Introduction to Galaxies and Cosmology
Exercises I

Magnitudes and colours

1. A star of type G5V is observed in two photometric filters (B and V). The measured magnitudes are: $m_B = 18.3$ and $m_V = 15.8$. A G5V star has a temperature of around 5500 K (also see figure 1).
   a) What is the observed colour index of the star?
   b) Calculate the interstellar extinction in V ($A_V$) towards the star. What is the extinction-corrected V magnitude of the star?

2. The Sun is observed from a distant star to have an apparent blue magnitude ($B$) of 14.4 and an apparent visual magnitude ($V$) of 12.8. The Sun has a temperature of 5800 K and its absolute visual magnitude is +4.8. How far away is the star?

3. A star is observed to have an apparent V magnitude of 9.8 and a parallax of 0.002". If you assume that the extinction from the interstellar medium in the visual increases by 2 magnitudes per kpc, what is the V absolute magnitude of the star?

4. Two stars in a close binary system have $m_B = 18.2$ and $m_B = 19.6$, respectively. The first star has a colour $B - V = -0.2$ and the second $B - V = 0.5$. If this system is observed in a telescope which cannot resolve the two components, what would the integrated $m_B$ and $(B - V)$ of this object be?

5. When doing astronomical observations to measure magnitudes of objects (this is called imaging, as opposed to spectroscopy) the apparent magnitudes are found by calibrating the magnitude scale to a reference flux (this reference is often the flux observed from Vega). The magnitude scale is then fixed with a zeropoint magnitude constant. Another factor that’s considered when doing observations is the effect of the atmosphere on the observed flux. Depending on how high on the sky the object is situated when observed, more or less of the light from it will be absorbed. To find the apparent magnitude astronomers then use the formula:

   $$m = m_{zp} - 2.5 \log F + c \cdot X,$$

   with $m_{zp}$ the zeropoint magnitude, $F$ the measured flux (in dimensionless units) from the object, $c$ the atmospheric extinction
coefficient and $X$ the airmass (the higher up on the sky an object is, the lower the airmass will be).

In table 1 results of an observation of a F0 star is shown. This table also contains information on the calibration that has been done. Use the information in the table to find:

- a) The apparent B magnitude of the star.
- b) The apparent V magnitude of the star.
- c) Assume that the star has an effective temperature of 7500 K. Calculate $A_V$ for the star.

6. Galaxies with luminosities above or around $L_\star \approx 2 \cdot 10^{10} L_\odot$ are characterized as bright galaxies. This corresponds to a blue absolute magnitude of $M_{B,\star} \approx -20$. If the detection limit for a galaxy survey is given by $m_B = 25.0$, what is the furthest distance an $L_\star$ galaxy can be detected from (assume that the extinction is negligible)?

7. a) Calculate the $B - V$ colour of a galaxy which has $F_\lambda$ constant with wavelength.
   - b) Calculate the $B - V$ colour of a black body with temperature 10000 K (assume that the filters are narrow, i.e. calculate the value of the Planck function at the central wavelengths of the two filters, see the magnitude compendium).
   - c) A galaxy has $B - V = 0.5$ and $V - I = 0.9$, what is the $B - I$ colour?

8. If the distance modulus to a star or galaxy has an uncertainty of 0.1 magnitudes, show that this correspond to an uncertainty of $\approx 5\%$ in distance.

**Stellar properties and physics**

9. The black body spectrum of stars is directly determined by their effective temperature. In figure 2 the blackbody spectrum of Betelgeuse is given.
   - a) What is the effective temperature of Betelgeuse?
   - b) Look at the blackbody spectrum again, if you consider the colour index $B - V$, will the star be red or blue?
   - c) Consider table 1 in the magnitude compendium. The cooler stars are also the reddest ones. Why is this?
   - d) The bolometric correction is smallest for stars of type $\sim$F0, why?

10. Show how the the measured angular diameters and observed energy fluxes of stars can be used to measure their effective temperatures.
11. The effective temperature of Betelguese was found in exercise ??.
The angular radius of the star is 0.045" (arcsec) and the distance from earth is 140 pc. Calculate the the radius and luminosity of the star. Express your result in solar units ($R_\odot$, $L_\odot$). How would you classify this star?

12. Assume that the fusion of Hydrogen into Helium that takes place in the centre of the sun converts 0.7% of the Hydrogen rest mass into light.
   a) At what rate would it have to convert H into He in order to supply the luminosity that we observe?
   b) Assuming that only the central 10% (by mass) is available for fusion, how long would the Hydrogen last?

13. The stability of a star is maintained by the so called hydrostatic equilibrium. The equation for this equilibrium is given by:
\[ dP = -\frac{GM(r)}{r^2} \rho dr, \tag{2} \]
where $dP$ is differential pressure outwards, $G$ the gravitational constant, $M(r)$ the mass contained inside the radius $r$ and $\rho$ the density inside the star.
Assume that the interior of a star of total radius $R$ has a constant density $\rho_0$.
   a) Find the expression for $M(r)$.
   b) Insert this expression into equation 2 and integrate over radius to find an expression for the pressure at radius $r$. The density is zero for $r > R$ (i.e. outside the star) which then also means that there is no pressure outside the star.
   c) What is the central pressure of the Sun using this expression?

**Surface brightness**

14. A flat, two-dimensional disk galaxy has an exponentially declining surface brightness distribution given by $I(r) = I_0 e^{-r/a}$, where $a$ represents the scale length. Derive an approximate expression for the total luminosity of the disk.

15. A galaxy with the same surface brightness distribution as in the previous exercise has a scale length of 5 kpc and a rest-frame B-band luminosity of $L_B = 10^{10} L_\odot$.B.
   a) Derive an expression for $r$ as a function of $a$, $I(r)$ (in units of $L_\odot$/pc$^2$) and $L$.
   b) Derive an expression which converts surface brightness in units
of \( \text{mag/arcsec}^2 \) to \( L_\odot/\text{pc}^2 \) and show that surface brightness is independent of distance (as long as redshift dimming is neglected). What does a surface brightness of \( \mu_B = 27 \text{ mag/arcsec}^2 \) correspond to in units of \( L_\odot/\text{pc}^2 \)?

**Spectral lines from stars and galaxies**

16. The level populations and ionisation fraction of atoms is dependent on the temperature. For hydrogen in thermodynamic equilibrium the *Saha* equation relates \( n_1 \), the number density of hydrogen in the lowest energy level (i.e. the ground state), to \( n_p \), the number density of protons (i.e. ionised hydrogen):

\[
\frac{n_1}{n_p} = 4.14 \cdot 10^{-22} n_e T^{-3/2} e^{\chi_I/kT},
\]

\( \chi_I \) is the ionization energy of Hydrogen (13.6 eV), \( n_e \) the electron density and \( k \) the Boltzmann constant. Assume that \( n_e \approx 10^{21} \text{ m}^{-3} \). What is the ionization fraction \( (n_p/n_1) \) for:

a) a G0 star
b) a B5 star
c) The line strength for the \( H/\alpha \) line in stellar atmospheres depends on the temperature. For temperatures below \( \sim 10000 \) K the line strength increases with temperature, but above 10000 the line strengths starts to drop off. Try to explain why (this is a quite difficult, check the course literature if you’re uncertain).

17. The galaxy ESO 114-G07 is a low surface brightness galaxy. Observations of one of its HII regions give the emission line fluxes listed in table . Theory predicts a ratio \( F(H_\alpha)/F(H_\beta) \approx 2.85 \) at densities and temperatures typical of HII regions. The extinction due to dust at wavelength \( \lambda \) is given by \( I_\lambda = I_{\lambda 0} e^{-\tau_\lambda} \), where \( \tau_\lambda \) represents the optical depth and may be approximated by \( \tau_\lambda = Cf(\lambda) \). Values of \( f(\lambda) \) for a standard extinction curve can be found in Table 2.

a) Calculate the constant \( C \) using the data and the predicted \( F(H_\alpha)/F(H_\beta) \) ratio.
b) Correct the observed emission line ratios for extinction.
c) The observed colours of this galaxy is \( B - V = 0.5 \). What is the intrinsic colour of the stellar population when corrected for dust reddening (use table ).
Figure 1: Theoretical \((B - V)\) colour index vs. stellar effective temperature (figure from Sekiguchi & Fukugita, 2000).

Figure 2: A blackbody curve with the same temperature as Betelgeuse.
Table 1: Observations of a F0 star

<table>
<thead>
<tr>
<th>Filter</th>
<th>$F$</th>
<th>$X$</th>
<th>$m_{zp}$</th>
<th>$c$</th>
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</thead>
<tbody>
<tr>
<td>$B$</td>
<td>92000</td>
<td>1.3</td>
<td>25.62</td>
<td>-0.28</td>
</tr>
<tr>
<td>$V$</td>
<td>212000</td>
<td>1.5</td>
<td>25.43</td>
<td>-0.18</td>
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</table>

Table 2: Observed emission line fluxes

<table>
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<tr>
<th>$\lambda$ (Å)</th>
<th>Element</th>
<th>Flux ($F(H_\beta)=100$)</th>
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</thead>
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<tr>
<td>3727</td>
<td>[OII]</td>
<td>127</td>
</tr>
<tr>
<td>5007</td>
<td>[OIII]</td>
<td>531</td>
</tr>
<tr>
<td>4861</td>
<td>$H_\beta$</td>
<td>100</td>
</tr>
<tr>
<td>6563</td>
<td>$H_\alpha$</td>
<td>307</td>
</tr>
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</table>

Table 3: Standard interstellar extinction curve

<table>
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<tr>
<th>$\lambda$ (Å)</th>
<th>$f(\lambda)-f(H_\beta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20000</td>
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<td>12500</td>
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<tr>
<td>10000</td>
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<tr>
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<tr>
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<td>-0.00</td>
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<tr>
<td>4545</td>
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<tr>
<td>4167</td>
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<tr>
<td>4000</td>
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<tr>
<td>3846</td>
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<tr>
<td>3727</td>
<td>0.33</td>
</tr>
<tr>
<td>3571</td>
<td>0.39</td>
</tr>
</tbody>
</table>
Answers, Exercise I

1. a) \((B - V)_{obs} = 2.5\)
   b) \(A_V = 5.4\) (using \((B - V)_0 = 0.70\) and \(m_{V,corr} = 10.4\))

2. \(d = 100\) pc

3. \(M_V = 0.31\)

4. \(m_{V,tot} = 17.94, (B - V)_{tot} = 0\)

5. a) \(m_B = 12.85\)
   b) \(m_V = 11.84\)
   c) \(A_V = 2.3\) (using \((B - V)_0 = 0.25\))

6. \(d = 10\) Gpc

7. a) \(B - V = -0.61\)
   b) \(B - V = 0.12\)
   c) \(B - I = 1.4\)

8. –

9. a) \(T_{eff} = 3500\) K
   b) –
   c) –
   d) –

10. –

11. \(R = 676 R_\odot, L = 6.1 \cdot 10^4 L_\odot, K/M\) supergiant

12. a) \(\dot{M} = 6.127 \cdot 10^{11}\) kg/s
   b) \(\tau = 7.3 \cdot 10^9\) yrs

13. a) \(M(r) = \frac{4\pi r^2 \rho_0}{3}\)
   b) \(P(r) = \frac{2 \pi G \rho_0 R^2}{3} \left(1 - \left(\frac{r}{R}\right)^2\right)\)
   c) \(P_c = 1.34 \cdot 10^{14}\) Pa

14. \(L = 2\pi I_0 a^2\)

15. a) \(r = -a ln \left(\frac{2\pi a^2 I(r)}{L}\right)\)
   b) \(I[L_\odot/(pc^2)] = \frac{1.6796 \cdot 10^{12}}{(2\pi a_{arcsec})^2} \cdot 10^\left(\frac{M_B + 5 - M_{B,\odot}}{-2.5}\right)\)

16. a) \(\frac{M}{M_1} = 4.23 \cdot 10^{-6}\)
   b) \(\frac{m}{m_1} = 140.1\)
17. a) $c = 0.2125$
   b) The corrected ratios for lines at 3727 and 5007 respectively are: 1.362 and 5.357
   c) –