1 Importance of AGB stars

a) Assume a Salpeter-like Initial Mass Function

\[ \frac{dN}{dM} \propto M^{-2.35} \tag{1} \]

for a group of stars forming at the same time (you can chose a total mass of \(10^4 M_\odot\) if you find that easier). Calculate the fraction of stars in the mass range 0.8—100 \(M_\odot\) that will not become supernovae. Assume that the upper mass limit for stars to end their lifes as White Dwarfs is 9 \(M_\odot\).

b) If the Main Sequence life time scales as \(\tau_{\text{MS}} \propto M^{-2.5}\), with an 8 \(M_\odot\) star having \(\tau_{\text{MS}} = 50\) Myr, find the percentage of stars that have evolved off the Main Sequence as a function of time between 50 Myr and 10 Gyr (take at least 10 logarithmically spaced time intervals).

c) Use the Initial-Final Mass Relation (approximate it from Fig. 20 in the notes) to find the cumulative fraction of mass returned to the Interstellar Medium for the time intervals from b). Compare this number to the mass returned by the supernova producing stars \((M > 9 M_\odot)\). To do this assume that those stars leave behind remnants of \(\sim 1 M_\odot\) and have all ended their evolution before 50 Myr.

2 Shell burning stability

a) Review the derivation of the gravothermal specific heat and find the origin of the factor 3 in the expression

\[ C_* = C_p \left( 1 - \nabla_{\text{ad}} \frac{4\delta}{4\alpha - 3} \right) \tag{2} \]
(you may need to look at some of the other lecture notes from the set of Robin Ciardullo; the first number in his equation numbering refers to the lecture number; see the course web site for a link to his collection of lecture notes).

b') From this show that it makes sense to replace this 3 by $r/D$ when considering a thin shell, and that none of the other factors (like the two “4”’s) have to be changed.

c) Check the stability of a H-core flash (start of H-fusion in an electron degenerate core). For the temperature dependence consider both cases: pp-chain and CNO-cycle.

d) Use Fig. 5 in combination with the gravothermal specific heat conditions to argue that we can expect He-shell flashes, but no H-shell flashes in AGB stars\(^1\).

3 Mass evolution

a) Use Fig. 12 from the notes together with the $L - M_c$ relation to find the increase of the core mass during the AGB for the three different stellar masses. Express this in $M_{\odot} \, yr^{-1}$ and compare this to the average mass loss rates during the AGB (derive these from Figs. 7 and 8 or the IFMR). Why does the answer for the 5 $M_{\odot}$ does not make sense?

4 On the origin of fluorine in the Milky Way\(^*\)

Read (at least) Sect. 2.5.7 as well as the last part of Sect. 2.5.13.2 from Lattanzio & Wood’s contribution in Asymptotic Giant Branch Stars by Habing & Olofsson. Then read Renda et al. (2004), MNRAS 354, 575.

a) What is the new result in this paper?

b) What are the formation and destruction processes of $^{19}$F in AGB stars? How is the $^{19}$F production related to s-process element nucleosynthesis?

c) Give an overview of the uncertainties in the $^{19}$F production.

d) What do you think is meant with post-processing nucleosynthesis models (Sect. 3.1)? (Hint: stellar evolution models which calculate the evolution of 100s of different isotopes may be very time consuming). Why is this a reasonable approach to use for $^{19}$F?

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\(^1\)Note the figure actually only gives you the central positions of the shells, not the actual widths; guesstimate a width on the basis of the numbers in the figure.
e) Explain why the AGB stars of spectral type MS, S, and C are not used in finding the $^{19}$F abundance of the Milky Way, LMC and ω Cen.

f) Does the O/Fe ratio increase or decrease with time? Explain how you came to your conclusion.

g) What is the problem with the globular cluster ω Cen? What is the solution, and what does this imply for its star formation history?