Early and late stellar evolution, part III: Star Formation

Problem set 3, June 13, 2008

All problems except those marked (*) are compulsory for pass. A report containing the solutions to these problem sets should be handed in **no later than 2008-06-20, 24:00** for grades higher than pass. The report can be submitted on paper, or as a PDF sent to `alexis@astro.su.se`, or both (as long as the versions are identical). If submitted only in paper form, still send an email notifying that the report has been submitted.

1. Small dust grains are sensitive to the pressure from radiation, which in dust disks around some stars can overcome gravity and thus efficiently eject the dust. Assume a grain can be modelled as a sphere of radius a, density ρ, and albedo η, independent of wavelength. The grain is subject to radiation pressure from a star of radius $R_\star$, mass $M_\star$, and effective surface temperature $T_{\rm eff}$.

   (a) Let the *critical blow-out radius* $a_{\beta=1}$ be the radius where the radiation force equals gravity. Derive $a_{\beta=1}$ as a function of ρ, η, $R_\star$, $M_\star$, and $T_{\rm eff}$.

   (b) If $M_\star = 2\,{\rm M}_\odot$, $R_\star = 2\,{\rm R}_\odot$, $T_{\rm eff} = 8\,000\,{\rm K}$, $\rho = 10^3\,{\rm kg\,m^{-3}}$ and $\eta = 0.5$, what is $a_{\beta=1}$?

   (c) * Use tabulated values for the mass $M_\star$ and luminosity $L_\star$ of stars on the main sequence, and plot $\left(\frac{\rho}{10^3\,{\rm kg\,m^{-3}}}\right)\left(\frac{\eta}{0.5}\right)^{-1} a_{\beta=1}$ as a function of spectral type between O and M.

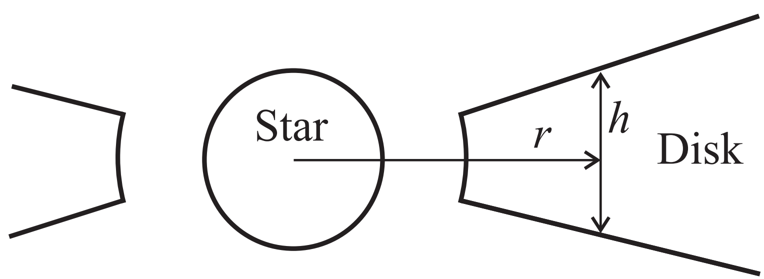

Figure 1: Illustration of disk with constant $\alpha = h/r$.

2. A passive circumstellar disk is heated solely by radiation from the central star, i.e., there is no additional accretion luminosity. Assume the disk has a constant opening angle as seen from the star, expressed as the $\alpha = h(r)/r$ being constant, where $h(r)$ is the height the disk extends at radius r, and r the distance to the star (see Fig. 1). If a disk is observed to emit the fractional luminosity $f_{\rm disk} = L_{\rm disk}/L_\star$, where $L_{\rm disk}$ is the luminosity of the disk and $L_\star$ is the luminosity of the star, what is the *minimum* α possible for the disk?

3. Study the thermal balance of dust grains in a passive disk. Again assume that dust grains are spherical with an absorption cross-section $\sigma = \pi a^2$ and an emitting area $A = 4\pi a^2$, and that they absorb and emit as perfect blackbodies. Let the central star be of radius $R_\star$ and have the effective surface temperature T_{eff}.

   (a) What is the equilibrium temperature T_{dust} of dust as a function of distance r from the star?

   (b) Assume $R_\star = 2\,\mathrm{R}_\odot$ and $T_{\text{eff}} = 8\,000\,\mathrm{K}$, and plot $T_{\text{dust}}(r)$ from the stellar surface out to 2 000 AU (preferably with logarithmic scale).

   (c) Let the inner radius r_{in} of the disk be set by the sublimation temperature of dust, $T_{\text{subl}} = 1\,500\,\mathrm{K}$ (i.e., dust inside this radius gets evaporated; $T_{\text{dust}}(r_{\text{in}}) = T_{\text{subl}}$), the density of grains decrease radially as $\rho(r) \propto r^{-3.5}$, and the opening angle be constant. Compute and plot the (arbitrarily normalised) spectral energy distribution (SED) from the disk, under the assumption it is optically thin (no radiation from the star is blocked). The SED L_ν is then the integrated dust luminosity over the whole disk,

   $$L_\nu \propto \int_{r_{\text{in}}}^{\infty} \rho(r) B_\nu[T_{\text{dust}}(r)] r^2 dr,$$

   where $B_\nu[T]$ is the Planck function and ν the frequency.

   (d) * Repeat the previous three sub-problems under the assumption that grains absorb and emit as *modified blackbodies*, with the opacity being $\kappa_\nu \propto \nu^\beta$, $\beta = 1$, the absorbtion cross-section $\sigma_\nu = \pi a^2 \kappa_\nu$ and the emitted flux being $F_\nu = \kappa_\nu B_\nu(T)$.

4. A triple star with components a, b, and c, is observed with adaptive optics in a star forming region. The extinction for the region in various wavebands is related through $A_V = 2.90 A_J$, $A_I = 1.83 A_J$, and $A_K = 0.37 A_J$. The observed magnitudes in the different bands are $I_a = 11.03$, $I_{b+c} = 11.73$, $J_a = 9.21$, $J_b = 10.55$, $J_c = 10.87$, $K_a = 7.03$, $K_b = 8.92$, $K_c = 9.32$. The a component is of spectral type K5, while b and c are of spectral type ~K7. The intrinsic $I - J$ colours of K5 and K7 atmospheres are $(I - J)_0 = 0.80$ and 0.92, respectively.

   (a) Estimate the extinction A_V towards the b and c components.

   (b) Assume the extinction A_V is the same towards component a, and compute the excess of a in the J and K bands.

   (c) * The bolometric correction in J for K5 and K7 is $\mathrm{BC}(J) = 1.41$ and 1.37, respectively. The distance modulus to the star forming region is 6.0, and the absolute bolometric magnitude of the Sun is $\mathrm{M}_{\text{bol},\odot} = 4.5$. Compute the luminosities of the three components.

   (d) * Using the temperature calibration $T_{\text{eff}} = 4340\,\mathrm{K}$ for K5 and $T_{\text{eff}} = 4040\,\mathrm{K}$ for K7, derive ages for the three components separately using evolutionary models by Baraffe et al. (1998), assuming solar metallicity. The models are available from Vizier, `http://cdsarc.u-strasbg.fr/viz-bin/Cat?J/A%2bA/337/403`.