Early and late stellar evolution, part III: Star Formation

Problem set 2, June 2, 2008

All problems except those marked (*) are compulsory for pass. Problems marked with (*) are for higher grades. Please motivate your answers carefully (when applicable). There will be 3 problem sets in total, for this part of the course. A report containing the solutions to these problem sets should be handed in **no later than 2008-06-20, 24:00** for grades higher than pass. The report can be submitted on paper, or as a PDF sent to `alexis@astro.su.se`, or both (as long as the versions are identical). If submitted only in paper form, still send an email notifying that the report has been submitted. Note that not all problems are explicitly treated in the lectures; most are in the Stahler & Palla book, however.

1. A spherically symmetric isothermal cloud satisfies the isothermal Lane-Emden equation

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\psi}{d\xi}\right) = \exp(-\psi)$$

(equation 9.7 in Stahler & Palla), where

$$\xi = r\sqrt{\frac{4\pi G\rho_c}{c_S^2}},$$

$\psi = \Phi/c_S^2$, $c_S = \sqrt{\mathcal{R}T/\mu}$ is the sound speed, Φ is the gravitational potential, $G = 6.67\times10^{-11}\,\mathrm{m^3\,kg\,s^{-2}}$ is the gravitational constant, $\mathcal{R} = 8.31\times10^{7}\,\mathrm{erg\,g^{-1}\,K^{-1}}$ is the gas constant, μ is the mean molecular weight, T is the gas temperature, and ρ_c is the central density.

Let $\mu = 2$, $T = 30\,\mathrm{K}$, $\rho_c = 7.6\times10^{-20}\,\mathrm{g\,cm^{-3}}$ and assume the equation of state $P = \rho c_S^2$, where P is the pressure and ρ the density. Assume an external pressure of $P_{\mathrm{ext}} = 7.60\times10^{-12}$ Pa from the interstellar medium is being applied on the sphere.

(a) What is the sound speed?

(b) What is the mass of the sphere? You may use Figs. 1 & 2 to read out values derived from the solution of the differential equation.

(c) Imagine that the external pressure starts to rise. How high can it rise before the sphere becomes unstable?

(d) Assume instead that the clouds starts to (isothermally) cool. How much can it cool before the sphere becomes unstable?

(e) * Solve the isothermal Lane-Emden equation numerically, and reproduce Figs. 1 & 2. To solve the differential equation, show that it is equivalent to solve the non-linear system of ordinary first-order differential equations

$$\begin{aligned} y_0' &= y_1 \\ y_1' &= \exp(-y_0) - \frac{2}{\xi} y_1 \end{aligned}$$

where the prime signifies derivation with respect to ξ, $y_0 = \psi$, and the initial condition is $y_0' = y_1' = 0$. This may then be solved by e.g. Euler's method, i.e. start from the initial condition, and iteratively compute

$$y_i(\xi_{n+1}) = y_i(\xi_n) + y_i'\left[\xi_n, y_1(\xi_n), y_2(\xi_n)\right] \Delta\xi_n,$$

where $\Delta\xi_n = \xi_{n+1} - \xi_n$ is the length step and $i \in \{1, 2\}$. Check that the solution converges as $\Delta\xi_n \to 0$ ($\forall n$). You are of course welcome to solve the differential equation by other means (e.g. by using MATHEMATICA, MAPLE, or MATLAB).

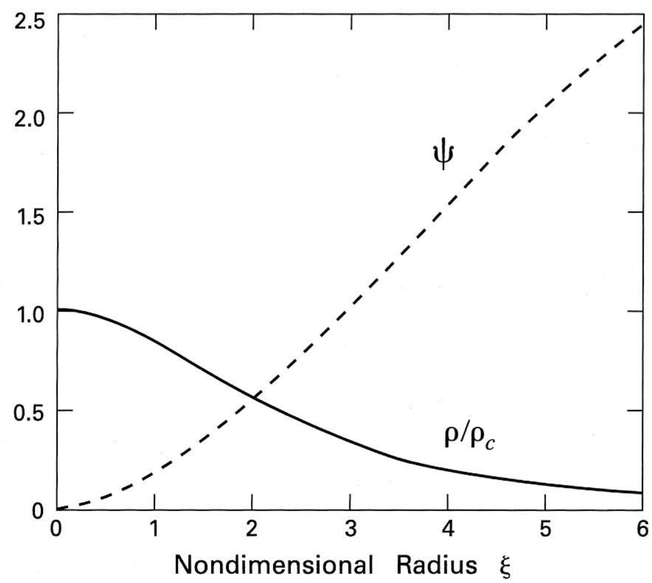

Figure 1: From Stahler & Palla (2004)

2. The collapse of a Bonnor-Ebert sphere gives rise to a system of differential equations that have to be solved numerically. To get a rough idea of how a cloud behaves under collapse, let us study an even simpler case: a homogeneous sphere of gas without pressure, with gravitation being the only relevant force. The equation of motion can then be written

$$\ddot{r} = -\frac{GM_r}{r^2}, \tag{1}$$

where r is the radius, M_r is the mass interior to r, and G is the gravitational constant (as above).

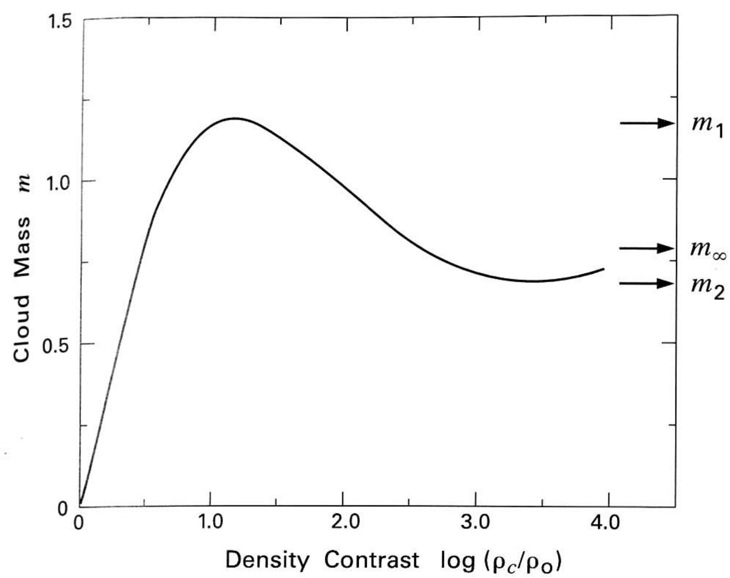

Figure 2: From Stahler & Palla (2004)

(a) Let $r(t=0)=r_0$ and $\dot{r}(t=0)=0$, and show that

$$\frac{\dot{r}}{r_0} = -\sqrt{\frac{2GM_r}{r_0^3}\left(\frac{r_0}{r}-1\right)}. \tag{2}$$

(b) Introduce

$$\cos^2(\xi) = \frac{r}{r_0} \tag{3}$$

and show that

$$\xi + \frac{1}{2}\sin(2\xi) = t\sqrt{\frac{2GM_r}{r_0^3}}. \tag{4}$$

(c) How long does it take a shell to free-fall to the origin? Express the time in the initial density ρ_0.

(d) If the uniform sphere had the density $\rho_0 = \rho_c$ of Problem 1, what would be the total time for the free fall to the origin?

(e) * A *wind* can sometimes be modelled as antigravity working on gas. The relevant equation of motion would then be

$$\ddot{r} = (\beta - 1)\frac{GM_r}{r^2},$$

where $\beta > 1$. What are the equations in this case, corresponding to equations 2–4?