Lecture 16 Pre-collapse phase II

A basic concept governing the physics of interstellar clouds, and the observations of them, is <u>Temperature</u>. There are many different kinds of temperatures defined in astrophysics, and we will discuss some of the following:

- Excitation temperature T_{ex}
- Gas (kinetic) temperature, T_{gas}
- Dust temperature, T_{dust}
- Antenna temperature, T_B.

These temperatures can be the same for a system, but are not in general. Often other temperatures are defined as well, but they generally fall into the above categories. (EX: Rotation temp, nuclear spin temp etc)

Excitation temperature T_{ex}

T_{ex} is defined for line transitions so that the line emission intensity corresponds to the emission from a blackbody of temperature T_{ex}. More specifically,

$$T_{ex}: \underbrace{B_\nu}_{\text{Planck}}(T_{ex}) = \underbrace{S_\nu}_{\text{line soure function}}(\text{line}),$$

$$\left[\begin{array}{l}\text{cf. radiative transfer eq.} \\ \dfrac{dI_\nu}{d\tau} = S_\nu - I_\nu\end{array}\right]$$

where

$$S_\nu(\text{line}) = \frac{2h\nu_{ij}^3}{c^2} \frac{1}{\frac{n_i}{n_j}\frac{g_j}{g_i}\frac{\varphi_{abs}(\nu)}{\psi_{em}(\nu)} - 1}.$$

Occupation number (n_i, n_j); statistical weight of level (g_j, g_i)

φ_{abs} and ψ_{em} are the absorption and emission profiles. When "complete redistribution" applies, $\varphi_{abs} = \psi_{em}$.

Note that it is not necessary for the emitting medium to actually be in thermal equilibrium for Tex to be well defined. In general, the population of energy levels are obtained from the statistical equilibrium equations:

$$\frac{dn_j}{dt} = \begin{Bmatrix} \text{transitions} \\ \text{to } j \end{Bmatrix} - \begin{Bmatrix} \text{transitions} \\ \text{from } j \end{Bmatrix} =$$

$$= \sum_k \sum_i n_i R_{ij}^{(k)} - n_j \sum_k \sum_i R_{ji}^{(k)} = 0$$

where $\{k\}$ enumerates different processes (e.g. spontaneous emission, collisional excitation... etc) and $R_{ij}^{(k)}$ is the <u>rate</u> for a single particle in state (i) to transit to state (j) by process (k). In general, some rates (e.g. radiative excitation) depend on the radiation field, and hence on the solution to the radiative transfer equation. Since the radiative transfer also depend on the level populations, we have to solve radiative transfer and level population simultaneously.

Gas and dust temperatures T_{gas} and T_{dust}

Because interstellar clouds are often transparent to thermal radiation, the heating and cooling processes for gas and dust can cause them to have different temperatures; $T_{gas} \neq T_{dust}$. The temperature is related to the energy density through the energy balance equation

$$\frac{d\mathcal{E}}{dt} = \sum_i \Gamma_i(n,T) - \sum_j \Lambda_j(n,T) - \vec{\nabla}\cdot(\sigma\vec{\nabla}T),$$

where Γ_i are heating terms (usually expressed in $erg\ cm^{-3} s^{-1}$), Λ_i are cooling terms, and the last term is conduction (with conductivity σ) which we usually neglect in the ISM.

In steady state, equilibrium is reached in a cooling time

$$\tau_{cool} \approx \frac{\mathcal{E}}{\sum_j \Lambda_j(n,T)} .\quad \text{With } \frac{d\mathcal{E}}{dt} = 0$$

and neglecting conduction, we equate heating with cooling, $\sum_i \Gamma_i(n,T) = \sum_j \Lambda_j(n,T)$.

Examples of important heating mechanisms in the ISM are photoelectric emission, cosmic rays, shocks, friction (gas-dust, can also act as cooler). Cooling mechanisms are forbidden atomic transition line emission, molecular emission, adiabatic expansion.

Heating Γ!

- Cosmic ray heating

$$\Gamma_{CR} = \sigma_0 \, n_{H_2} \, \Delta E .$$

$\sigma_0 \Delta E$ is determined empirically to be $\sigma_0 \Delta E \sim 10^{-28}$ erg s^{-1} molecule^{-1}.

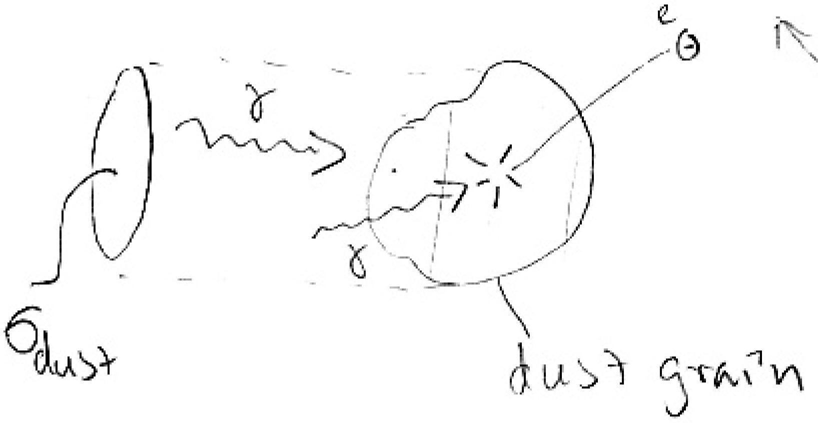
$\uparrow$ H^0

- Photoelectric heating, due to photons kicking out electrons from dust and thus heating the gas

$$\Gamma_{PE} = 4\pi \, n_{dust} \, \sigma_{dust} \underbrace{\varepsilon_{PE}}_{\text{efficiency}} \int_{FUV} J_\nu \, d\nu$$

$\uparrow$ mean intensity

FUV is $\gamma \gtrsim 6$ eV



If the only source of UV photons is the stellar background (the "Habing flux") then $\Gamma_{PE} = 3 \times 10^{-11} \left(\frac{n_H}{10^3 \text{cm}^{-3}} \right)$ eV cm^{-3} s^{-1}.

- Gas - dust collisions

$\Gamma_{g\text{-}d} \propto (T_{dust} - T_{gas})$

If $T_{dust} > T_{gas}$, the gas is heated by the dust, if $T_{dust} < T_{gas}$ it is cooled.

Cooling Λ_i

- Spectral line: $\Lambda_{line} = h\nu_{ij}\, n_i C_{ij}$

where C_{ij} is the collision rate coefficient from state i to j. The assumption is that all collisional transitions from i to j result in a radiative transition from j to i, whith the energy beins lost from the cloud through the photon (also assuming the cloud is optically thin!).

Alternatively, if the levels $\{n_i\}$ are determined from statistical equilibrium, the cooling rate can be determined as $\Lambda_{line} = h\nu_{ij} n_i A_{ij}$, where A_{ij} is the Einstein coefficient for spontaneous emission.

- Dust cooling: Dust radiates from its surface as a modified blackbody

$$\Lambda_{dust} = \frac{1}{c} \oint \int \underbrace{\kappa_\nu}_{\text{opacity}} B_\nu (T_{dust}) \, d\nu \, d\Omega$$

- Expansion:

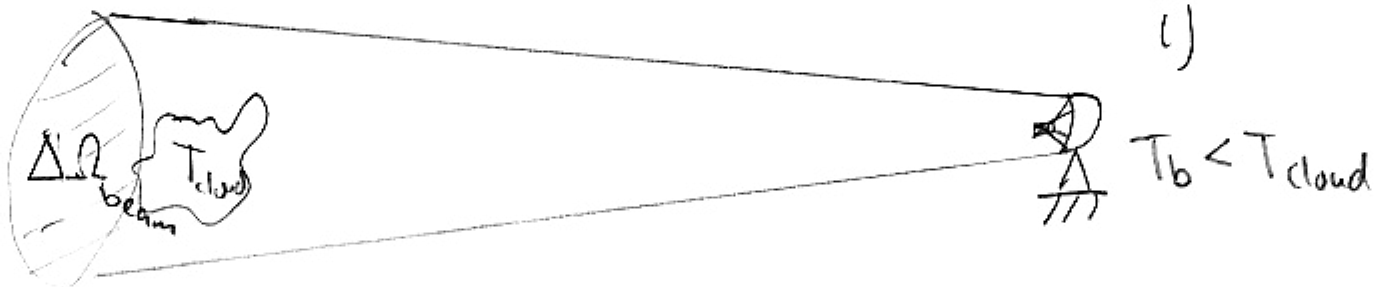
$$\Lambda_{exp} = -\frac{DU}{Dt} = -\frac{P}{\rho}\frac{D\rho}{Dt} - U \bar{\nabla} \cdot \bar{v} \approx kT \frac{dn}{dt} = \underbrace{nkT\left[\frac{a}{v} + \frac{2v}{r}\right]}_{\text{wind expansion}}$$

energy density (U), pressure (P), density (ρ), velocity ($\bar{v}$), acceleration (a)

"convective" derivative:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \bar{v} \cdot \bar{\nabla}$$

Antenna temperature



The antenna temperature is the temperature of a black body that would produce the measured intensity (averaged over the beam). In case (ii) the cloud fills the beam, and $T_b = T_{cloud}$. In case (i), the beam is larger than the solid angle of the cloud, introducing "beam dilution"; the antenna temperature is then derived from

$$B(T_{cloud}) \cdot \Delta\Omega_{cloud} = B(T_b) \cdot \Delta\Omega_{beam}.$$

The virial theorem

Why don't giant molecular cloud collapse onto themselves, what gives them the support to counteract gravity?

To approach the answer to this question we study the virial theorem.

Recall the equation of hydrostatic equilibrium:

pressure, gravitational potential

$$-\nabla P - \rho \nabla \Phi_g = 0.$$

density

We are now generalising it to allow for time-dependent moving fluid elements, in analogy with Newton's law $F = ma$:

fluid velocity

$$\rho \frac{D\bar{u}}{Dt} = -\nabla P - \rho \nabla \Phi_g + \frac{1}{c} \bar{j} \times \bar{B},$$

where $\frac{D}{Dt} = \frac{\partial}{\partial t} + \bar{u} \cdot \bar{\nabla}$ again is the "convective" derivative, and $\frac{1}{c} \bar{j} \times \bar{B}$ is the Lorentz force from a magnetic

field $\bar{B}$ acting on a current density $\bar{j}$.
These are related through Ampère's law

$$\bar{\nabla} \times \bar{B} = \frac{4\pi}{c} \bar{j} ,$$

so

$$\frac{1}{c} \bar{j} \times \bar{B} = \frac{1}{4\pi} (\bar{\nabla} \times \bar{B}) \times \bar{B} =$$

$$= \frac{1}{4\pi} (\bar{B} \cdot \bar{\nabla}) \bar{B} - \frac{1}{8\pi} \bar{\nabla} |\bar{B}|^2 ,$$ where

we have used the vector identity for triple cross products,

$$\bar{A} \times (\bar{B} \times \bar{C}) = (\bar{A} \cdot \bar{C}) \bar{B} - (\bar{A} \cdot \bar{B}) \bar{C} .$$

The first term expresses the tension from the curvature of magnetic field lines, while the second is the gradient of the <u>magnetic pressure</u> $\frac{|B|^2}{8\pi}$, resulting from the crowding of magnetic field lines.

Summarising, we have the equation

$$\rho \frac{D\bar{u}}{Dt} = -\vec{\nabla} P - \rho \nabla \phi_g + \frac{1}{4\pi}(\bar{B}\cdot\vec{\nabla})\bar{B} - \frac{1}{8\pi}\vec{\nabla}|\bar{B}|^2$$

which describes the local behaviour of a fluid under various forces. To study the global properties of the fluid, we integrate the equation over the full volume. This is a bit tedious but is shown in Appendix D of S&P. The resulting equation has the same terms but in integrated form:

$$\frac{1}{2}\frac{\partial^2 I}{\partial t^2} = 2T + 2U + W + M,$$

where $I \equiv \int \rho |\bar{r}|^2 dV$ is reminiscent of "moment of inertia"

$T = \frac{1}{2}\int \rho |\bar{u}|^2 dV$ is the total kinetic energy ("bulk" motion)

$U = \frac{3}{2}\int n k_B T \, dV$ is the total thermal energy

$W = \frac{1}{2}\int \rho \Phi_g \, dV$ is the gravitational potential energy

$\mathcal{M} = \frac{1}{8\pi}\int |\bar{B}|^2 dV$ is the magnetic energy. Note: only W negative.

If $|2T + 2U + \mathcal{M}| < |W|$ we have a gravitational collapse. Assuming $|W|$ is completely dominating, we have free fall. With M being the mass of the cloud, and R its characteristic radius, $W \approx -\frac{GM^2}{R}$.

Approximating $I \approx MR^2$, then

$$\frac{1}{2}\frac{\partial^2 I}{\partial t^2} = W \Rightarrow \frac{1}{2}\frac{\partial^2 (MR^2)}{\partial t^2} = -\frac{GM^2}{R},$$ (16:14)

or $$\frac{\partial^2 R^2}{\partial t^2} = \frac{2GM}{R}.$$ Sloppy hand-waving

$$\Rightarrow \frac{\partial^2 R^2}{\partial t^2} \approx \frac{R^2}{\tau_{ff}^2} = \frac{2GM}{R} \Rightarrow \tau_{ff} = \sqrt{\frac{R^3}{2GM}}$$

$$= 7\times10^6\,\mathrm{yr}\left(\frac{M}{10^5 M_\odot}\right)^{-\frac{1}{2}}\left(\frac{R}{25\,\mathrm{pc}}\right)^{\frac{3}{2}}.$$

This hints that a GMC should
= giant molecular cloud
collapse within a few Myr, while they are observed to last longer.
The condition for long-term stability is $\frac{\partial^2 I}{\partial t^2} \approx 0$, which results in the

<u>virial theorem</u>: $\boxed{2T + 2U + W + \mathcal{M} = 0}$

In stars, the thermal pressure balances gravity. Could this be the case for

GMCs? If we focus on the term (16:15)

$$U = \frac{3}{2}\int P\,dV = \frac{3}{2}\int n k_B T\,dV \approx$$

$$\approx \frac{M \mathcal{R} T}{\mu} \quad (\mathcal{R}, \text{gas constant})$$

then

$$\left|\frac{U}{W}\right| \approx \frac{M\mathcal{R}T}{\mu}\left(\frac{GM^2}{R}\right)^{-1} =$$

$$= 3\times10^{-3}\left(\frac{M}{10^5 M_\odot}\right)^{-1}\left(\frac{R}{25\,\mathrm{pc}}\right)\left(\frac{T}{15\,\mathrm{K}}\right).$$

Thus, for typical GMC values, $|U| \ll |W|$, and <u>cannot</u> be responsible for the support. What about magnetic support? $\mathcal{M} \approx \frac{|B|^2 R^3}{6\pi}$, so

$$\left|\frac{\mathcal{M}}{W}\right| \approx \frac{|B|^2 R^3}{6\pi}\left(\frac{GM^2}{R}\right)^{-1} =$$

$$= 0.3\left(\frac{B}{20\,\mu G}\right)^2\left(\frac{R}{25\,\mathrm{pc}}\right)^4\left(\frac{M}{10^5 M_\odot}\right)^{-2}.$$

Quite close! But magnetic pressure has the peculiarity of only working <u>across</u> the field lines, thus being highly anisotropic. Along the field lines the cloud could in principle still collapse, forming a huge sheet. A solution might be that the magnetic field is not well ordered on large scales, so that there is no preferred direction. These distortions of a global magnetic field can arise from MHD waves, something we will get back to later.

Finally, let's check the last term T, corresponding to bulk motion of the cloud. $T \propto \frac{1}{2} M \Delta v^2$, so

$$\frac{T}{|W|} = 0.5 \left(\frac{\Delta v}{4\,\mathrm{km\,s^{-1}}}\right)^2 \left(\frac{M}{10^5 M_\odot}\right)^{-1} \left(\frac{R}{25\,\mathrm{pc}}\right)$$

(4 km/s is typical dispersion in GMCs).