Astrophysical Spectra

Practical exercise

Stockholm Observatory, 2006

1 Introduction

Almost everything we know about the universe outside our solar system comes from information provided by light, or more precisely, electromagnetic radiation. The science of astronomy is thus intimately connected to the science of analysing and interpreting light. The purpose of this exercise is to provide some insight into how information can be extracted from observations of spectra. Several spectra are studied; first from stars of various types, and then a spectrum from the nebula surrounding the supernova remnant SN 1987A. Sect. 2 introduces some background theory and general concepts that are used in the exercises. Sect. 3 gives some preparatory exercises. Sect. 4 shortly describes the functionality of the program *Spectrix*, a tool to assist in the analysis of spectra. In the last section, the exercises to be solved are detailed.

1.1 The report

To pass this assignment you have to write a report that shall be handed in within two weeks of the lab. Remember to put your name and personal number on it. Regarding the level of detail in the report, a good rule of thumb is that the report should be comprehensible to yourself even in five years from now. Answer every exercise fully, and be careful to estimate errors of your figures when asked to. If you wish, you may write in English, but that is not at all necessary. Finally, there is no need for the report to be machine written, although it may be easier to edit than a hand written report.

2 Background theory

2.1 Radiation processes

Radiation can be both emitted and absorbed in a number of processes. Here we concentrate on mainly two processes: free-free and bound-bound. “Free” and “bound” refer to the state of a particle, in our case an electron, interacting with a photon. The electron is considered to be bound if it is bound to an atomic nucleus, and free if it is not. Thus a process is called “free-free” when the electron is free both before and after the energy exchange with a photon, and “bound-bound” correspondingly. As you may have guessed, there are also “bound-free” processes etc.

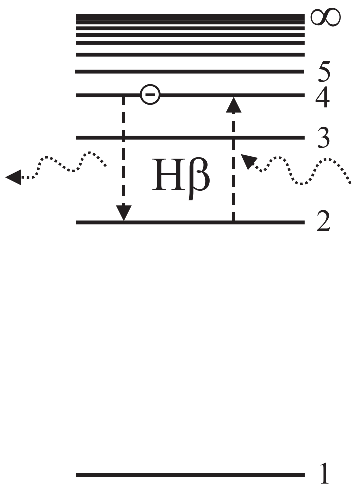

Figure 1: A level diagram of the hydrogen atom. Along the vertical direction, the energy of the different numbered levels are shown, to scale. As an example, an Hβ absorption line arises when an electron in level 2 interacts with a photon and is raised to level 4. An Hβ emission line correspondingly occur when an electron transits from level 4 to level 2, thereby emitting an photon corresponding to the energy difference.

If bound, the electron can only exist in particular states with particular energies. According to quantum mechanics, the possible states of the electron has energies $\{E_0, E_1, E_2, \dots\}$, where $E_n < E_{n+1}$. This is called to be a discrete set of energies, as opposed to a continuum set of energies; in a discrete set, there are always energies E such that $E_n < E < E_{n+1}$, for all n. It is sometimes useful to draw an energy level diagram. In Fig. 1, a schematic energy level diagram is shown.

By switching energy level (called a level transition), an electron can either absorb the energy of a photon, or produce a photon with the excess energy from the switch. Since the energy of a photon is inversely proportional to the wavelength of the corresponding light wave, $\Delta E = hc/\lambda$ (where $h = 6.6261 \times 10^{-34}\,\mathrm{J\,s^{-1}}$ is Planck's constant, $c = 2.9979 \times 10^{8}\,\mathrm{m\,s^{-1}}$ is the speed of light, and is λ the wavelength), a level transition gives rise to an absorption/emission at the frequency corresponding to the energy difference between the two levels (Fig. 1). Every transition thus has a particular wavelength $\lambda = hc/\Delta E$ associated with it, and by studying light at that wavelength, we may be able to deduce something about the physical conditions of the responsible matter (see Sect. 2.3).

When an electron interacts with a photon in a free-free transition, there are no discrete energies the electron has to adhere to. Consequently, the photons may be of any wavelength from a continuous distribution.

2.2 Properties of matter in thermal equilibrium

Atoms and ions with bound electrons can change their state by collisions with other particles. If these collisional transitions are more frequent than the radiative transitions, then the levels populated by electrons will be determined by the statistical properties of the collisions. A special kind of statistical state of matter is called thermal equilibrium; that is the state a closed system of particles eventually will tend to. When the system is not closed, but closed enough to reach equilibrium before disturbed, the state is called local thermal equilibrium (LTE). In LTE, electrons in bound states are distributed among levels according to Boltzmann's equation:

$$\frac{n_j}{n_i} = \frac{g_j}{g_i} \exp\left(-\frac{E_j - E_i}{k_B T}\right), \tag{1}$$

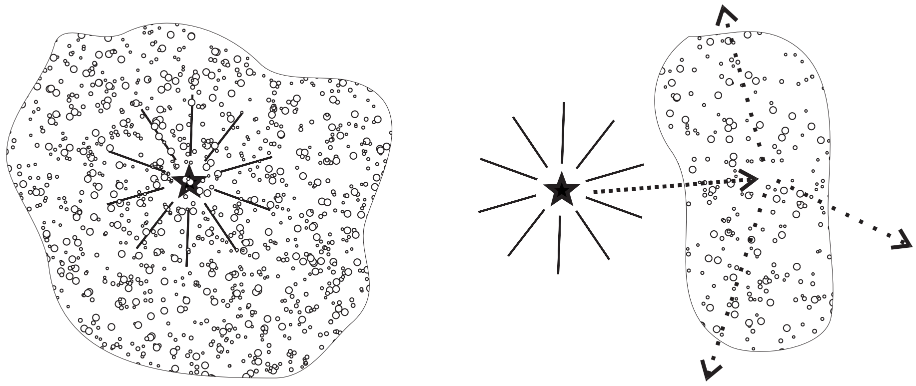

Figure 2: The light from a continuum source (like a star) behind a gas cloud gives rise to absorption (when observed directly), while a cloud illuminated from the side shows emission lines (arising from starlight absorbed and re-emitted by the gas). If the star is hot enough, most of its radiation will be ionising, causing recombination emission to a higher degree than purely re-emitted light (see Sect. 2.3.2).

where n_j/n_i is the relative population of electrons in level j compared to level i, g_j and g_i are statistical weights of those levels, T is the temperature (in Kelvin) and $k_B = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$ is Boltzmann's constant. The level population thus only depends on the temperature, a very important property of thermal equilibrium.

Gas in LTE emits light due to free-free processes, with a spectrum according to the Planck distribution:

$$B_\lambda \propto \lambda^{-5} \left[\exp\left(\frac{hc}{\lambda k_\mathbf{B} T} \right) - 1 \right]^{-1}, \tag{2}$$

where B_λ is the intensity of light as a function of wavelength. This distribution also depends solely on temperature.

If the temperature is high enough, the collisions between the atoms may be energetic enough to strip the atoms of electrons; that is, the atoms get ionised. The ions then later recombine with a free electron to become a neutral atom again. In thermal equilibrium, the number of ionisations and recombinations are balanced and well described by the Saha equation. The Saha equation describes what the degree of ionisation is for a particular temperature (see preparatory exercise 2 below); the higher the temperature, and the lower the density, the more strongly particles become ionised. As an example, at typical densities of a stellar atmosphere, about half of the hydrogen becomes ionised at 10 000 K.

2.3 Line formation

This section is concerned with how spectroscopic lines are formed, and why lines are seen in absorption sometimes, and in emission at other times. The simple picture of a cold gas being illuminated by a background source (Fig. 2) is seldom valid in astrophysical applications, as the gas itself often either is warm and gives rise to line emission, as in a stellar atmosphere, or is strongly ionised and produces *recombination line emission*. The nature of recombination line emission is shortly reviewed in Sect. 2.3.2.

2.3.1 Stellar atmospheres

A star, being a fuzzy ball of hot gas, does not have the same kind of well-defined surface as, for instance, a rocky planet. Instead, the density of a star drops more or less smoothly from the centre and outwards, until it far out approaches the interstellar density. The radius of a star is merely a matter of definition. Fortunately, there is a sharp transition region in stars that can serve as the defining radius, and that is the photosphere. The photosphere is the region where the star, seen from the outside observer, suddenly becomes optically thick, i.e. non-transparent. The radius of the photosphere is only a weak function of wavelength, so most photons we see come from approximately the same region in the star. The photosphere is the region where the continuum from a star is emitted, the radiation due to free-free processes in the hot gas that is well approximated by blackbody radiation.

Even though the star suddenly becomes essentially transparent above the photosphere, there is still a substantial mass of gas left. The gas may be too sparse for free-free processes to be efficient, but bound-bound processes are certainly still important, though limited to certain wavelengths in contrast to free-free processes (bound-free processes are also important, but we do not consider them here). This means, that while photons from the photosphere generally travel unhindered to the observer, photons that happen to reside at wavelengths where important bound-bound transitions prevail, only reach a certain distance l on average before they are absorbed by the stellar atmosphere (Fig. 3). l is usually called the mean free path of the photon, and depends on how strong the transition in an absorbing species is, and its density. The higher the probability that a photon will be absorbed by a single particle, and the more such particles there are, the shorter the mean free path of the photon.

As the density of absorbing particles drops further out from the star, the mean free path becomes longer, and eventually long enough for the photon to travel more or less freely to the observer (Fig. 3). The radius from the stellar centre at which this happens depends on the particular transition, so it varies strongly. This means that the photons an observer sees come from different layers in the star. If the local temperature of the layer is lower than the layer where the continuum is emitted, then the LTE emission in the line is lower, and an absorption line is seen (upper row, Fig. 4). If, on the other hand, the layer temperature is higher, then an emission line can arise (lower row, Fig. 4). In a real stellar atmosphere, the radial temperature structure is fairly complicated and gives rise to line emission both below (absorption) and above (emission) the background continuum, depending also on the detailed ionisation structure.

2.3.2 Gaseous nebulae

The reason why the gas in stars is kept in (or at least close to) thermal equilibrium is that densities are high. Interstellar gas clouds, however, are typically so sparse ($n_e \sim 10^6 - 10^{11}$ m^{-3}) that collisions between particles are very rare. This gives time for radiative transitions to re-assemble the energy levels of the atoms and ions so that they no longer are distributed according to the Boltzmann equation. Instead, particles are expected to reside most of their time in the state of lowest energy. For these objects thermal equilibrium is far from a good description, and the emitted spectrum is very unlike a blackbody spectrum attenuated by absorption features. Instead, the spectrum is dominated by *emission* lines sitting on top of a very weak continuum. If there is a hot star radiating energetically enough to ionise atoms in the neighbourhood, then several of the species may be strongly ionised, and the occasional capture of an electron by one

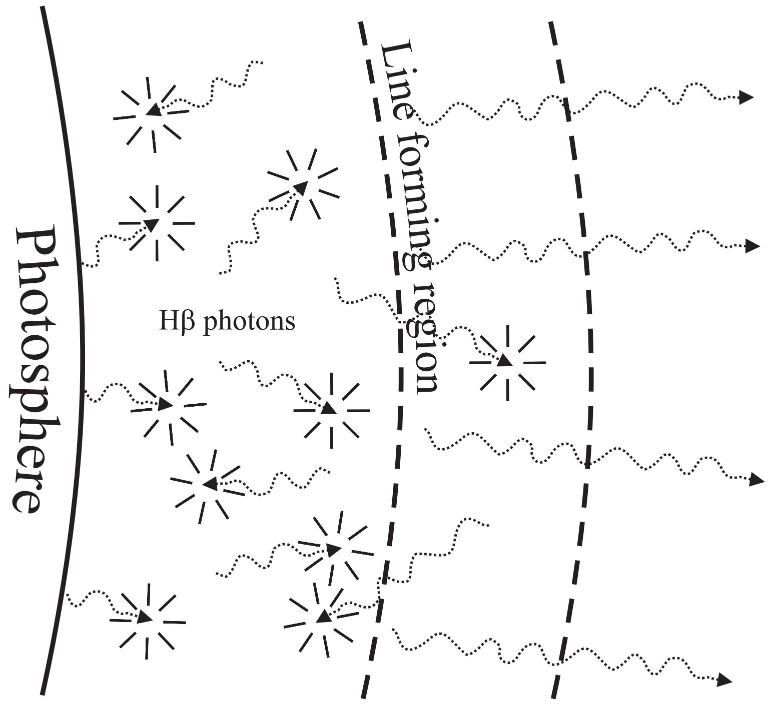

Figure 3: When a photon at a wavelength corresponding to a strong bound-bound transition (like Hβ in this figure) leaves the photosphere, its mean free path is short and it is soon absorbed. As the density of the absorbing particles decrease outwards, however, the mean free path grows in length to ultimately become essentially infinite. The intensity of the flux of photons in the line at that point, depends on the local temperature of the region. If the temperature happens to be higher than the continuum temperature, then the flux of photons in the line will exceed the number of photons originally emitted by the photosphere and we get an emission line. Conversely, if the temperature is lower, we get an absorption line (see also Fig. 4).

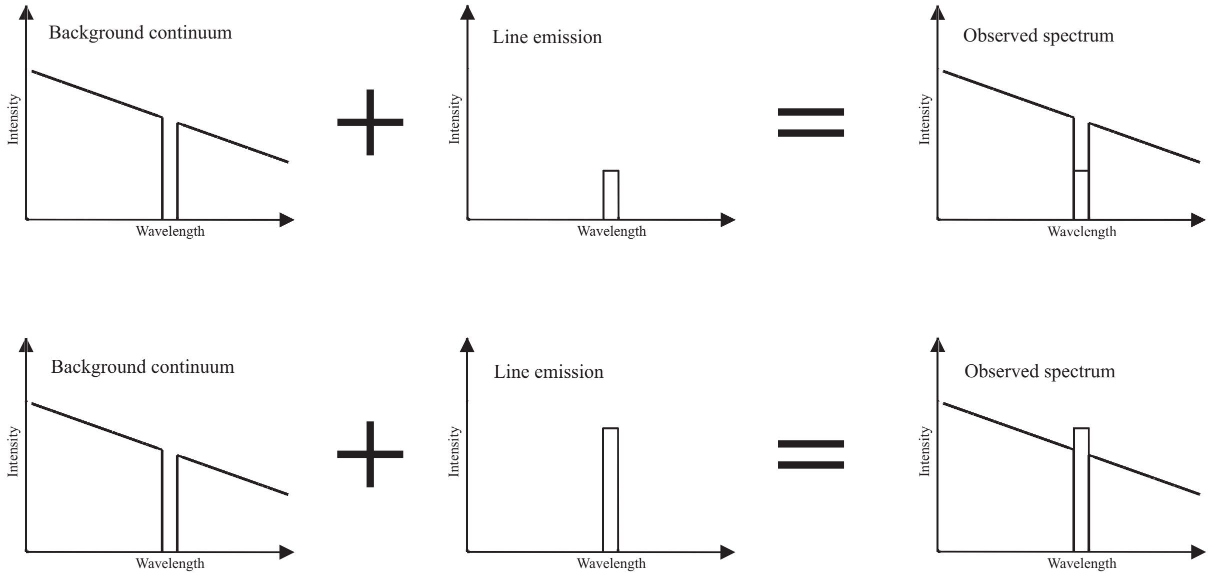

Figure 4: The upper row corresponds to a temperature profile like the left panel of Fig. 5, where the temperature of the continuum is higher than the line forming region, and the lower row corresponds to a temperature profile like the right panel of Fig. 5, where the line forming region has the highest temperature. The light an observer sees comes mostly from the photosphere (leftmost column), where the continuum is stripped of all photons from the spectral line because of the small mean free path in that line. Further out, where the mean free path is high enough for photons to finally escape in the spectral line (middle column), we see photons in the line, but no continuum, since almost no continuum is emitted here. The total spectrum we observe (rightmost column) is the sum of the continuum and line emission, and if the temperature in the line forming region is lower than the region where the continuum comes from, then the line emission will not quite fill out the gap created by the photons absorbed further in (upper row), while a higher temperature in the line forming region (lower row) will cause an effective emission.

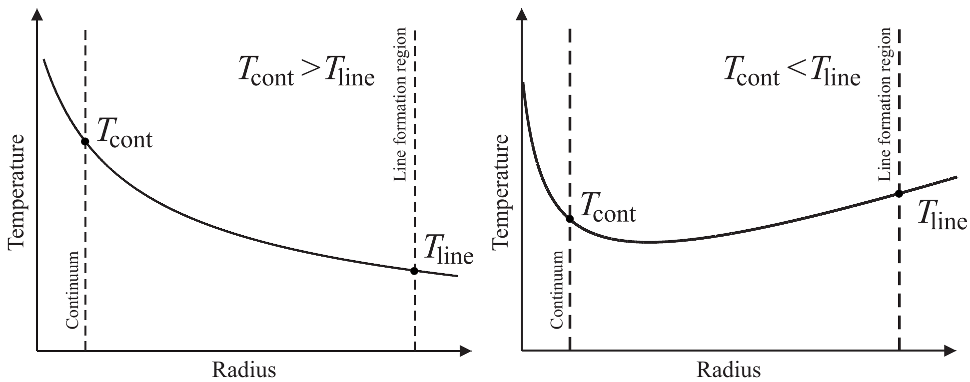

Figure 5: The figures show examples of temperature profiles in stellar atmospheres, where the radius increases to the right. If the continuum forms in a region of the atmosphere where the temperature is higher than the region where the line is formed, then we get absorption (left panel). In contrast, if the photons we see in the line come from a region where the temperature is higher, we get effective line emission (right panel). Se also Fig. 4.

of these ions gives rise to recombination radiation, as the electron cascades down to the ground state, emitting radiation on its way. Another source of emission is by scattered (or *re-emitted* light), where photons from a nearby source excite the particles of the nebula that subsequently re-radiate the photons, but then in a possibly different direction and different wavelength. Finally, if there is a background source, one may see the gas in absorption against the source, much like in Fig. 2; this is commonly seen against stars as interstellar absorption lines, and may be distinguished from the atmospheric absorption lines of the star by their comparatively small line width, and usually different radial velocity relative to the star (i.e. the interstellar lines are Doppler shifted differently from the atmospheric lines). One good thing about gaseous nebulae is that it is relatively simple to use them to derive abundances of elements like H, He, C, N, O, Ne and S.

2.4 Line broadening

Since bound-bound processes give rise to a photon with the specific energy of the difference between the transitioning energy levels, one might expect that the photon would always have exactly this energy, and that the corresponding spectral lines thus would be infinitely sharp. This is clearly not the case, and for several reasons. First, due to fundamental quantum mechanical limitations, the energy can never be exact, but must have a measure of uncertainty associated with it. Second, and usually more importantly, since the interacting particles almost never are at rest relative to the observer, the energy of the photon, and thus the wavelength, will be Doppler shifted. If we add up light coming from particles moving in different directions, this will effectively broaden the spectral line. One example when this happens is the intrinsic thermal motions of the particles, another is the rotation of a star, where different parts of the star move with different velocities relative to the observer, and thus gives rise to a rotationally broadened line.

3 Preparatory exercise

To be allowed to start with the actual computer exercise, the preparatory exercises below must be completed and handed in, which can be done in connection with the lab. This is mainly to ensure that sufficient preparations for the lab has been made to allow for the best use of the lab time.

The strength of an absorption line depends mainly on the temperature and its gradient in the stellar atmosphere. The following two problems are intended to illustrate the role of these physical parameters for the formation of an absorption line. The first problem focuses on the temperature gradient by making the (unrealistic) assumption that the number of atoms that contribute to the absorption be independent of the temperature. In the second, and more realistic problem, the temperature gradient is kept fixed. Instead, it is emphasised that the fraction of all atoms that contributes to a spectral line is determined by the temperature through Saha's equation. In the case of the Hβ line, not all hydrogen atoms have an electron in the appropriate level (i.e., the principal quantum number n = 2, henceforth denoted n_2). The electron can be in any other level or even separated from the proton (ionisation).

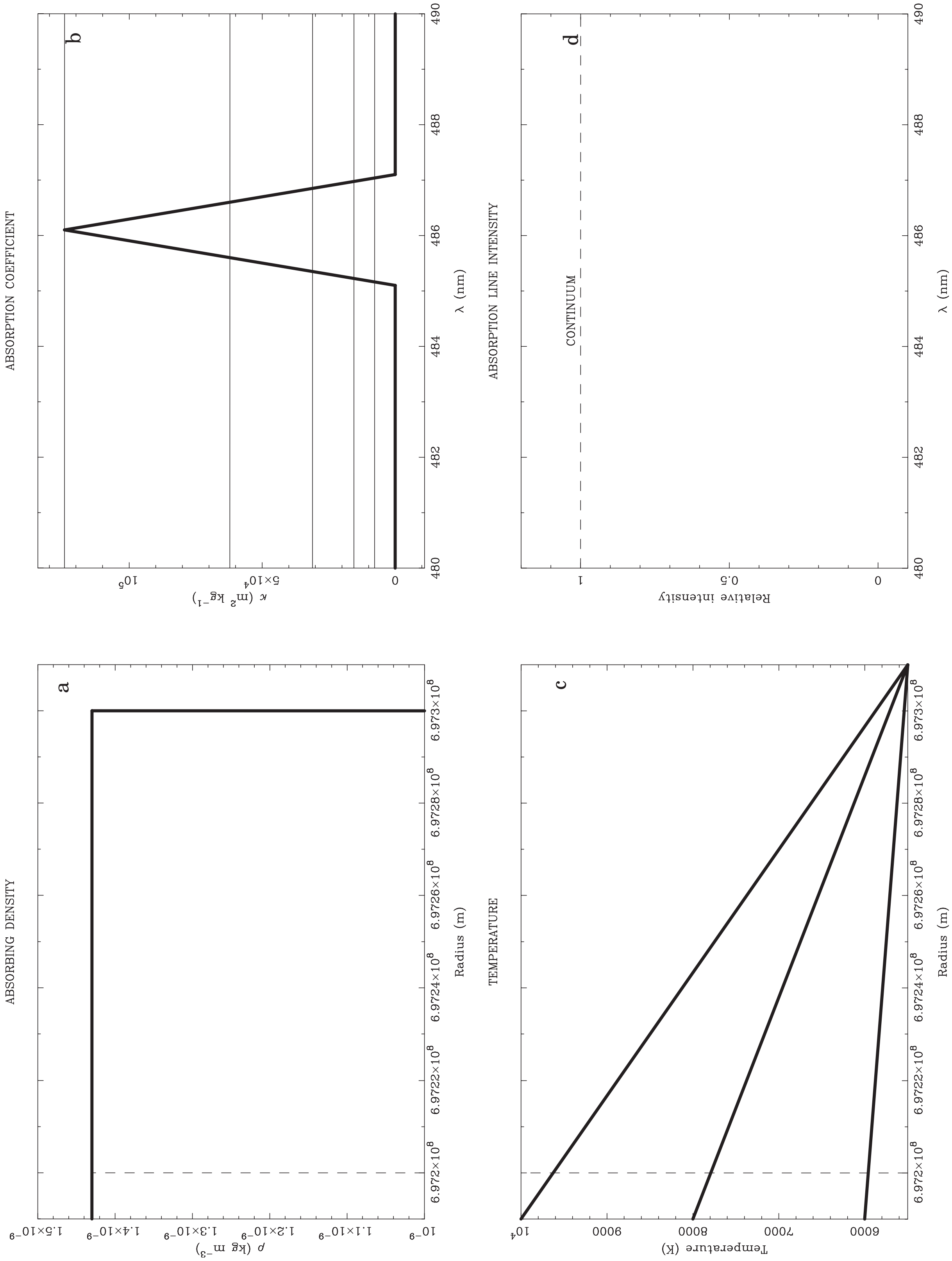

Figure 6: Plots for preparatory exercise 1

1. The aim of this problem is to show how an absorption line forms, and that different temperature gradients lead to different strengths of the absorption line. In Fig. 6a we plot the density (ρ) as a function of radius for the gas in the region where the Balmer lines are formed. In order to simplify the mathematics, ρ is assumed to be constant. Furthermore, the star is assumed to be of pure hydrogen. Only the outer part of the star, i.e., the absorption line region, is represented in the diagram. The dashed line marks the radius of the photosphere, i.e., the radius from where the continuum radiation is emitted. In Fig. 6b we show the mass extinction coefficient (sometimes denoted as the absorption coefficient) for Hβ as a function of wavelength. In this plot we have also drawn five horizontal lines, each representing the value for the mass extinction coefficient at the two wavelengths where the horizontal line intersects the mass extinction coefficient profile. In Fig. 6c we plot three different temperature gradients in the stellar atmosphere to simulate three different stars.
   Before we start, we need to introduce a new quantity, the *mean free path*, l, which gives the average distance a photon can travel before being absorbed. It is defined as

$$l = \frac{1}{\kappa\rho} \tag{3}$$

   where κ is the mass extinction coefficient. We proceed as follows:

   - Read the values of the mass extinction coeffcient κ for the corresponding wavelengths. Compute the mean free path of the photons according to Eq. 3.
   - Subtract the obtained mean free path from the stellar radius in order to obtain the depth in the star from where the observed photons emerge. The five different values of κ correspond to five different mean free paths, and thus five different depths.
   - For each of these depths, estimate the temperature for the three different temperature gradients.
   - Compute the blackbody radiation intensity from these various depths using the Planck function

$$B_\lambda = \frac{2hc^2}{\lambda^5}\left(\frac{1}{e^{(hc/\lambda kT)} - 1}\right) \tag{4}$$

     It is sufficient to calculate the relative intensities, i.e., to divide the obtained intensities by those of the continuum for the respective stellar model. Plot the relative intensities in Fig. 6d.

   Your results should show three different absorption lines, of which the strongest is associated with the steepest temperature gradient.

2. To illustrate the temperature dependence of the level population/ionisation in a stellar atmosphere, we consider the number density in the lowest level of hydrogen, n_1. In the case of thermodynamic equilibrium (T.E.) this is related to the number density of protons, n_p, according to the *Saha equation* (note that the Saha equation usually is written as the inverse of this)

$$\frac{n_1}{n_p} = 4.14 \times 10^{-22} n_e T^{-3/2} e^{\chi_I/kT} \tag{5}$$

where n_e is the electron density in m^{-3}, T the temperature in K, $\chi_I = 13.6$ eV the ionisation threshold of level 1 and Boltzmann's constant $k_B = 1.38\times10^{-23}$ J K^{-1}. Assume $n_e = 1021$ m^{-3} and calculate for what temperature $n_1 = n_p$. (1 eV = 1.6022×10^{-19} J). Note that the equation has no analytical solution. It has to be solved by iteration, using for example the Newton-Raphson method. It can also be solved graphically. Assuming the *total* stellar density to be constant (see Fig. 7a) we plot in Fig. 7b three different stellar models with the same temperature gradient, but with different temperatures in the region where the continuum arises.

To account for variations of n_2/n_{tot} with temperature we should use, instead of Eq. 3,

$$l = \frac{1}{\kappa\rho_2} = \frac{1}{\kappa\rho}\left(\frac{n_{tot}}{n_2}\right) \tag{6}$$

where n_2/n_{tot} is given by an expression similar to Eq. 5 and is plotted in Fig. 7c.

The mean free path, for Hβ photons say, can now be computed for the different temperature structures by assuming that the density of the absorbing material is constant, but different for the individual temperature structures. The constant value is taken as the mean value in the absorption line region. The mean values are given below, together with the temperatures of the continuum photosphere:

T1 = 9000 K, $< n_2 = n_{tot} >= 9.82\times10^{-7}$
T2 = 13000 K, $< n_2 = n_{tot} >= 1.50\times10^{-5}$
T3 = 18000 K, $< n_2 = n_{tot} >= 4.56\times10^{-6}$

Calculate from Eq. 6 the mean free path for photons in the line centre. Use this value to determine (just as in the previous problem) the depth in the atmosphere from where Hβ radiation can escape and, thus, be observed by us. Estimate, from the figure, the temperature at these depths for the three stellar models respectively.

A crude estimate of the relative strengths of the absorption line in the three different stars can be made by computing the difference between the blackbody intensity at the continuum photosphere, B_C, and the blackbody intensity at the depth from where the Hβ photons emerge, $B_{H\beta}$. Normalise this difference by dividing with B_C to obtain an estimate of the absorption line strength.

$$f_\lambda = \frac{B_C - B_{H\beta}}{B_C} \tag{7}$$

Do this, and plot the ratio as a function of the temperature of the continuum photosphere. How do you think this ratio would change for lower (T $\leq$ 9000 K) and higher (T $\geq$ 18000 K) temperatures, respectively?

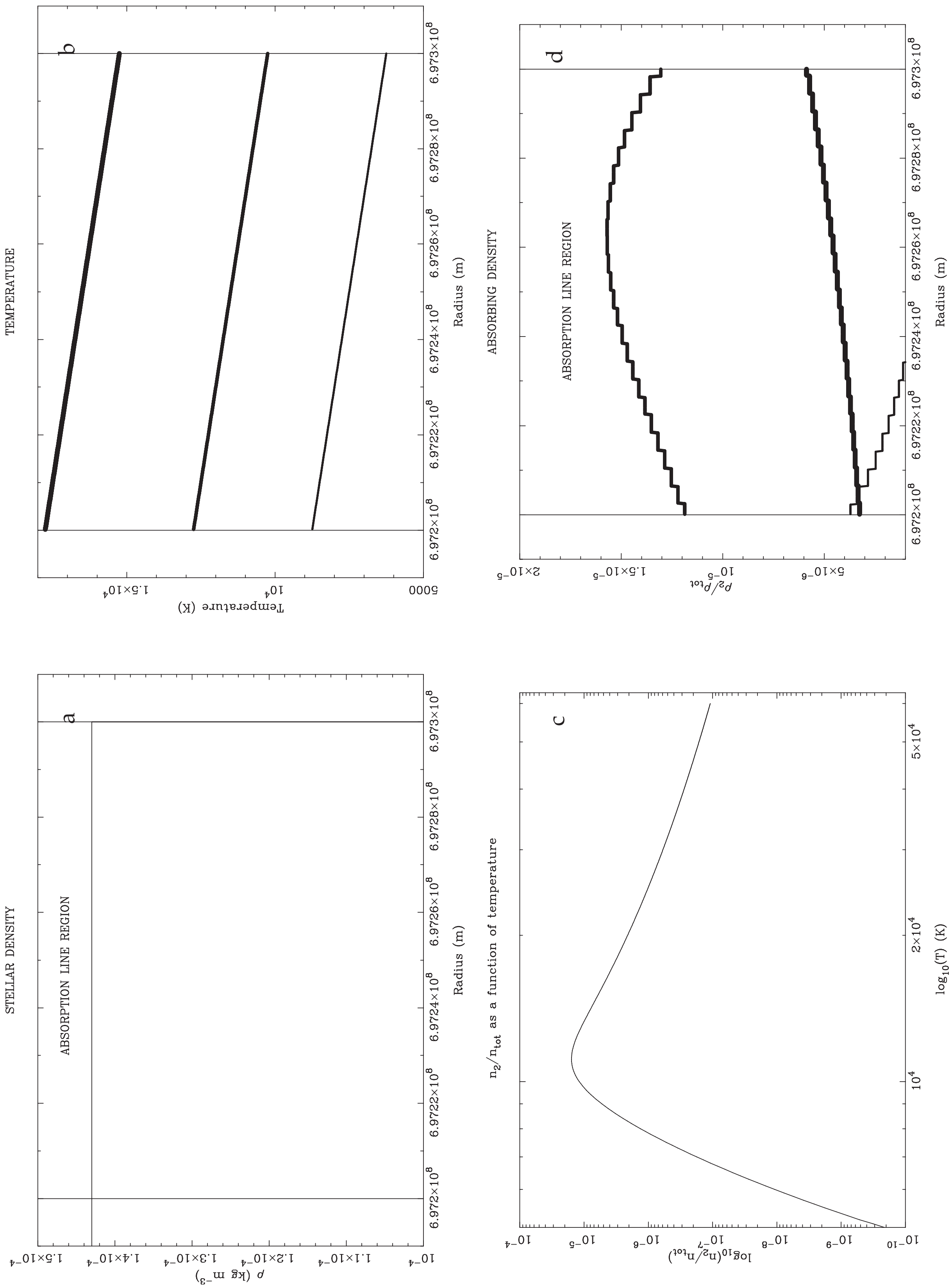

Figure 7: Plots for preparatory exercise 2

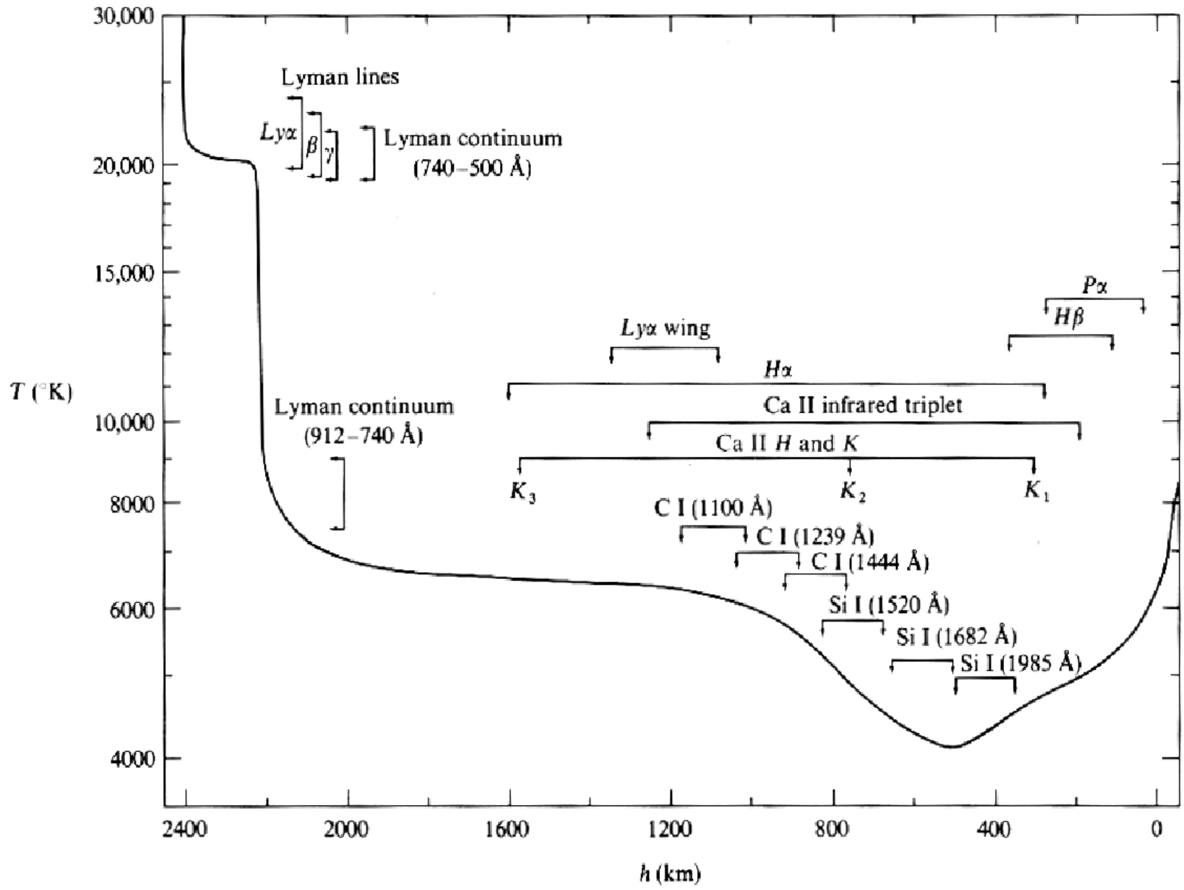

Figure 8: Temperature structure of the Sun. The height, h, is put to zero roughly at the continuum photosphere.

Note that normally the strength of an absorption line is given as the area of the absorption line below the continuum. See Fig. 10 for the definition of the so called equivalent width.

3. The Planck function (Eq. 4) has the property that when λ is kept fixed, it increases monotonically with T, so that

$$\frac{dB_\lambda}{dT} > 0. \tag{8}$$

This has important consequences for stars; if some parts of the spectrum are emitted by regions cooler than others, this causes dips in the spectrum at these particular wavelengths. In general, stars have a temperature that decreases with radius. This is shown in Figure 8 for the Sun; the temperature drops from $\sim$ 6 500 K at a radius close to the continuum photosphere to $\sim$ 4 000 K, 550 km above this radius.

Figure 8 shows that spectral lines form above the continuum photosphere, so it is expected that the spectral lines are emitted by gas cooler than the gas emitting the continuum. Stellar spectral lines are therefore mainly absorption lines. In particular, this is the case for the Balmer lines of hydrogen which arise as a result of transitions between level 2 and higherlying bound levels of hydrogen. Figure 8 also shows that far out in the solar atmosphere (at h > 1000 km), the temperature starts to increase with radius. Lines forming this high up in the atmosphere are therefore emission lines. However, since density drops rapidly with radius, the mass of the gas that gives rise to the emission lines is low, and these lines are therefore weak. (During solar eclipse, the spectrum of the Sun is mainly an emission line spectrum. Why?) Which lines are expected to show the strongest absorption features in the case of the Sun, according to Figure 8? (Note that wavelengths in Fig. 8 are given in Ångström, not in nanometers.)

4 Spectrix

Spectrix is a Java tool designed to aid in the analysis of the spectra of the present exercises. Fig. 9 shows the layout of the program as it appears on a Windows XP machine, but due to the portable nature of Java, Spectrix will work on other platforms as well, with a slightly different layout. If you have access to a computer at home, you can download and run Spectrix.[1] Note that you need Java installed as well, but this is usually installed by default on modern operative systems. If you do not have Java installed, you may download the latest Java platform from Sun Microsystems.[2] The main functionalities of Spectrix are:

- Display different parts of a spectrum to a variable degree of detail.
- Integrate a spectrum over a specified interval.
- Overlay a blackbody function with an adjustable temperature.

The detailed functions of the graphical user interface are described in the caption to Fig. 9.

5 Computer exercises

5.1 Stellar spectra

Depending mainly on the temperature of the photosphere, the spectra of different stars are dominated by different lines. Important lines are shown in Table 1.

The Hϵ line and the Ca II H line nearly coincide in wavelength, 3970 Å. The Ca II H & K lines are usually only seen in cold and medium hot stars, however, and always both at the same time, while the Hϵ line is only seen in hot stars, so there is usually no problem in distinguishing between these two. In addition to these lines, there are also molecular bands due to CN (around λ 414.4-421.5), and due to TiO (many bands at wavelengths larger than $\sim$ 476.0 nm.), but in this exercise we will focus on those of Table 1. In the following exercises, you will determine some properties of the stars from their spectra. We will take a closer look on the seven stars of spectral types K5, G9, G7, F8, A2, B3 and O5.

[1] Find Spectrix at `http://www.astro.su.se/~alexis/Spectrix.zip`

[2] `http://java.sun.com`

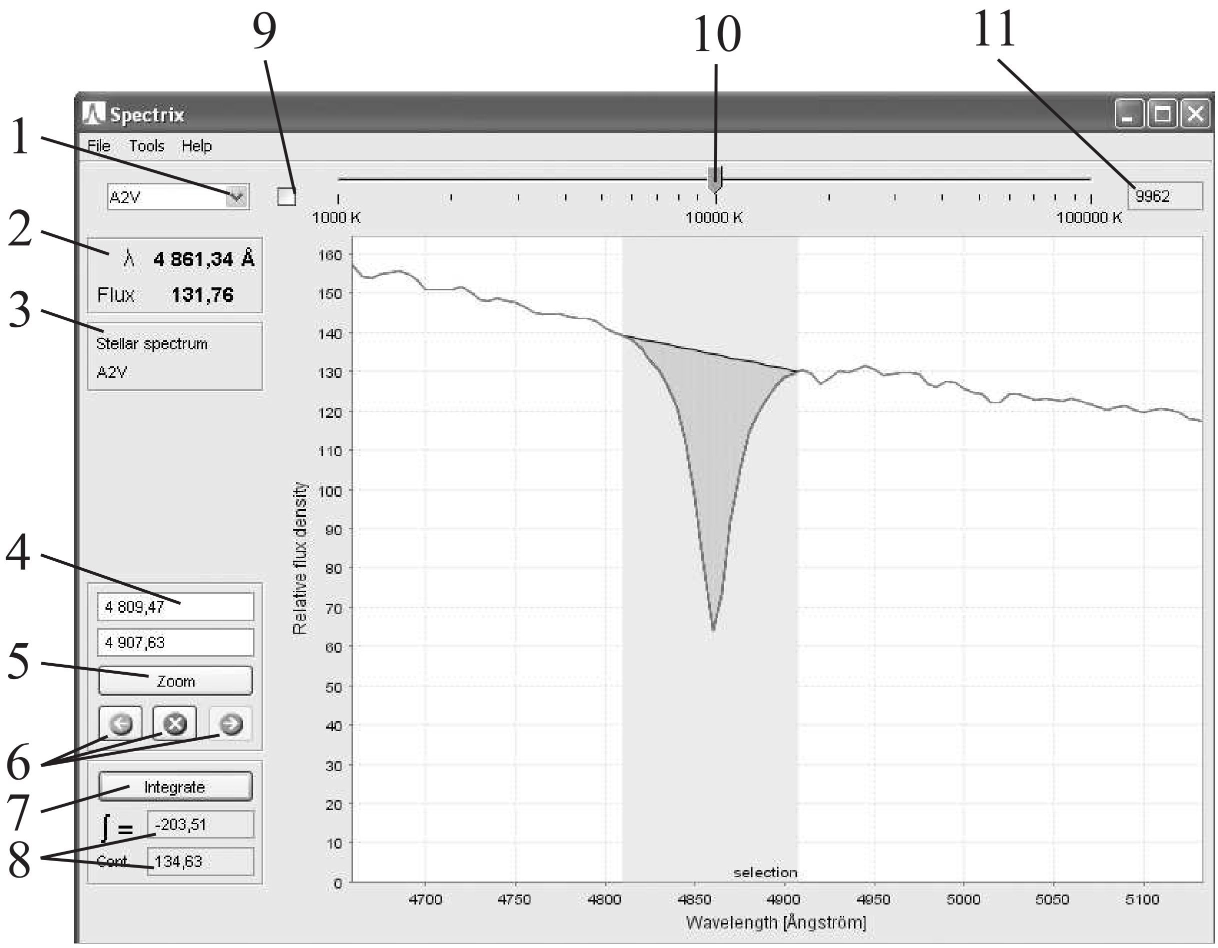

Figure 9: The Spectrix guide user interface. This is the window that appears when you start Spectrix. The different areas are: (1) A selectable combo box that allows you to switch between available spectra. (2) An information box that tells you what the precise coordinates of the mouse pointer is in terms of wavelength and intensity in the graph of the spectrum. (3) Name and class of the spectrum. (4) Two editable fields with the wavelength of the presently selected interval. An interval may be selected either by entering it directly into these fields, or by dragging the mouse pointer over an interval in the graph. (5) Zoom button, zooms into the presently selected field. (6) The arrows go back and forth in the zoom history, while the cross deselects the present interval selection. (7) Integrate button, integrates over the presently selected interval. (8) Two fields showing the integrated area and the continuum level of the integration, respectively. The integrated area is negative for absorption and positive for emission. (9) If this box is checked, then a blackbody curve is overlaid the spectrum, with a temperature as selected by the slider (10) or entered directly into the editable field (11).

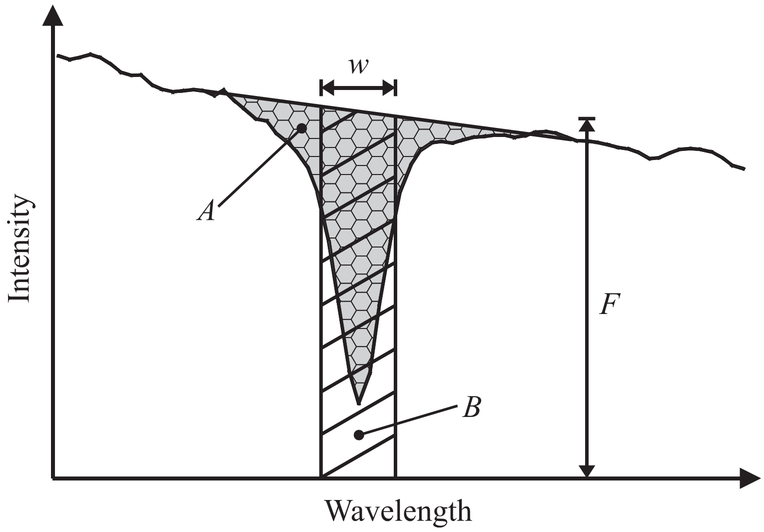

Figure 10: The equivalent width of an absorption line is defined so that the integrated grey area in the line A equals the striped area B, which is the equivalent width w times the continuum level F.

Exercise 1 *Use Spectrix to determine which three or four of the lines in Table 1 that are the strongest in the spectra of the K5, G7 and O5 stars. Try to rank the lines. You will find that there are some strong lines in the spectra that are not listed in Table 1. For this comparison however, please ignore those lines. Why are not the same lines the strongest for all stars?*

Exercise 2 *A stellar surface emits light that can be roughly approximated by blackbody radiation. As the shape of the blackbody curve depends only on the temperature, the shape of a stellar spectrum provides a simple indication of its surface temperature. Find the temperature that fits best a blackbody spectrum to the stellar spectrum. Mark the box left of the temperature slider in Spectrix, and slide the temperature until the shape of the blackbody curve roughly corresponds to the stellar spectrum. In general, the blackbody fit is not so good in the blue part of the spectrum where there are many lines which 'block' the radiation from the photosphere. The stellar spectrum is thus deficient of photons in this region, compared to a blackbody. Because the energy from the star has to come out somewhere in the spectrum, the green/red part of the stellar spectrum lies above the blackbody spectrum (when the proper stellar temperature has been chosen.) Make a list of blackbody temperatures as a function of the type of the stars. . Estimate the error in your fit by finding what range of temperatures still yield an acceptable fit. N.B! The fit doesn't give adequate temperatures for the hot stars (i.e the 0-, B- and A-stars). For these stars you may take values from the literature*

Spectral lines provide a rich source of information, and in the following exercises you will take a closer look at the Hβ absorption line to study its dependence on stellar properties.

Exercise 3 *Use the integrator in Spectrix and calculate the equivalent width* $w_{\mathrm{H}\beta}$ *of the H*β *line in each of the seven stars (see Fig. 10 for an example of an integration). Estimate the error you make in this measurement by noting how the result varies when you integrate the line over slightly different intervals. In your report, plot in a diagram* w_λ *against the blackbody temperature for all of the stars. Note that the H*β *line will be strong in some stars, but extremely faint in others (maybe even invisible).*

Element	λ [Å]	Name
Ca II	3934	K
Ca II	3968	H
H I	3970	H ϵ
Ca I	4227	
Si IV	4089	
H I	4102	H δ
H I	4340	H γ
He I	4471	
O II	4650	
He II	4686	
H I	4861	Hβ
H I	6563	H α

Table 1: Lines to look for in the stellar atmospheres.

Element	λ [Å]	Name
N III]	1750	
C III]	1909	
[O II]	3728	
He II	4686	
H I	4861	Hβ
[O III]	4959	
[O III]	5007	
He I	5876	
N II]	6548	
H I	6563	H α
[N II]	6583	

Table 2: Lines to look for in the nebula. Due to the infrequent collisions between particles in nebulae, some particles reach states not attainable in stars. The emission related to those transitions are called forbidden or semi-forbidden, and are marked by [] and], respectively.

Exercise 4 *Since the Hβ absorption line arises from a transition between level 2 and 4 in the hydrogen atom (Fig. 1), the observed equivalent width should tell us something about the number of H atoms that have an electron in level 2.*

Use the temperatures derived for the stars in Exercise 2 to estimate the fraction of hydrogen in the stellar atmospheres that have an electron in level 2, n_2/n_{H}, where n_2 is the number of hydrogen atoms with electrons in level 2, and $n_{\mathrm{H}} = n_{\mathrm{H\,I}} + n_{\mathrm{H\,II}}$ is the total number of both neutral and ionised hydrogen. In principal, one can calculate this ratio by using the fact that $n_{\mathrm{H\,I}}/n_2 = (n_1+n_2+n_3+\ldots)/n_2 = n_1/n_2+n_2/n_2+n_3/n_2+\ldots = \sum_i n_i/n_2 = \sum_i \frac{g_i}{g_2}\exp\left(\frac{E_2-E_i}{k_{\mathrm{B}}T}\right)$, where the Boltzmann equation (Eq. 1) has been used, and then continue to use the Saha equation to calculate the degree of ionisation, that is, $n_{\mathrm{H\,II}}/n_{\mathrm{H\,I}}$. This, however, is beyond the scope of the present exercise, so for your convenience there is a tool in Spectrix that does this calculation for you. Look in the Tools menu. Plot in your report the fraction of hydrogen in level 2 against the Hβ equivalent width in a logarithmic diagram.

Exercise 5 *Discuss the results obtained so far. In particular, why do the equivalent widths change with temperature in the way they do in the plot? Explain also the behaviour of the equivalent width vs. n_2/n_{tot}. If you find the relations confusing, you may want to consult the background theory section again.*

5.2 Emission line spectrum

You will now turn to study an emission line spectrum, that from the nebula around supernova 1987A. The spectrum was obtained from the Hubble space telescope. One important advantage of observing from space is that light normally blocked by Earth's atmosphere, like ultraviolet radiation, can be observed. That is the reason this spectrum reaches down as far as 1640 Ångström in wavelength when ground based observatories normally are limited to wavelengths longer than 3000 Å.

The nebula is moving with respect to Earth at a considerable velocity. The spectrum therefore gets shifted in wavelength, according to the Doppler shift law for non-relativistic velocities: $\lambda_{obs} = (v/c + 1)\lambda_0$, where v is the radial relative velocity between the source and the observer and λ_0 is the rest wavelength. This means that a spectral line that resides at the wavelength λ_0, will be shifted to λ_{obs} if the relative velocity between the source and the observer is v along the line of sight.

Exercise 6 *Determine the relative velocity v between the observer and the nebula by measuring the difference between the rest wavelengths of spectral lines in Table 2 and the corresponding observed wavelengths in the spectrum. Use at least three different spectral lines, preferentially at different parts of the spectrum, and estimate the errors of your measurements by comparing the derived velocities for the different lines.* Hint: The difference between the tabulated wavelengths and measured wavelengths will not be huge; a few tens of Ångströms at most. Also, according to the Doppler shift law, the wavelength difference will increase for increasing wavelengths; it is not constant.

Exercise 7 *Identify which five lines in Table 2 are the strongest in the spectrum. It is enough to use the peak fluxes of the lines to judge which of the lines are the strongest. Again, there are many lines in the spectrum that are not listed in the table, but concentrate on those lines that are. Lines marked] or [ ] are called semi-forbidden and forbidden, respectively. They are only seen in the low-density gas of nebulae, and never in stars, where they are destroyed by the high frequency of collisions among the particles. Take care to properly compensate for the Doppler shift when you identify the lines, as you otherwise risk to misidentify lines that are close in wavelength.*

In the following exercises, you will determine abundances of elements by measuring line strengths. This can be done because the strength of a line is proportional to the number of elements, as long as the line is not optically thick, that is, as long as the mean free path for a photon in the line is greater than the size of the radiating medium.

Exercise 8 *Measure the flux in the Hβ line of hydrogen, the 5876 Å line of He I, and the 4686 Å line of He II. Be careful to identify the lines correctly. Vary the integration intervals slightly to understand how sensitive the integrals are to the particular choice of interval, and estimate the error of your measurements.*

The hydrogen and helium lines are recombination lines that arise when an electron is captured by an ion (when it "recombines"). The electron is then often captured to an excited state, that is, an energy level above the ground level, and subsequently transits down to levels of lower energy and ultimately to the ground level. During those transitions photons are emitted, the photons now observed in the spectrum, and the intensity of the line can be written $F_{\rm line} \propto C_{\rm line} n_{\rm ion} n_{\rm e}$, where $C_{\rm line}$ is a constant that quantifies how much energy is emitted by that particular line on average, for each recombination.

Since these lines are recombination lines, Hβ line measures the amount of H II, the 5876 Å line of He I measures the amount of He II, and the 4686 Å line of He II measures the amount of

He III. The measurements are thus not sensitive to the amount of neutral hydrogen and helium, but since this nebula around the supernova is highly ionised, we may assume that the neutral particles are negligible in comparison to the number of ionised particles. Thus we have $n_{\mathrm{H}} = n_{\mathrm{H\,I}} + n_{\mathrm{H\,II}} \approx n_{\mathrm{H\,II}}$ and $n_{\mathrm{He}} = n_{\mathrm{He\,I}} + n_{\mathrm{He\,II}} + n_{\mathrm{He\,III}} \approx n_{\mathrm{He\,II}} + n_{\mathrm{He\,III}}$.

Exercise 9 *Calculate the ratio $n_{\mathrm{He}}/n_{\mathrm{H}}$ from your measured line strengths. Use $C_{\lambda 5876}/C_{\mathrm{H}\beta} = 1.35$ and $C_{\lambda 4686}/C_{\mathrm{H}\beta} = 12.0$. Estimate the error of your derived ratio. Compare this ratio with the "normal" cosmic ratio $n_{\mathrm{He}}/n_{\mathrm{H}} \approx 0.09$. Does your ratio differ significantly from the cosmic ratio, and in that case, do you have any suggestion as to why?*