Exercise 3

1. Classify the spectra in Fig 1 – 6 in the separate document. Motivate your choices. *

2. The ring around SN 1987A has a minor axis of 1.2" and a major axis of 1.62". The light curve of the N III] λ1750 line, as observed with the IUE satellite, shows a maximum at 399 days (see Sonneborn et al, ApJ 477, p.856, or figure in the hand-outs). This emission has been explained as a result of the ionization of a circumstellar ring around SN 1987A by the radiation from the shock breakout during the first minutes after the explosion. The increase in the emission is explained as a light echo of this, as we see more and more of the ring. Estimate the distance to the ring from the SN, and the distance to the SN from us. *

3. Calculate the constant velocity surfaces used in the Sobolev approximation for V= constant. *

4. Write a Monte Carlo program that calculates the escape of photons from a sphere with radius R by diffusion.

   Steps:

   (a) Emit a photon from the center in a random direction. Choose the direction from a random value of the azimuthal angle between 0 and 2π and a random value of $\cos\theta$ between -1 and 1.

   (b) The probability to be scattered (or absorbed) when propagating a distance, d_i, corresponding to an optical depth $\tau_i = \kappa\rho d_i$ is $P = 1 - \exp(-\tau_i)$, so that $0 \leq P \leq 1$. Therefore, pick a random number r and calculate the optical depth as $\tau_i = -\ln(1-r)$. The length corresponding to this is then $d_i = \tau/\kappa\rho$.

   (c) Propagate the photon a step equal to d_i in the chosen direction.

   (d) If the photon crosses the surface of the sphere let it escape, and determine the number of scatterings, N.

   (e) Otherwise, let the photon scatter at this new position in a new random direction, as before.

   etc.

   The total time since emission is then $t = \Sigma_i^n d_i/c$.

Use a random generator, either from Numerical Recepies or the internal in e.g., Matlab or compiler. In Fortran (or other software) initialize first with 'call srand(seed)' where 'seed' is an integer and then 'rn = rand()' is a random number between 0 and 1. For this exercise you may use non-dimensional units, normalized to e.g., the values at time $t = 1$, or more specific numbers, as you prefer. Cartesian coordinates are probably convenient.

(a) Assume that the total optical depth is $\tau = 1, 10, 100$, plot the distribution of the time when the photons escape. *

(b) Explain the relation between τ and the time of the peak flux, and compare it with the formulas in the lecture notes. *

(c) What is the relation and difference between a random walk and the diffusion of radiation? *

(d) Instead of starting the photons at the center, emit them uniformly in the sphere. Compare the distribution with that in a. *

(e) Now let the sphere with initially $\tau = 1000$ and $R = 1$, expand with a velocity V as the photons are emitted continuously with time from the center. Because all radii $R \propto Vt$ and density $\rho \propto R^{-3} \propto (Vt)^{-3}$, you can calculate how τ depends on time. Assume that this and the radius are constant during each individual scattering. Assume also that the photons are emitted $\propto \exp(-t/t_{\rm decay})$, to simulate radioactive decay. Calculate the light curve. When does the maximum occur in relation to the parameters of the problem? How does it depend on $t_{\rm decay}$? ** -***

5. Assume that the exploding star is a white dwarf, consisting mainly of carbon, and that half of the mass is transformed into ^{56}Ni. Also assume that it has constant density.

(a) Calculate the energy released. (You may use a previous exercise for this.) *

(b) Calculate the velocity of the ejecta, assuming that you can neglect the gravitational binding energy and neutrino losses and that it exppands homologously. *

(c) Assuming that all ^{56}Ni is at the center, estimate the optical depth of the gamma-rays as function of time. *

(d) Estimate when the positrons dominate the gamma-ray input to the ejecta. *

(e) Use the results above as input to your Monte-Carlo code. **–***

If you feel you need it you may use $\kappa = 0.2$, corresponding to Thompson scattering.