5 Pulsations

Most AGB stars are variable stars, due to the fact that they pulsate. The periods are long, from 10 to 1000s of days, and they are therefore called *Long Period Variables*, or LPVs. These long periods make it time-consuming to collect accurate light curves. In recent years a lot of progress has been made in this area due to several large area surveys programs (aimed at the more fashionable subject of micro-lensing) such as MACHO, OGLE, EROS and MOA.
Note however that the first ever variable star discovered in modern astronomy was an AGB star, o Ceti (Fabricius, 1596), from then on known as *Mira*. When later stars with similar variability characteristics were discovered these were called Mira variables.
The LPVs are observationally divided into several classes (see Fig. 15)

Mira variables They have very regular light curves, with amplitude $\Delta V > 2.5^{\mathrm{m}}$, and periods $\Pi > 100$ days.

Semi-regular variables (SRVs) These have fairly regular light curves with $\Delta V < 2.5^{\mathrm{m}}$, and $\Pi > 20$ days.

Irregular variables (Irr) These have irregular light curves, with small ΔV and no clear period.

Note that these definitions are based on the *optical* light curves. The bolometric variations in luminosity are often much less dramatic, see Fig. 16. It is also true that many Irr's just suffer from a poor sampling of their light curves, and on closer inspection do show signs of specific period(s).
Some AGB stars are so heavily obscured by their circumstellar material that they have no observable optical emission. The so-called OH-IR stars are only observable in the infrared (and from their OH maser emission), and are variable with very long periods, more than 500 days.

5.1 Period-Luminosity Relations

As expected from the theory of pulsations, relations exist between period and luminosity. A major breakthrough was made after the MACHO results allowed the determination of Π and L for a large sample of AGB stars in the Large Magellanic Cloud (LMC), see Fig. 17. A series of relations can be discerned, denoted by A, B, C, D, E in the figure. The ratios between the A, B, and C periods suggest that

C fundamental mode radial pulsations (Miras)

B first overtone mode radial pulsations (SRVs)

A second overtone mode radial pulsations (SRVs)

Relation D is a mystery, either it represents another type of pulsation (non-radial), or it is connected to binarity. Relation E is definitely associated with binaries (Derekas et al. 2006).

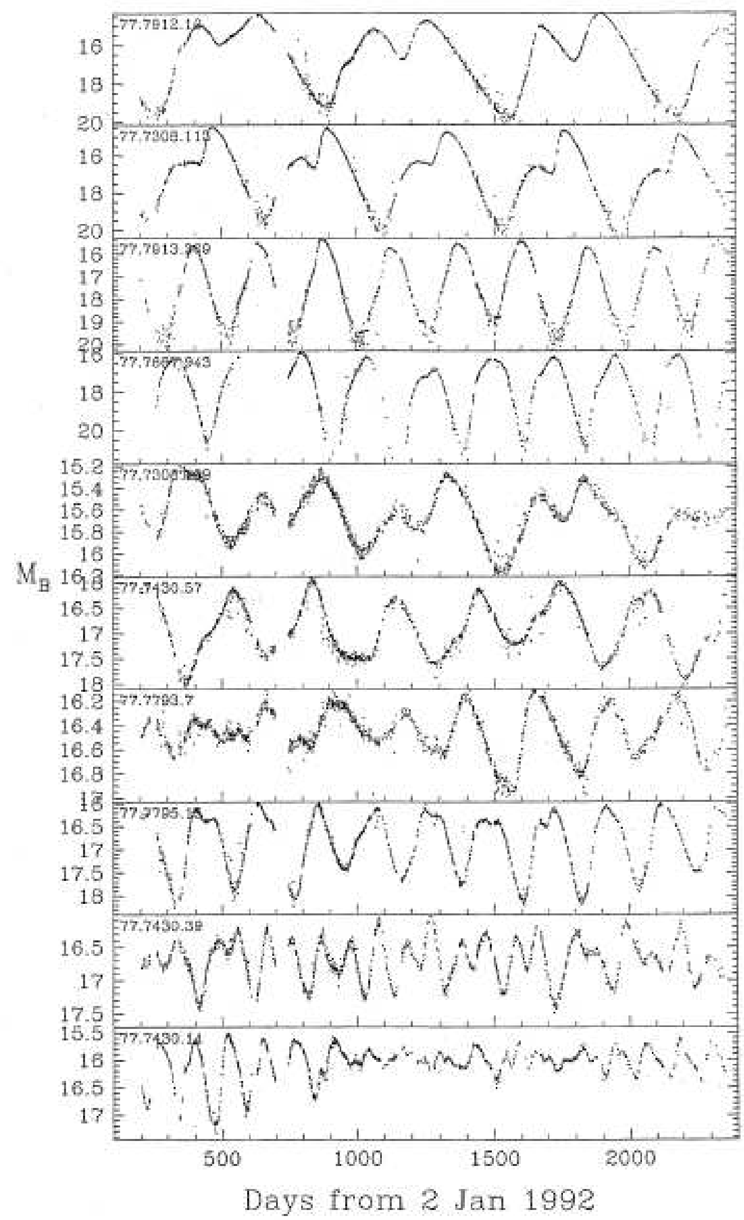

Figure 15: Examples of light curves of Mira variables (top four panels) and semiregular variables (bottom six panels) in the LMC from the Macho database. The bandpass of the lightcurves is MACHO blue, which is centered near 0.53μm, similar to the visual bandpass. All red giants in the MACHO database seem to show distinct periodicities at some times, but without high quality light curves such as these, they would possibly be classified as irregular.

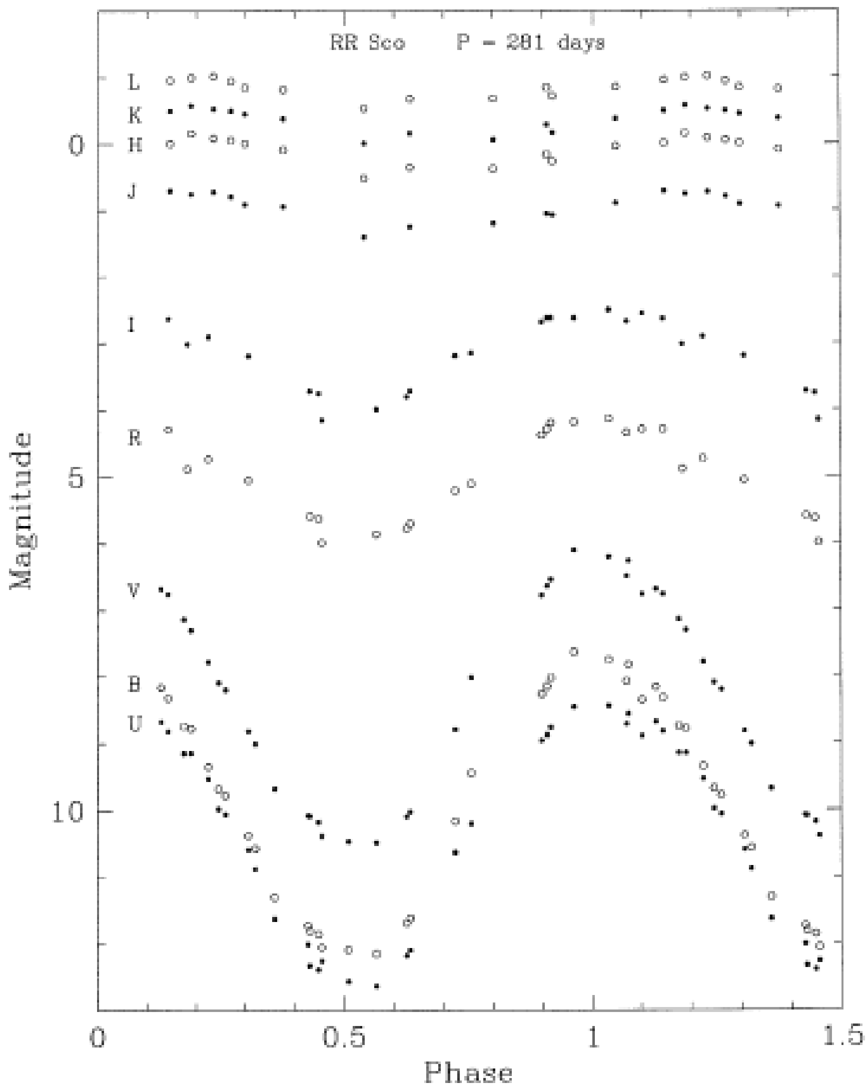

Figure 16: Lightcurve for the Mira variable RR Sco in the bandpasses UBVRIJHKL. The UBVRI light curve come from Eggen (1975), while the JKHL light curves are from Catchpole et al. (1979).

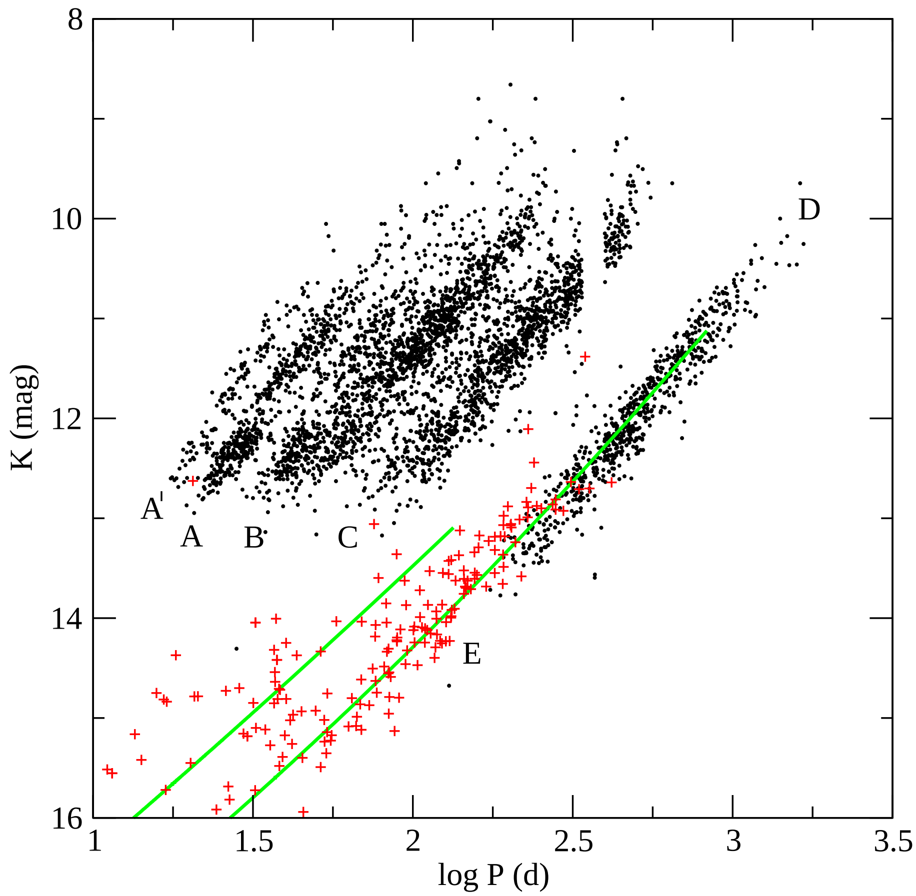

Figure 17: The period-luminosity relations for a sample of MACHO observations. Several sequences can be discerned, showing that AGB stars pulsate in different modes. The stars indicated with plus signs are eclipsing binaries. From Derekas et al. (2006)

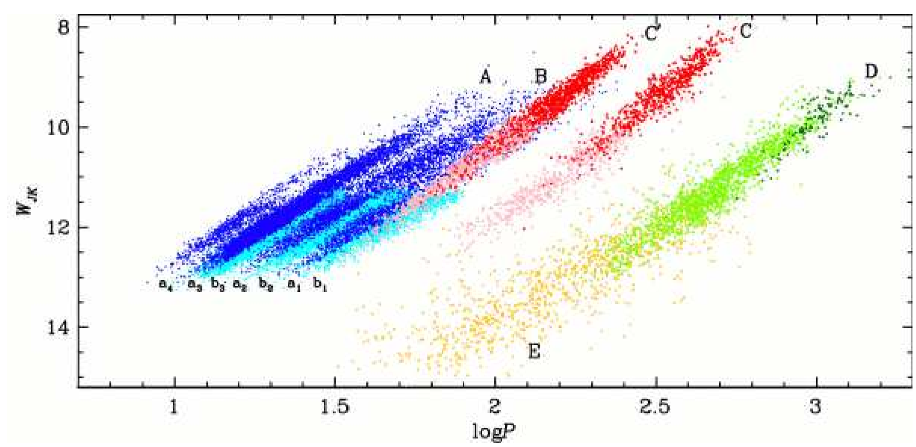

Figure 18: The period-luminosity diagram for a sample of red giants in the Large Magellanic cloud (LMC), observed in the OGLE project. OSARG variables are shown as blue points (RGB as light blue, AGB as dark blue). Miras and SRVs are marked with pink (O-rich) and red (C-rich) points. Light and dark green points refer to O-rich and C-rich Long Secondary Period variables, respectively (binaries?). Yellow points indicate ellipsoidal red giants. From Soszyński et al. (2007).

Before the MACHO results, there was very little data to work with, and one tried to derive stellar radii using interferometry, so as to establish the mode of the pulsations. Those measurements showed Miras to be first overtone pulsators. Since the MACHO results show them to be fundamental mode pulsators, the conclusion is that there is something wrong with the interferometric measurements of stellar radii. Recent results show that some AGB stars may be aspherical (Ragland et al. 2006).

The story does however not end here. More recent data from the OGLE survey has revealed a large group of variable red giant stars that have been called OSARGs (OGLE Small Amplitude Red Giants). Some of these are on the RGB (showing that also those stars are variable), some on the AGB. These stars occupy most of the A and part of the B sequence from the MACHO data (Soszyński et al. 2007, see Fig. 18). The proper Mira and SRVs lie on two sequences indicated by C and C′ in Fig. 18, corresponding to fundamental mode and first overtone pulsators. Differences between C- and M-stars can also be seen. Clearly the interpretation of all this variability data is still variable itself.

5.2 Theory of Pulsations

There is a large body of work on understanding stellar pulsation, the foundations of which were layed by Eddington. Much of this work concentrates on hotter stars with radiative stellar envelopes, which simplifies the theory compared to the convective envelopes of the cool AGB stars. The hotter stars are for example the Cepheid variables (important for the cosmological distance scale) and RR Lyrae stars.

The theory of pulsation is basically a stability analysis for stars, i.e. studying how a star reacts to perturbations. From such an analysis one obtains

- frequencies of modes
- whether or not the star is stable against these modes.

Stability analysis usually starts with linear perturbations: small variations around an equilibrium solution, neglecting the higher order (non-linear, quadratic and higher) terms in the equations. For AGB stars matters are complicated by the fact that one tries to calculate the stability of a convective stellar envelope, but no fully self-consistent theory for convection is available.

5.2.1 Estimate of luminosity-period relations

The fundamental mode is a radial pulsation with its wavelength equal to the stellar diameter

$$\Pi = 2R_*/c_{\rm s} \tag{28}$$

where $c_{\rm s}$ is the (adiabatic) sound speed, $c_{\rm s}^2 = \gamma_{\rm ad} p/\rho$.
By assuming that the variations are around an equilibrium solution, one can use the Virial Theorem

$$-\Omega_{\rm grav} = 2K_{\rm thermal} = 3\int_0^{R_*} 4\pi r^2 p \mathrm{d}r\,. \tag{29}$$

Since $\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2\rho$, this can be rewritten as

$$2K_{\rm thermal} = 3\int_0^{R_*} \frac{c_{\rm s}^2}{\gamma_{\rm ad}}\mathrm{d}m \approx 3\frac{c_{\rm s}^2}{\gamma_{\rm ad}}M_* \tag{30}$$

The graviational energy of the star is

$$-\Omega_{\rm grav} = \alpha\frac{GM_*^2}{R_*} \tag{31}$$

where α is a dimensionless number which depends on the internal density structure of the star. Combining the expression for the gravitational and thermal energies through the Virial Theorem gives

$$\Pi = 2\sqrt{\frac{3}{\gamma_{\rm ad}\alpha G}}R_*^{\frac{3}{2}}M_*^{-\frac{1}{2}} \equiv QR_*^{\frac{3}{2}}M_*^{-\frac{1}{2}} \approx (G\langle\rho\rangle)^{-\frac{1}{2}} \tag{32}$$

which means that the pulsation period for the fundamental mode will depend on the average density of the star. Using typical parameters for a Mira, $M = 1\ M_\odot$, $R = 200\ R_\odot$, $\alpha = 2$, one obtains $\Pi \approx 0.3$ years.
The above analysis also leads immediately to a period-luminosity relation

$$L = 4\pi R^2\sigma T_{\rm eff}^4\,, \Rightarrow L \propto \left(\frac{\Pi}{Q}T_{\rm eff}^3 M^{1/2}\right)^{4/3} \tag{33}$$

A more elaborate perturbation calculation, assuming adiabatic conditions, but using

$$p(m,t) = p_0(m)\,[1 + \delta p(m) \exp i\omega t] \tag{34}$$
$$\rho(m,t) = \rho_0(m)\,[1 + \delta \rho(m) \exp i\omega t] \tag{35}$$
$$r(m,t) = r_0(m)\,[1 + \delta r(m) \exp i\omega t] \tag{36}$$

leads to a series of frequencies ω_0, ω_1, ω_2, etc., which are the fundamental mode, the first overtone, second overtone, etc. Expressed as a period, the relation between the Π values of these modes is

$$\Pi_n \propto \sqrt{n+1}\langle\rho\rangle^{-\frac{1}{2}} \tag{37}$$

5.2.2 The κ mechanism

For a star to be actually pulsating in one or more of these modes, energy needs to be fed in at the right time during a pulse cycle. As first realized by Eddington, the key is to block energy transport during the compression phase. This raises the pressure *after* compression, leading to expansion and restoration of the state at the start of the cycle.
If energy transport is radiative, it is inversely proportional to the opacity κ, so in order to achieve the effect described above, we need to increase κ upon compression. A reasonable approximation for the opacity inside a star is Kramer's opacity

$$\kappa \propto \rho T^{-3.5} \tag{38}$$

For this opacity law, the density increase in a compression goes in the right direction of increasing the opacity, but this effect will be more than offset by the decrease of the opacity caused by the temperature increase during compression. So, standard stellar opacity will not trigger pulsations.
However, in zones of partial ionization (of either He or H), the heat added by compression will be mostly used to increase the ionization fraction, and will hardly raise the temperature. Then as the density goes up, and the temperature remains constant, the opacity will go up. This is known as the κ mechanism for stellar pulsations, and it explains the pulsational instabilities in Cepheids and RR Lyrae stars. In fact, the requirement of having these zones while still keeping a radiative stellar envelope, defines a region in the HR-diagram known as the instability strip.

5.2.3 Pulsation models for AGB stars

For AGB stars energy transport is dominantly *convective*, and thus independent of the opacity, so in the simplest interpretation the κ mechanism should not work. The observations show this conclusion to be wrong. However, to explain the pulsational instability in AGB stars one needs a dynamic theory of convection, for which currently only approximations exist ('mixing length theory'). Models based on this approximation of convectice energy transport are the only ones that are available, and should hopefully give at least a qualitative understanding of the pulsational behaviour of AGB stars.
Figure 19 shows some results from such a model. The top panel shows that most energy is transported convectively, and also indicates the the H and He ionization zones, which are still important for the model. The bottom panel shows how the radial excursion is

roughly proportional for the fundamental mode, while the first overtone is concentrated towards the photosphere. The third panel shows the so-called partial work integral W_r which is the sum of all the work done interior of r during one pulsation cycle. The regions where W_r increases with radius contribute to exciting the pulsation, the regions of negative W_r gradient are damping pulsations. Clearly the H and He ionization zones contribute to exciting the pulsation. The second panel shows the quantity $J\Sigma_r^2$, the absolute gradient of which shows which regions contribute most to the period. For the fundamental mode it are the top layers of the H ionization zone which contributes most to determining the pulsation period. For the first overtone no regions clearly dominates.

5.3 Non-radial pulsations

Stars may also pulsate non-radially: angular patterns of pulsation in addition to the radial patterns. Not much is known about non-radial pulsations in AGB stars, but they are sometimes invoked to explain observed asymmetries (for example in the mass loss).

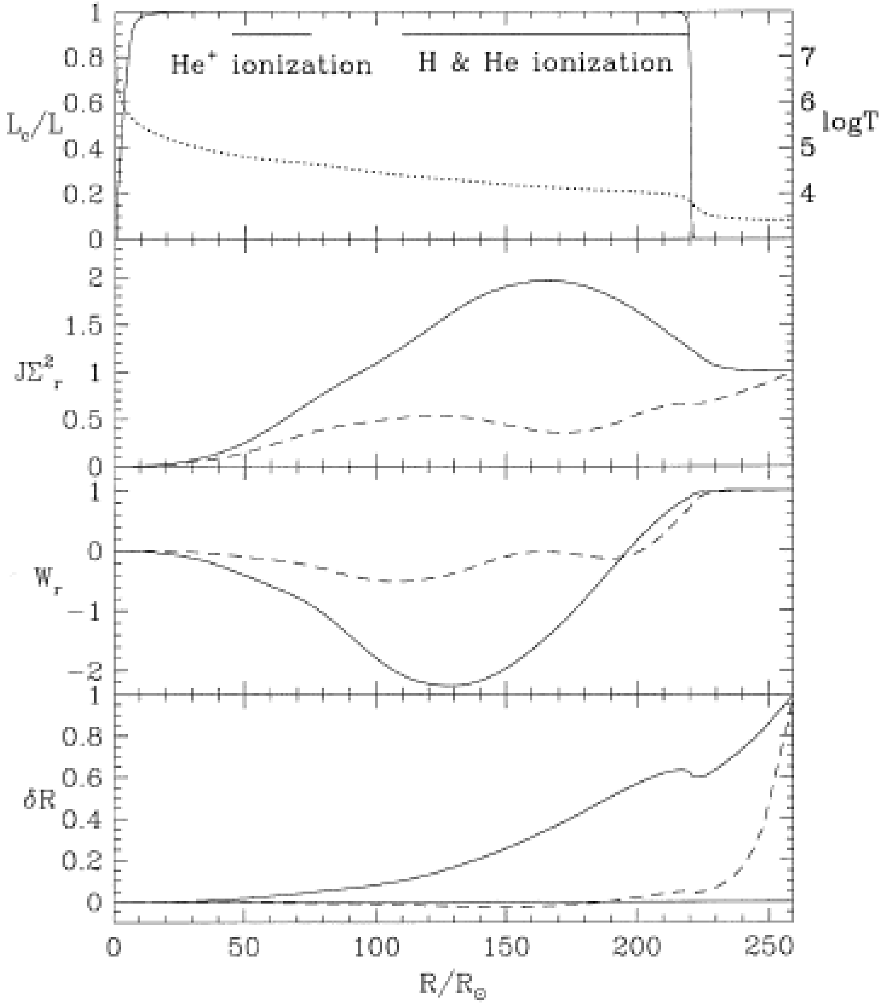

Figure 19: Various properties of a model Mira variable with $L = 5000\ L_\odot$ and $M = 1\ M_\odot$ and solar metallicity plotted against radius within the star. The fundamental mode period of the model is 333 d, similar to that of the prototypical Mira o Ceti. Top panel: log T (dotted line) and the fraction of the energy flux carried by convection (solid line); second panel: the partial integral $J\Sigma_r^2$ for the fundamental mode (solid line) and the first overtone (dotted line); third panel: the partial work integral W_r for the fundamental mode (solid line) and the first overtone (dotted line); bottom panel: the real part δR of the eigenfunction for the fundamental mode (solid line) and the first overtone (dotted line), where the eigenfunction is defined to have complex amplitude (1.0,0.0) at the stellar surface. Based on Fox & Wood (1982).