# 1 1

## Key concepts

- The definitions of the length units AU, parsec and light-year.
- The celestial sphere, and its coordinates Right Ascension and Declination.
- The coordinates for a local obersever; hour angle, elevation and siderial time.
- Keplers laws
- Elliptical orbits and their parameters; semi-major axis, semi-minor axis, apoapsis and periapsis.

## D1

One **Astronomical Unit** (AU) is the (average) distance between the earth and the sun. This distance has a value of 150 million kilometers, which is a good number to remember when doing astronomy problems.

One **parsec** (pc) is the distance at which 1 AU has an angular size of 1 arc second (=1/3600 degree)

$$1AU = 150 \cdot 10^6 \cdot 10^3 m = 1.5 \cdot 10^{11} m$$

$$\tan\theta = \frac{1\ AU}{1\ pc} \rightarrow 1pc = \frac{1\ AU}{\tan\theta}$$

Now use the approximation

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \sim \frac{\theta}{1} = \theta$$

which is valid for small $\theta$. Our $\theta$ is 1/3600 of a degree so its ok. We get

$$1pc \approx \frac{1AU}{\theta} = \frac{1AU}{\frac{1}{3600}\frac{2\pi}{360}} = 206,000\ AU \approx 3.09 \cdot 10^{16} m$$

That one parsec is about 200,000 times the distance between the earth and the sun is another good relation to remember.

A **light-year** (ly) is the distance that light (or any other electromagnetic wave) travels in one year. The speed of light is 300,000 km/s. Since one year has $60 \cdot 60 \cdot 24 \cdot 365 = 3.15 \cdot 10^7$ seconds, a light-year becomes

$$1\ ly = 300,000 \cdot 3.15 \cdot 10^7 = 9.45 \cdot 10^{12}\ km = 9.45 \cdot 10^{15} m = 0.31\ pc$$

## D2

The sidereal time is the hour angle of the RA=0 direction. If the sidereal time is 20 h, it therefore means that RA=0 passed above 20 hours ago. Vega has RA=$18^h 36^m 56.3s$, which means that it passes above exactly that time after RA=0 passes above. Therefore, it must have passed above $20^h - 18^h 36^m 56.3^h = 1^h 23^m 3.7^s$ ago. The hour angle is then positive with this value.

## D3

The **Scientific Revolution** is traditionally said to have begun in 1543 with the publication of 'On the Revolutions of the Heavenly Spheres' by Copernicus and 'On the Fabric of the Human body' by Andreas Vesalius. Up until then, the same natural philosophies had dominated for almost 2,000 years (!); the ones developed in ancient Greece.

The new philosophy of science emphasized *systematic observations* as the foundation for all progress. The first astronomer to put this into action was the Danish **Tycho Brahe** (1546-1601). Using only his eyes (the telescope was invented only around 1610), he systematically began noting positions of the planets over long periods of time. Tycho's student was **Johannes Kepler** (1571-1630), who after Tycho's death[1] found mathematical relations for how the planets moved. His three laws are

1. The orbit of each planet is an **ellipse** with the Sun at one of the (two) focal points.
2. The 'slice' of this ellipse that the planet cuts per unit time **always has the same area**.
3. The **period** of the orbit is related to the **semi-major axis** of the ellipse as $P^2 \propto a^3$.

This was all before Newton (1643-1727) came along and developed a theory for gravitation and planet motion, so the laws are just observational facts.

a) Now, when we want to use Kepler's third law, we dont need to remember the proportionality constant. We can compute it by using earth's values; the semi-major axis is approximately 1 AU and the period is a year. Thus

$$1^2 = const \cdot 1^3 \rightarrow const = 1$$

If we now look at the comet Halley instead, we get

$$76^2 = 1 \cdot a^3 \rightarrow a = 76^{2/3} AU = 17.9\ AU$$

b) For an elliptical orbit, the position where the distance to the closest focal point reaches a minium is called the **peri-apsis**. If the ellipse refer to an orbit around the sun, the derivative word **peri-helion** is used. In the same way, the point where the distance to the furthest away focal point is reaches is called the **ap-apsis** for the general case, and the **ap-helion** for the planetary case.

The **eccentricity** (e) is defined as

$$e = \sqrt{1 - \left(\frac{r_{minor}}{r_{major}}\right)^2}$$

For nearly circular orbits $r_{minor} \approx r_{major}$ and so $e \approx 0$. On the other hand, for very elongated orbits $r_{major} \gg r_{minor}$ and $e \approx 1$.

b) We know the perihelion and the semi-major axis. How to do this?

[1] Tycho almost died young when his nose was chopped off in a duel. However, he glued on a gold nose and went on to become a legend!

## 4

As the earth moves around the sun, the angle towards an astronomical object changes with respect to a constant reference direction. *Half* (!) the size of the angular change is called the **parallax**.

Since the stars are much farther away than the earth-sun distance, the tri-

Figure 1: The direction to a star changes as the earth is at different places in its orbit around the sun.

angle is very close to begin 'liksidig', and we can use

$$\tan\theta/2 = \frac{1\ AU}{D} \rightarrow D = \frac{1\ AU}{\tan\theta/2}$$

The angle $\theta/2$ is the parallax $p$. Furthermore, since $D \gg 1\ AU$ we again use the small angle approximation and get

$$D \approx \frac{1\ AU}{\theta} = \frac{1\ AU}{0.77 \cdot 1/3600 \cdot 2\pi/180} = 268,000 AU = 1.3 \text{ parsecs}$$

## 5

a) We first transform the distance to parsecs.

$$1pc = 3.26ly \rightarrow 427ly = 131\ pc$$

Then we use the relation between parsecs, AU and angular size in arcseconds; one AU takes up an angular size of one arcsecond at a distance of one parsec.

Thus

$$x[AU] = D[pc]\theta[arcsec] = 131 \cdot 0.125 = 16.4\ AU$$

b) Is it reasonable for a star to be this large; sixteen times the earth-sun orbit? For some, yes. These are the red supergiants. The size is similar to the orbit of Uranus at 19 AU.

The sun's radius is about 0.01 AU. A red supergiant thus has volume that is $\sim (16/0.01)^3$ or four billions times the sun's volume! Does it also have a mass this large, four billion solar masses? It does not. Stellar theory predicts that stars more massive than 100-150 solar masses cannot exist. The red supergiants have a density about 10 million times lower than in the sun.

## 6

The stars move above us because the earth is rotating. If it was not, they would be at the same spot for very long times. We can therefore give them 'absolute' coordinates that will be accurate over any astronomers lifetime.

To specify the direction to something we need two angles; $(\theta, \phi)$ in spherical coordinates. In astronomy this coordinates system is defined in the following manner; the $z$ axis is parallell to the earth's spin axis, so that the $(x, y)$ plane is coplanar with the earth's equator. Then, the $x$ axis is chosen as the direction of then sun at a particular time of the year; the March equinox.

Any point of a sphere can then be expressed as the angle from the x-axis and the angle from the xy-plane. These two angles are called **Right Ascension** (RA) and **Declination** (Dec). They could both be given in normal degrees, but traditionally slighly other units are used. The RA angle is given as a time, where the full 360 degrees correspond to 24 hours. For example, an angle of 30 degrees would become

$$RA = \frac{30}{360} \cdot 24 = 2\ h$$

And an angle of 200.1 degrees would become

$$RA = \frac{200.1}{360} \cdot 24 = 13.34\ h = 13\ h \text{ and } 20.4 \text{ min} = 13\ h\ 20 \text{ min } 24 \text{ sec}$$

The declination is expressed in degrees, arcminutes and arcsecond. One arcminute is 1/60 of a degree and one arcsecond is 1/60 of an arcminute. Thus, an angle of 10.12 degrees can be expressed as

$$Dec = 10^\circ \text{ and } 0.12 \cdot 60 \text{ arcmin} = 10^\circ \text{ and } 7.2 \text{ arcmin} =$$

$$= 10^\circ \text{ and } 7 \text{ arcmin and } 0.2 \cdot 60 \text{ arcsec} = 10^\circ \text{ and } 7 \text{ arcmin and } 12 \text{ arcsec}$$

For a local observer, the direction to an object is given by the **Hour Angle** and the **Elevation**. The elevation is simply the angle between the object and the horizon. The Hour Angle is the horizontal angle (in time units just as RA) between the object and the direction of maximum elevation (local meridian). In the northern hemisphere, that direction is south. The hour angle is negative the

the object has not yet passed the meridian, and positive if it has. For example, if the hour angle of a star is -2 h it means that it will pass the local meridian in 2 hours.

a) The sidereal time is the hour angle of the RA=0 direction. Thus, when the sidereal time is zero, the hour angle of any other object equals minus its Right Ascension. Conversily, if the hour angle of an object is zero, a time equal to its RA must have passed since RA=0 passed overhead.
b) The Mean Solar Time is the hour angle of the mean Sun.

### 7

The sun, with its special position, continously change its declination and right ascension. When the northern hemisphere has summer, it lies above the earth's equatorial plane and has thus a positive declination. In winter, it lies below and has negative declination. With a figure we easily show that the values it moves between must be plus and minus the angle of the earth's rotation axis. This has a value of 23.5 degrees.

### 8

a) Pisces has $\alpha = +1h, \delta = +15$ and Gemini has $\alpha = +7h, \delta = +20^o$. The relative directional separation is thus six hours, which is equivalent to $6/24 = 1/4$ of the arc. Then exactly half the moon will be reflecting light in earth's direction.
b) As six months pass, the sun will have moved to the exact opposite side of the celestial sphere, i.e. it will have $\alpha = +13h, \delta = -15^o$. The moon, which circles the earth with a period of one month, will be back in the constellation Gemini with $\alpha = +7h$. There is thus, also now a 90 degree separation between sun and moon, and we will have half-moon.

### 9

a) The eccentricity as function of apo-apsis and peri-apsis is given by

$$e = \frac{r_a - r_p}{r_a + r_p} = \frac{1861 - 1839}{1861 + 1839} = 0.0059$$

b) The period is related to the semi-major axis (which is the same as the apo-apsis) by Kepler's law

$$P^2 = R^3$$

where $P$ is in years and $R$ is AU. We get

$$P = \left(\frac{1861}{150 \cdot 10^6}\right)^{3/2} = 4.37 \cdot 10^{-8}\ y =$$

## 2

## Key concepts

- The resolution of a telescope : its theoretical formula and the effect of the atmosphere.

## D1

If you toss a ball up into the sky it comes back after a while. But if you would be able to throw it fast enough it wouldnt; there is a critical limit called the **escape velocity**.

To understand escape velocities, we should understand that gravity is a force that binds objects together, just as the electric force binds atoms together. To tear apart such a bond requires adding an amount of energy equal to the binding energy.

The binding energy of a gravitational bond is given by

$$E_g = G\frac{Mm}{r}$$

where $G$ is the gravitational constant and has value $6.67 \cdot 10^{-11}$

Now, to break the bond we need to give the object a kinetic energy $E_k$ at least as large as this graviationa bdingin energy. Since $E_k = \frac{1}{2}mv^2$ we get

$$\frac{1}{2}mv_{esc}^2 = G\frac{Mm}{r} \rightarrow$$

$$v_{esc} = \sqrt{\frac{GM}{r}} \tag{1}$$

The important insight here is that *the escape velocity does not depend of the mass of the escaping object.*

Let us consider now an object sitting on the surface of the earth, which has mass $M = 6 \cdot 10^{24}$kg. $r$ is then the earth's radius, which is 6400 km. We get

$$v = \sqrt{\frac{6.67 \cdot 10^{-11} \cdot 6 \cdot 10^{24}}{6.4 \cdot 10^6}} = 11,200\ m/s = 11.2\ km/s$$

## D2

a) For $1/\lambda$ to become a positive quantity, we need $n_1 < n_2$. The Lyman series are all the transitions to $n = 1$, so we get $n_1 = 1, n_2 = 2, 3, ....$ The Balmer series are all the transitions to $n = 2$ so here $n_1 = 2, n_2 = 3, 4, ....$ The Paschen series is in the same manner $n_1 = 3, n_2 = 4, 5, ...$

b) The letter $H$ is reserved for the Balmer line, so we know that $n_1 = 2$. The greek letter then tells us the value of $n_2$. The first possible $n_2$ is called $\alpha$, the second $\beta$ etc.

$$\frac{1}{\lambda_{H\alpha}} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 1.097 \cdot 10^7 \cdot 0.139\ m^{-1} =$$

$$= 1.52 \cdot 10^6\ m^{-1} \rightarrow \lambda_{H\alpha} = \frac{1}{1.52 \cdot 10^6} = 6.56 \cdot 10^{-7}\ m = 656\ nm$$

In a similar way

$$\frac{1}{\lambda_{H\beta}} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) \rightarrow \cdots \lambda_{H\beta} = 486\ nm$$

$$\frac{1}{\lambda_{H\gamma}} = R\left(\frac{1}{2^2} - \frac{1}{5^2}\right) \rightarrow \cdots \lambda_{H\beta} = 434\ nm$$

c) As $n_2$ increases, the line wavelengths converge to the value

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{\infty}\right) = \frac{R}{n_1^2}$$

For the Balmer series ($n_1 = 2$) there is apparantly a lower limit to the wavelengths at $4/R = 365\ nm$.

The hydrogen can, however, also make transitions from an *unbound* state to $n_1$. Since such unbound states can be anything, they are referred to as the **continuum**. A transition from somewhere in the continuum to $n_1 = 2$ will this produce a photon with wavelength shorter than 365 nm.

## D3

## D4

Refraction limits how fine details any telescope can observe. It can be shown that the resolution is given by

$$\theta = 1.22\frac{\lambda}{D}$$

where $D$ is the diameter of the telescope. We see that bigger telescopes and/or observations at shorter wavelengths favor making sharp images.

a) The Keck telescope on Hawaii is one of the largest in the world. Its resolution at 500 nm is

$$\theta = 1.22\frac{500 \cdot 10^{-9}}{10\ m} = 6.1 \cdot 10^{-8}\ radians = 3.5 \cdot 10^{-6}\ deg = 12.6\ milliarcseconds$$

What size of object is this on the moon? $X = 384,000\ km \cdot 6.1 \cdot 10^{-8} = 0.023\ km = 23\ m$. The smallest detail the world's best telescopes could see on the moon is thus about 20 meters across. It was thus not possible to inpsect the moon landings by telescope from earth in 1969 (when telescopes were much worse) and it is not possible to see any stuff they left today.

b) Radio waves have much longer wavelength than optical light, so a radio telescope has much worse resolution. For observations at 20 cm we get

$$\theta = 1.22\frac{0.2}{10} = 1.4°$$

## 5

We clearly need to compute the escape velocity from the sun and compare to 150 km/s. Using formula 1

$$v_{esc} = \sqrt{\frac{6.67 \cdot 10^{-11} \cdot 2 \cdot 10^{30}\ kg}{7 \cdot 10^{8}\ m}} = 437\ km/s$$

The flare will thus not escape but will fall back into the sun.

## 6

We have seen (exercise 2) that the wavelength of an H$\beta$ photon is 486 nm. The energy is relatived to wavelength as

$$E = h\nu = h\frac{c}{\lambda}$$

where $h$ is Planck's constant ($6.63 \cdot 10^{-34}$) and $c$ is the speed of light. We get

$$E = 6.63 \cdot 10^{-34} \cdot \frac{3 \cdot 10^{8}}{486 \cdot 10^{-9}} = 4.1 \cdot 10^{-19}\ J$$

One electron volt (ev) equals $1.602 \cdot 10^{-19}\ J$, so the answer is eV is

$$E = \frac{4.1 \cdot 10^{-19}}{1.602 \cdot 10^{-19}} = 2.56\ eV$$

## 7

The wavelength of peak emission is related to the temperature of the blackbody; hotter bodies emit shorter wavelengths. The exact relation is called **Wien's displacement law**

$$\lambda_{peak} = \frac{2.9 \cdot 10^{-3}}{T} \tag{2}$$

In figure 5 we see that the peak occurs at a wavelength of $0.8 \cdot 10^{-6}$m. Then

$$T = \frac{2.9 \cdot 10^{-3}}{0.8 \cdot 10^{-6}} = 3625\ K$$

## 8

The two edges are moving at 2km/s with respect to the center of the sun. The circumference if the sun is

$$C = 2\pi R_{\odot} = 2\pi \cdot 6.96 \cdot 10^{1}0\ cm = 4.37 \cdot 10^{11}\ cm$$

With a velocity of 2 km/s the period becomes

$$P = \frac{4.37 \cdot 10^{11}}{2 \cdot 10^{5}\ cm} = 2.19 \cdot 10^{6}\ s = 25.3\ days$$

## 9

All stars are too good approximation blackbodies; this means that their gas is in local thermal equilibrium and that they are optically thick at all wavelengths. We can then use Stefan-Boltzmann's law to find the luminosity of any star:

$$L = \sigma R^2 T^2$$

The luminosity depends only on the star's radius and surface temperature! For Betelgeuse

$$L = 5.67 \cdot 10^{-8} \cdot (16.4\ AU)^2 \cdot 3625^4 = 5.67 \cdot 10^{-8} \cdot 6.0 \cdot 10^{28} \cdot 1.7 \cdot 10^{14} = 5.9 \cdot 10^{35}\ J/s$$

## 10

a) 1 AU at 1 parsec subtends an angle if 1 arcsec. Then 10 AU at 1 parsec subtends 10 arcsec, and 10 AU at 200 parsec subtends 0.05 arcsec. Since this is the semi-major axis of the orbit, the separation of the stars is $0.1''$. Since this is only 1/10 of the quoted resolution of $1''$, the binary is not resolved.

b) The resolution is $1''$ because the atmosphere blurs the images. The theoretical resolution of a 2.56 m telescope not disturbed by an atmosphere is at optical wavelengths ($\lambda \approx 500\ AU$) is

$$\theta = 1.22 \frac{\lambda}{D} = 1.22 \frac{500 \cdot 10^{-9}}{2.56} = 2.4 \cdot 10^{-7}\ rad = 1.37 \cdot 10^{-5} deg = 0.049''$$

If the atmosphere is removed, the NOT could thus indeed resolve the binary system.

# 3

## D1

a) The output from the star spreads out and at a distance $D = 7.76$ parsecs covers a spherical surface of area

$$A = 4\pi D^2 = 4\pi \cdot (7.76 \cdot 3.09 \cdot 10^{16}\ m)^2 = 7.23 \cdot 10^{35}\ m^2$$

The power per unit area is then

$$F = \frac{L}{A} = \frac{2.1 \cdot 10^{28}\ W}{7.23 \cdot 10^{35}\ m^2} = 2.90 \cdot 10^{-8}\ Wm^{-2}$$

b) The apparent magnitude is given by

$$m = -2.5 \log_{10}(F/F_{ref}) + m_{ref}$$

where $F_{ref}$ and $m_{ref}$ are the flux and apparent magnitude of any other object. Here we use the values of the sun $F_\odot = 1366\ W/m^2$ and $m_\odot = -26.81$. This gives us

$$m = -2.5 \log_{10} \frac{2.90 \cdot 10^{-8}}{1366} - 26.81 = -0.13$$

c) The power output derived above is the total power over all wavelengths that the star is emitting at. Only part of this will be at visible wavelengths. The **bolometric correction** specifies how much larger the visual magnitude is compared to the bolometric magnitude.

$$\text{Bolometric correction} = m_v - m_b = M_V - M_B$$

We then get

$$m_v = BC + m_b = 0.15 - 0.13 = 0.02$$

## D2

The color of a star can be specified by the relative brightness at two different wavelengths or wavelength bands. The wavelength bands are almost always chosen to be the blue (B) and the visual (V) bands. Then

$$Colorindex(B - V) = m_B - m_V$$

The magnitudes become smaller with increasing brightness, a blue object will have a small color index and a red object a large color index.

a) Here we get

$$B - V = 18.3 - 15.8 = 2.5$$

b) Since we know the star's temperature (5500 K), we know its spectrum and thus the value of the color index that it *should* have. The center wavelengths for B and V are 442 nm and 540 nm. Then Plancks blackbody formula gives us

$$\frac{I_B}{I_V} = \left(\frac{\lambda_V}{\lambda_B}\right)^5 \cdot \frac{\exp \frac{hc}{\lambda_V k \cdot 5500} - 1}{\exp \frac{hc}{\lambda_B k \cdot 5500} - 1} = 0.92$$

Then

$$M_B - M_V = -2.5 \log_{10} F_B - (-2.5 \log_{10} F_V) = 2.5 \log_{10} \frac{F_v}{F_B} = 0.09$$

## D3

The **distance modulus** is defined as

$$\text{distance modulus} = m - M = -2.5 \log_{10} \frac{F}{F_{ref}} + m_{ref} + M$$

Now, let the reference object be the object itself at 10 parsecs. Then

$$m - M = -2.5 \log_{10} \frac{F}{F_{10}} + M - M = -2.5 \log_{10} \left(\frac{D}{10\ pc}\right)^{-2}$$

We now solve this for $D$:

$$m - M = 5 \log_{10} \left(\frac{D}{10\ pc}\right) \rightarrow D = 10^{\frac{m-M}{5}+1}\ pc$$