



Late Stages of Stellar Evolution

Low to Intermediate Mass Stars

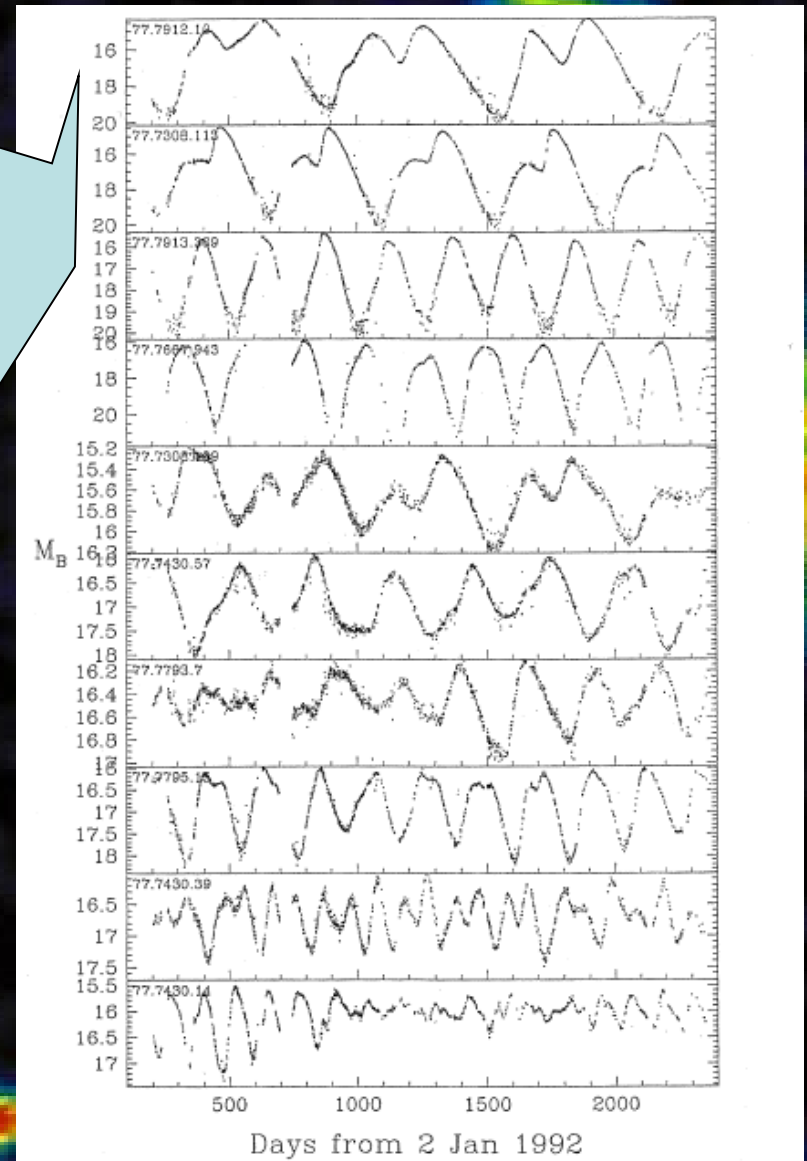
Section 5: Pulsations

Pulsations

- AGB stars are variable stars, due to the fact that they are pulsating.
- The periods are long, hence the designation *Long Period Variables* (LPVs): $P=10 - 1000\text{s}$ days.
- Until the arrival of large photometric surveys (MACHO, OGLE), the long periods made it difficult to derive light curves.
- The first historic variable star, α Ceti, was discovered in 1596 by David Fabricius, and is now known as *Mira*, the prototype of a class of variable AGB stars.

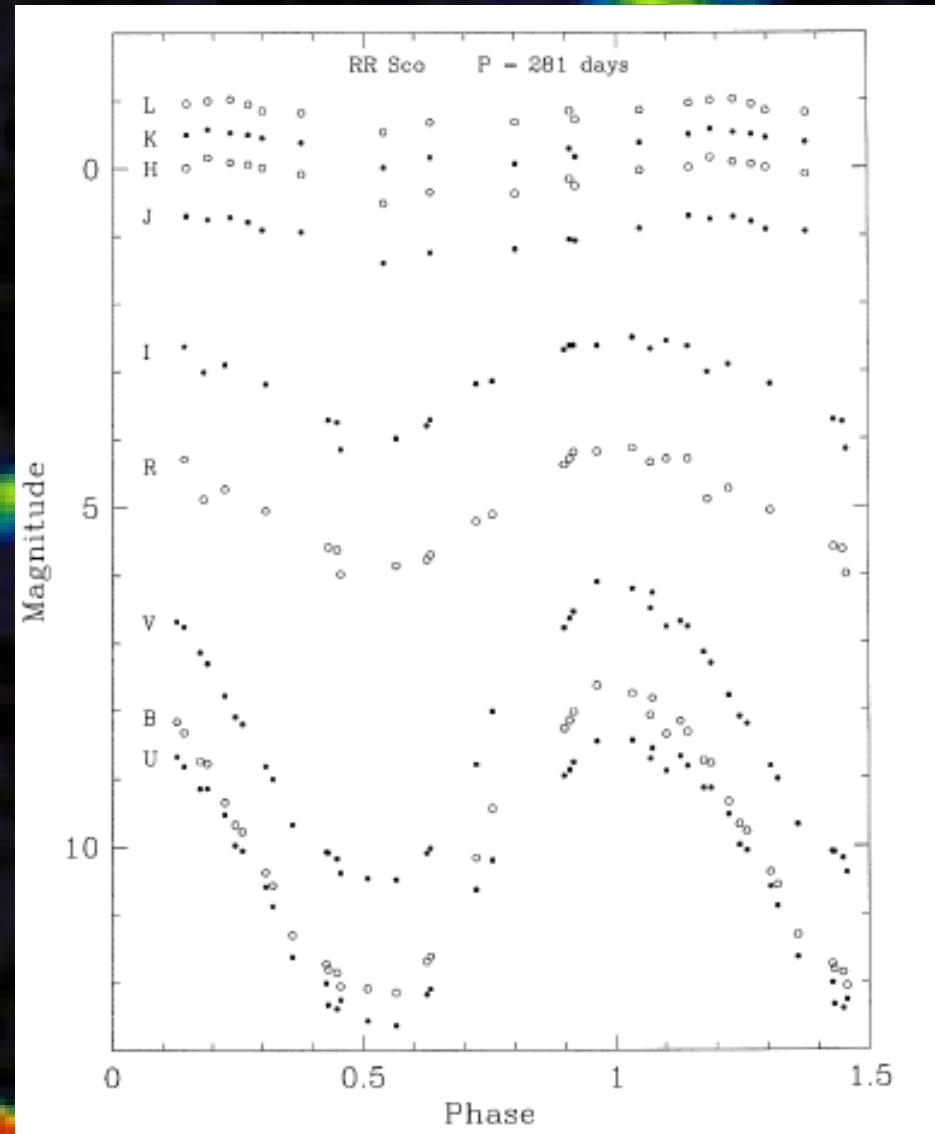
Types

- **Mira variables:** regular light curves, with amplitudes $\Delta V > 2.5^m$ and $\Pi > 100$ days.
- **Semi-regular variables (SRVs):** fairly regular light curves, $\Delta V < 2.5^m$ and $\Pi > 20$ days.
- **Irregular variables (Irr):** irregular light curves, small ΔV and no clear period.



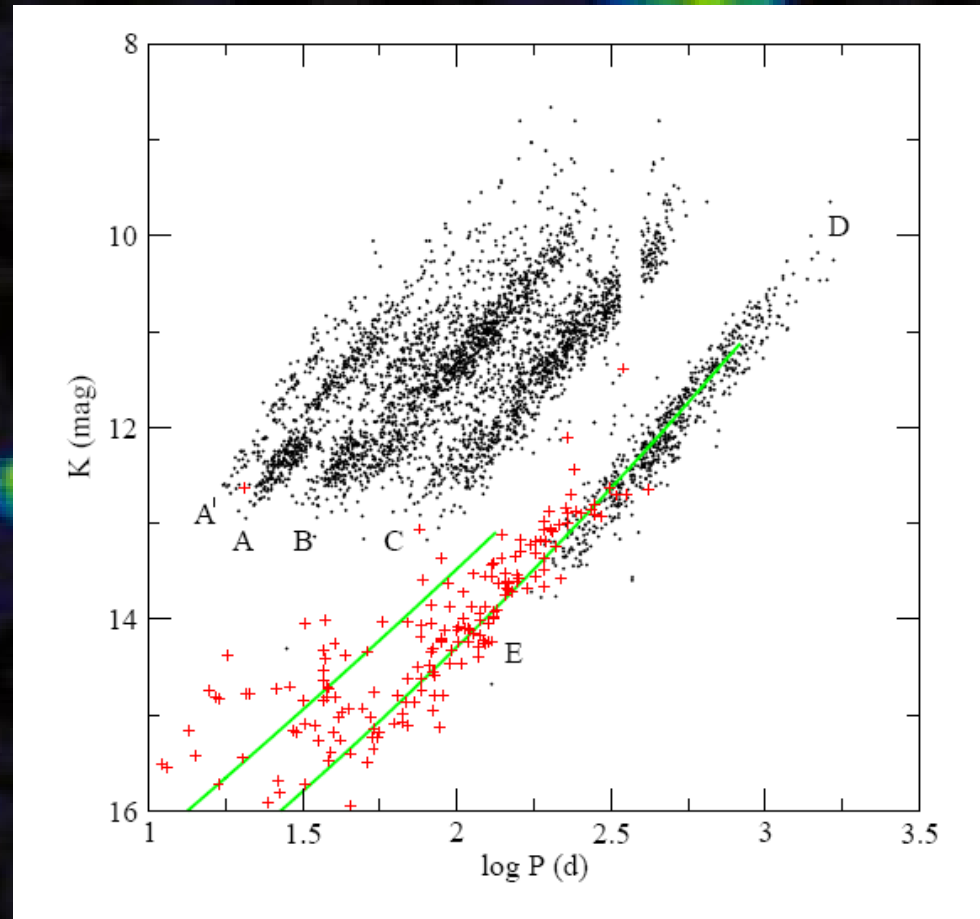
Classification Caveats

- The amplitudes used in the definitions are based on observations in the V band. The bolometric variations and the variations in the IR bands are in fact much smaller.
- Many Irr's just suffer from badly sampled light curves.
- Some AGB stars do not have optical emission (e.g. OH-IR stars), but are still variable!



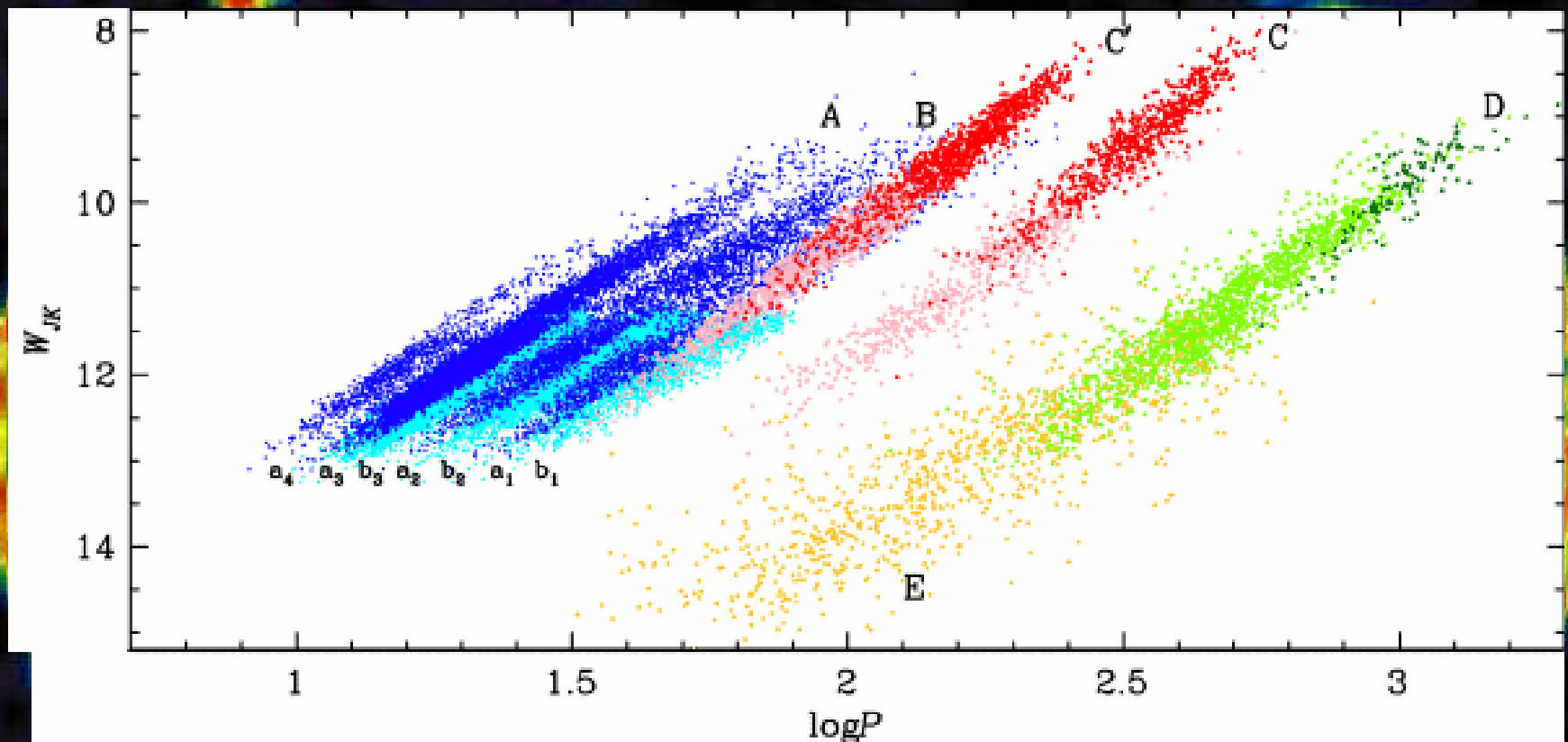
Period-Luminosity Relations

- Photometric survey data has allowed the mapping of a number of period-luminosity relations.
- The ratios between periods suggest:
 - C: fundamental mode pulsators (Miras)
 - B: first overtone pulsators (SRVs).
 - A, A': higher overtone pulsators (SRVs).
- Sequence D & E are probably related to binaries.



Macho data

OGLE data



- Better quality data shows more lower Π variables, including RGB stars (OGLE Low Amplitude Red Giants, OLARGs).
- Miras in C (fundamental), SVRs in C' (1st overtone).

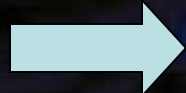
Period-Density Relation

- The fundamental mode is a radial pulsation with its wavelength equal to the stellar diameter:

$$\Pi = 2R_*/c_s$$

- If the pulsations are around an equilibrium solution, we can use the Virial Theorem to find the Π -L relation:

$$-\Omega_{\text{grav}} = 2K_{\text{thermal}}$$



$$2K_{\text{thermal}} = 3 \int_0^{R_*} 4\pi r^2 p dr$$

$$\frac{dm}{dr} = 4\pi r^2 \rho$$

$$-\Omega_{\text{grav}} = \alpha \frac{GM_*^2}{R_*}$$

$$2K_{\text{thermal}} = 3 \int_0^{R_*} \frac{c_s^2}{\gamma_{\text{ad}}} dm \approx 3 \frac{c_s^2}{\gamma_{\text{ad}}} M_*$$

$$\Pi = 2 \sqrt{\frac{3}{\gamma_{\text{ad}} \alpha G}} R_*^{\frac{3}{2}} M_*^{-\frac{1}{2}} \equiv Q R_*^{\frac{3}{2}} M_*^{-\frac{1}{2}} \approx (G \langle \rho \rangle)^{-\frac{1}{2}}$$

Period-Luminosity Relation

$$\Pi \approx (G\langle\rho\rangle)^{-\frac{1}{2}}$$

$$L = 4\pi R^2 \sigma T_{\text{eff}}^4$$

$$L \propto \left(\frac{\Pi}{Q} T_{\text{eff}}^3 M^{1/2} \right)^{4/3}$$

- More detailed analysis considering higher harmonics we find the relation for the periods of different pulsation modes:

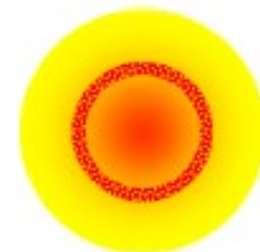
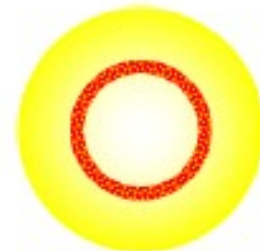
$$\Pi_n \propto \sqrt{n+1} \langle\rho\rangle^{-\frac{1}{2}}$$

Stellar Pulsations as Heat Engine

At one point in the pulsation cycle, a layer of **stellar** material loses support against the star's **gravity** and falls inwards.

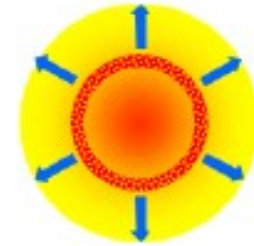
This inward motion tends to compress the layer, which heats up and becomes more opaque to radiation.

Since radiation diffuses more slowly through the layer (as a consequence of its increased opacity), heat builds up beneath it.

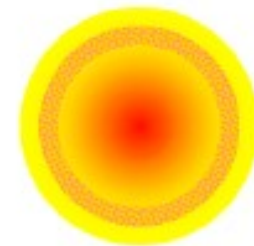


Stellar Pulsations as Heat Engine

The pressure rises below the layer, pushing it outwards.



As it moves outwards, the layer expands, cools, and becomes more transparent to radiation.



Energy can now escape from below the layer, and pressure beneath the layer drops.

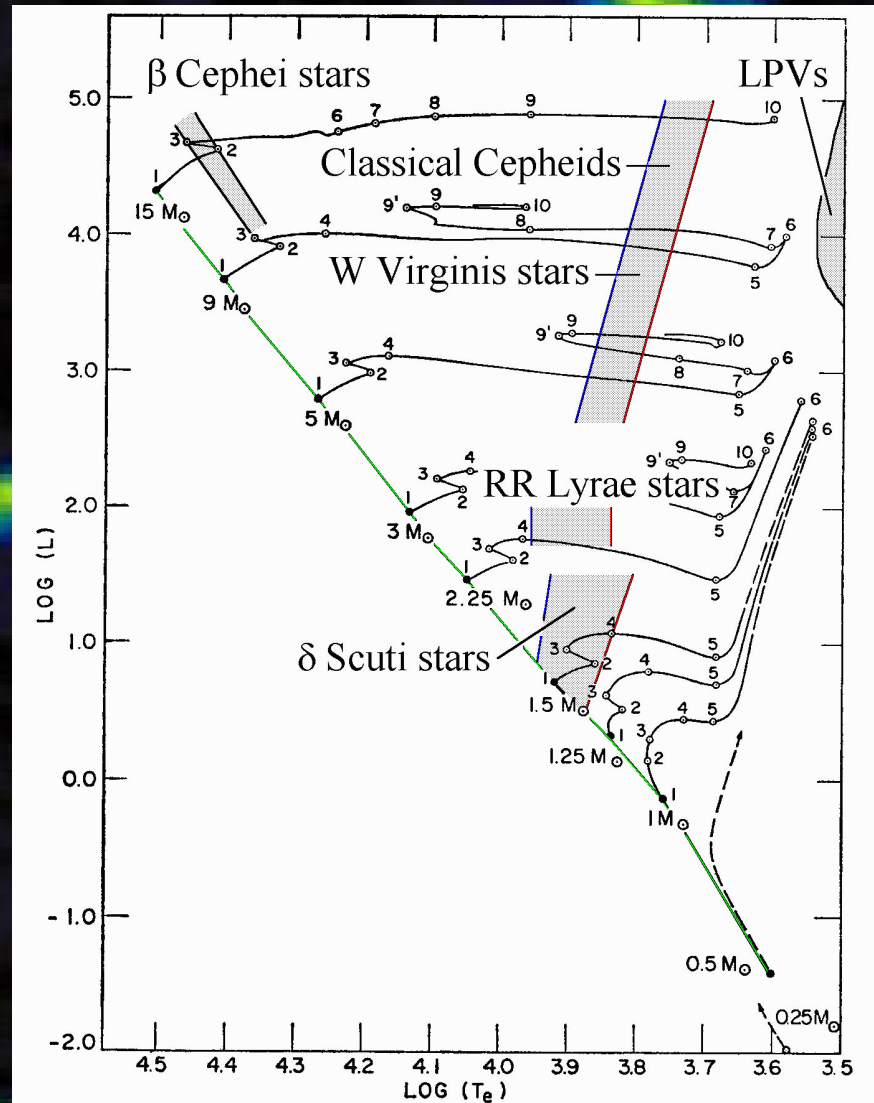


κ -mechanism

- For this to work, we need the opacity to increase upon compression, this is known as the κ -mechanism
- Typical opacity (Kramer's opacity): $\kappa \propto \rho T^{-3.5}$
- This opacity law does *not* have the desired effect: (adiabatic) compression raises the density, but also the temperature, so the opacity *drops*.
- Solution: Partial ionization zones ($\text{H} \rightarrow \text{H}^+$, $\text{He} \rightarrow \text{He}^+$, $\text{He}^+ \rightarrow \text{He}^{2+}$): compression energy used to ionize, rather than increase the temperature.
- This works well for classical variables such as Cepheids and RR Lyrae stars.

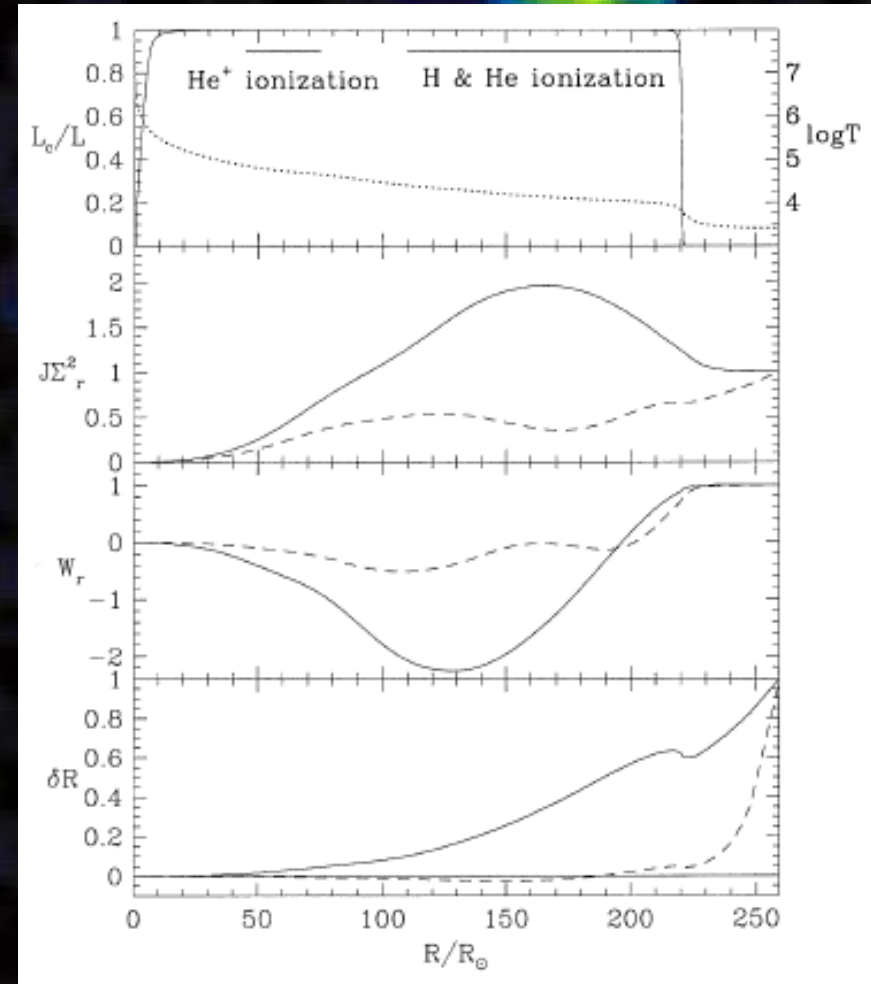
Instability Strip

- Instability strip boundaries:
- **Blue edge** ($T_{\text{eff}} \approx 7500$ K): partial ionization zone too close to the surface to drive pulsations.
- **Red edge** ($T_{\text{eff}} \approx 5500$ K): energy transport mostly through convection, so insensitive to opacity.
- So, how does this work for our cool AGB stars??



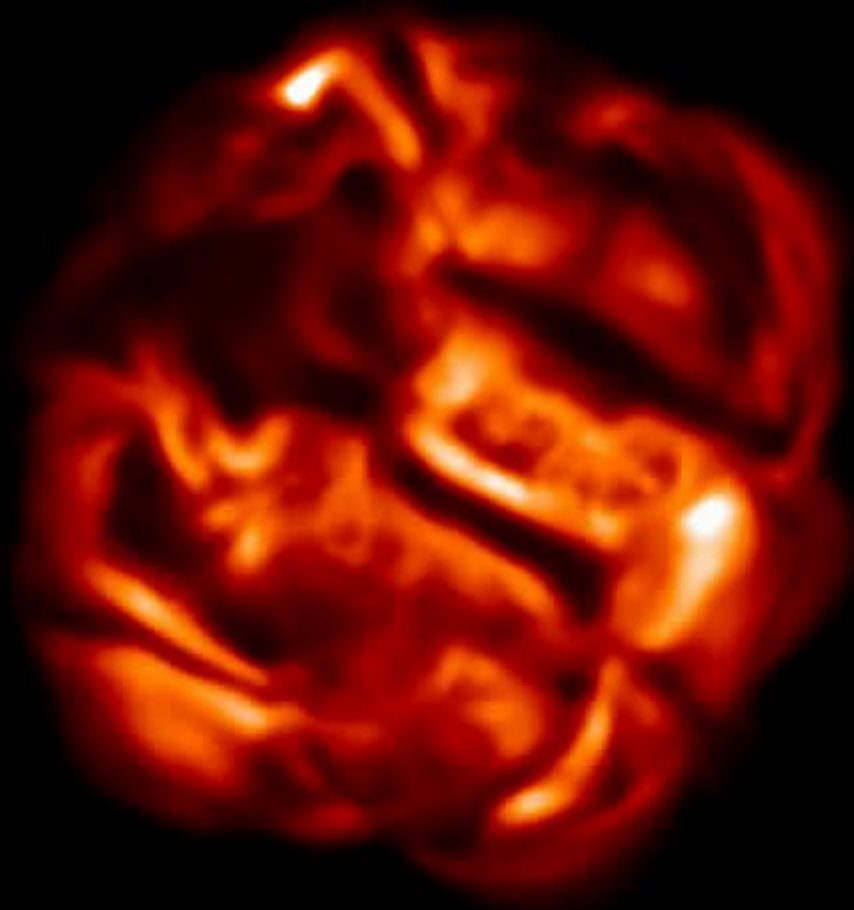
Pulsation Models for AGB Stars

- Modelling the pulsational behaviour of AGB stars requires a description of the changes in the convection zone during pulsations.
- We only have approximate models for convection ('mixing length theory'), so all these pulsational models are flawed.
- The existing models still point to the importance of the partial ionization zones.



Maybe We Need This?

st35gm04n26: Surface Intensity(3l), time(44.5)=30.952 yrs



- Bernd Freytag, 3D radiation-hydrodynamic models.
- 'Betelgeuse' (red supergiant), $5 M_{\odot}$
- $R=600 R_{\odot}$
- $L=41400 L_{\odot}$
- Duration of movie: 7.5 yrs