Determination of orbital elements

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Introduction

To specify an orbit in space-time you need 6 parameters - the orbital elements. Our project was about exactly this, to determine the orbital elements given certain observations. We divided the project into two separately parts, the first to determine the orbit of an artificial satellite and the second to determine the orbit of a comet, in our case the comet of Hale-Bopp that was visual with binoculars and expected to be the great comet of 1997.

Orbit of a comet

Given the six orbital elements it's a tedious but straight forward calculation to determine the apparent position of the comet on the celestial sphere, as seen from earth at a given instant. To do it the other way, that is determine the orbital elements from given observations, is much more difficult. Why is easy to understand if you consider that you only observe the angular position on the celestial sphere and not the distance to the object. So how do we do it then? Traditionally there are several more or less complicated methods, as Kepler's, Laplace's or Gauss'. They only need three observations to yield the orbital elements but have the serious disadvantage of being very sensitive to errors in the observations. There are also difficulties in incorporating more than three observations into those methods, making them unsuitable for more accurate orbit determinations. Fortunately there is a more straight forward way.

Model fitting

Consider the function from the seven-dimensional space of the orbital elements plus time to the two-dimensional space consisting of the celestial co-ordinates right ascension and declination, i.e. given any point in the seven-dimensional space the function spits out right ascension and declination. As written above this function is not difficult to implement for an object obeying the keplerian laws. Now, to determine the set orbital elements, start with a guess. Evaluate the function for a given instant and compare the result with an observation from the same instant, for instance by calculating the angular separation between the observed and predicted values. Do this for all your observations and sum the squared differences so that you get an *error* function, depending only on your choice of orbital elements. The orbital elements that best fits the observations are accordingly the ones that minimises the error function. Minimising the error function can be done with several methods, one is the simplex (or polytope) method, another is the differential correction method. The simplex method is described in the 1992 IAYC report page 54 (by Gregor Krannich) and is not very intuitive while the differential correction is very straight: approximate the function with a linearisation and solve the equations with a combination of Newton's method and the method of least squares. Unfortunately the differential correction method is more sensitive to local minima than the simplex method whilst it is more accurate for values close to the minimum.

The Model

The first thing you have to do, of course, is to construct this function. You don't only need to determine the comet's position relative to the sun, but also the sun's position relative to the earth. To calculate the earth's position I used an abridged version of the VSOP87 theory described in chapter 31 in Jean Meeus' book *Astronomical Algorithms.* Using it included several hours of typing in periodic terms of a series expansion, which is a tremendous overkill, of course, since it yields results with an accuracy on the order of arc seconds and you never can expect observations to be that good.

The rest of the function included such technicalities as co-ordinate transformations, calculations of Julian day number, obliquity, mean anomaly etc. etc. which as previous stated all are straight forward calculations.

Procedure

To determine the orbital elements of a comet you first need a couple of observations of it. In my case I used some observations of comet Hale-Bopp kindly supplied by Hermann Gumpp and some I did myself during the camp. The positions were determined from photographs using Uranometria making the accuracies on the order of five arc minutes. The uncertainties in time were usually about one minute which makes them completely neglectible compared to the angular uncertainties.

Because of the need of an initial guess in the calculations I was forced to consider one of the classical methods. The one I chose is called the method of Gauss-Encke-Merton (GEM) and is, as the name suggests, an improved version of Gauss' method. When the initial guess was generated using three of the observations I switched to the simplex method making use of all available observations. Only this optimisation took quite a while, approximately 30 minutes on a 66 MHz 486 computer.

Results

To see how well the procedure worked I compared the predictions made from the determined orbital elements with some down loaded directly from JPL via internet, and here is the result:

Days after last observ-ation	Angular distance error	α_{JPL}	δ_{JPL}	α_{det}	δ_{det}
1	15'	17h 53m 46.87s	-7° 43' 40.5"	17h 52m 49s	-7° 39' 3"
2	29'	17h 52m 43.23s	-7° 39' 6.5"	17h 51m 46s	-7° 34' 35"
4	14'	17h 50m 39.97s	-7° 30' 9.1"	17h 49m 45s	-7° 25' 50"
8	13'	17h 46m 50.02s	-7° 12' 56.4"	17h 45m 59s	-7° 9' 5"
16	11'	17h 40m 18.97s	-6° 41' 13.7"	17h 39m 38s	-6° 38' 29"
32	4'	17h 31m 59.01s	-5° 47' 2.9"	17h 31m 44s	-5° 47' 6"
64	11'	17h 32m 45.03s	-4° 12' 25.7"	17h 33m 18s	-4° 19' 22"

The length of the observation arc was 33 days extending from the first observation 12:th of July to the last 14:th of August.

References

"Astronomical Algorithms", Jean Meeus, 1991 Willmann Bell, Inc.

"Fundamentals pf celestial mechanics", JMA Danby, 1992 Willmann Bell, Inc.

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