freezeout has just been completed, and the neutron-to-proton ratio is $n_n/n_p = 1/5$. Big Bang Nucleosynthesis takes place through a series of two-body reactions, building more massive nuclei step by step. The essential first step is the fusion of a proton and a neutron to synthesize a deuterium nucleus:

$$p + n \leftrightarrow D + \gamma \; .$$

(Compare with the equation that describes recombination: $p + e^- \leftrightarrow H + \gamma$.) The energy released is equal to the binding energy of a deuterium nucleus:

$$B_D = (m_n + m_p - m_D) c^2 = 2.22\,\mathrm{MeV} \; .$$

The relative numbers of particles are given by an exact analog of the Saha equation:

$$\frac{n_D}{n_p n_n} = 6 \left(\frac{m_n kT}{\pi \hbar^2} \right)^{-3/2} \exp\left(\frac{B_D}{kT} \right) \; .$$

Deuterium is favored as $kT \to 0$, and free protons and neutrons are favored as $kT \to \infty$. We define T_{nuc} as the temperature at which $n_D/n_n = 1$.

Since 83% of all the baryons in the universe were in the form of unbound protons before nucleosynthesis and 75% is today, we can write

$$n_p \approx 0.8 n_{\mathrm{bar}} = 0.8 \eta n_\gamma = 0.8 \eta \left[0.243 \left(\frac{kT}{\hbar c} \right)^3 \right] \; ,$$

$\Rightarrow$

$$\frac{n_D}{n_n} \approx 6.5\eta \left(\frac{kT}{m_n c^2}\right)^{3/2} \exp\left(\frac{B_D}{kT}\right) ,$$

$\Rightarrow$ the temperature of deuterium nucleosynthesis is $kT_{\rm nuc} \approx 0.066\,{\rm MeV}$ ($T_{\rm nuc} \approx 7.6 \times 10^8\,{\rm K}$) and $t_{\rm nuc} \approx 200\,{\rm s}$. Remember that a free neutron is unstable

$$n \longrightarrow p + e^- + \overline{\nu}_e ,$$

with decay time $\tau_n = 890\,{\rm s}$ ($f = \exp(-t/\tau_n)$) (a bound neutron is stable). Thus, by the time nucleosynthesis actually gets underway, neutron decay will have decreased the neutron-to-proton ratio, from a value $n_n/n_p = 1/5$ to a value

$$\frac{n_n}{n_p} \approx \frac{\exp(-200/890)}{5 + [1 - \exp(-200/890)]} \approx 0.15 ,$$

$\Rightarrow Y_d(\max) \sim 0.27$.

Once a significant amount of deuterium forms, a vast array of possible nuclear reactions available.
Deuterium can fuse with a proton to form ^{3}He,

$$D + p \leftrightarrow {}^3{\rm He} + \gamma ,$$

or it can fuse with a neutron to form ^{3}H ('tritium'),

$$D + n \leftrightarrow {}^3{\rm H} + \gamma .$$

Deuterium nuclei can also fuse with each other to form ^{4}He,

$$D + D \leftrightarrow {}^4{\rm He} + \gamma .$$

or a ^{3}H nucleus

$$D + D \leftrightarrow {}^3{\rm H} + p ,$$

or they can form a ^{3}He nucleus, by emitting a neutron:

$$D + D \leftrightarrow {}^3{\rm He} + n .$$

As soon as ^{3}H or ^{3}He are formed, they convert to ^{4}He by

$${}^3{\rm H} + p \leftrightarrow {}^4{\rm He} + \gamma ,$$

$$^3\mathrm{He} + n \leftrightarrow {}^4\mathrm{He} + \gamma \ ,$$
$$^3\mathrm{H} + D \leftrightarrow {}^4\mathrm{He} + n \ ,$$
$$^3\mathrm{He} + D \leftrightarrow {}^4\mathrm{He} + p \ .$$

They all have large cross-sections and fast reaction rates $\Rightarrow$
deuterium efficiently converts to ^{4}He.

^{4}He is exceptionally tightly bound and there are no stable nuclei with 5 baryons. However, small amounts of ^{6}Li and ^{7}Li are made by

$$^4\mathrm{He} + D \leftrightarrow {}^6\mathrm{Li} + \gamma$$

and

$$^4\mathrm{He} + {}^3\mathrm{H} \leftrightarrow {}^7\mathrm{Li} + \gamma \ .$$

By the time the thermal energy has fallen to $kT \sim 0.04\,\mathrm{MeV}$ (at $t \sim 15\,\mathrm{min}$), the number density have dropped to the point where fusion no longer takes place at a significant rate and BBN is essentially over. Practically all the baryons are in the form of protons or ^{4}He nuclei. The small residue of free neutrons decays into protons. Small amounts of D, ^{3}H, and ^{3}He are left over. (The ^{3}H later decays to ^{3}He.) Very small amounts of ^{6}Li, ^{7}Li, and ^{7}Be are produced, but nothing heavier. (The ^{7}Be later decays to ^{7}Li.)

Comparing to observations

The yields of D, ^{3}He, ^{4}He, ^{6}Li, and ^{7}Li are sensitive to the baryon-to-photon ratio, η. Since we know the photon density from CMB, we can constrain the baryon density by comparing the predictions of BBN computer codes to the observed density of especially deuterium but also helium and lithium. *A high baryon-to-photon ratio increases the efficiency with which nucleosynthesis occurs $\Rightarrow$ more ^{4}He, and less of the leftovers like D and ^{3}He.*

We can measure the primordial value of D/H by looking at absorption by cool intergalactic gas cloud between us and quasars. The strength of the Lyman-α absorption lines ($\lambda_{\mathrm{obs}} = 555.9\,\mathrm{nm}$) yields a ratio of deuterium to ordinary hydrogen of

$$D/H = 3.0 \pm 0.4 \times 10^{-5}$$

$\Rightarrow$

$$\eta = 5.5 \pm 0.5 \times 10^{-10} \ .$$

Since

$$n_{\mathrm{bary},0} = \eta n_{\gamma,0}$$

and

$$\epsilon_{\mathrm{bary},0} = (m_p c^2)\eta n_{\gamma,0}$$

we get

$$\Omega_{\mathrm{bary},0} = \frac{(m_p c^2) n_{\gamma,0}}{\epsilon_{c,0}}\eta = 0.04 \pm 0.01 \ .$$

Baryon-antibaryon asymmetry

Why is there more matter than anti-matter?
Why is there more photons than baryons?

At $T \sim 10^{12}\,\mathrm{K}$ ($t \sim 10^{-4}\,\mathrm{s}$), the energy per particle was $\sim 100\,\mathrm{MeV}$. At this temperature, quarks are not bound into individual baryons $\Rightarrow$ "quark soup":

$$\gamma + \gamma \leftrightarrow q + \vec{q} \ .$$

Assume small asymmetry:

$$\delta_q = \frac{n_q - n_{\vec{q}}}{n_q + n_{\vec{q}}} \ll 1 \ ,$$

$\Rightarrow$ quark residue

$$\frac{n_q}{n_\gamma} \sim \delta_q \sim 3\eta \ .$$

(three quarks per baryon) $\Rightarrow$ a small asymmetry gives a small η.

Summary

The physics in the early universe is well understood $\Rightarrow$ we can predict the abundances of (light) elements which are synthesized during the first ~ 3 minutes. Most of the matter is in hydrogen and helium. Elements heavier than lithium is made in exploding stars (including us!).