Facts and formulae, revised 2009-11-19

This section contains some basic facts and formulae that will be useful when solving the exercises. These pages may be brought to the exam/midterm (at the discretion of the responsible teacher). However, you may not make notes at these pages if you plan to bring them to the exam.

Facts

Physical constants:

- Gravitational constant, $G = 6.673 \cdot 10^{-11}$ N m^2 kg^{-2}
- Planck constant, $h = 6.626 \cdot 10^{-34}$ Js
- Speed of light, $c = 2.998 \cdot 10^8$ m/s
- Rydberg constant, $R = 1.097 \cdot 10^7$ m^{-1}
- Boltzmann constant, $k_B (= k) = 1.38 \cdot 10^{-23}$ JK^{-1}
- Stefan-Boltzmann constant, $\sigma = 5.67 \cdot 10^{-8}$ Wm^{-2}K^{-4}
- Mass of the electron, $m_e = 9.1094 \cdot 10^{-31}$ kg
- Mass of the proton, $m_p = 1.6726 \cdot 10^{-27}$ kg
- Mass of the neutron, $m_n = 1.6749 \cdot 10^{-27}$ kg
- Mass of the Helium nucleus, $m_\alpha = 6.6447 \cdot 10^{-27}$ kg
- 1 Å(Ångström) = 10^{-10} m
- 1 eV = $1.602 \cdot 10^{-19}$ J
- 1 radian = $\frac{180}{\pi}^\circ$

Astronomical quantities

- The Astronomical Unit, 1 AU = $1.496 \cdot 10^{11}$ m
- The Lightyear, 1 l.y. = $9.454 \cdot 10^{15}$ m
- The Parsec, 1 pc = 3.26 l.y. = 206265 AU = $3.086 \cdot 10^{16}$ m.
- 1 y = 365.2564 days = $3.156 \cdot 10^7$ s.
- 1 siderial day = $23^h\, 56^m\, 4.091^s$

Properties of the Sun and planets:

- Mass of the Sun, $M_\odot = 2.0 \cdot 10^{30}$ kg
- Luminosity of the Sun, $L_\odot = 3.90 \cdot 10^{26}$ W
- Radius of the Sun, $1R_\odot = 6.9599 \cdot 10^8$ m
- Radius of the Earth, $R_\oplus = 6378$ km

- Inclination of Earth's orbit (the inclination of the Ecliptic): $23° 27'$
- Table of properties of the eight planets (and one dwarf planet) in the solar system:

Planet (Dwarf planet)	Mass (kg)	Siderial period (years)
Mercury	$3.302 \cdot 10^{23}$	0.241
Venus	$4.868 \cdot 10^{24}$	0.615
Earth	$5.974 \cdot 10^{24}$	1.000
Mars	$6.418 \cdot 10^{23}$	1.88
Jupiter	$1.899 \cdot 10^{27}$	11.86
Saturn	$5.685 \cdot 10^{26}$	29.46
Uranus	$8.682 \cdot 10^{25}$	84.10
Neptune	$1.024 \cdot 10^{26}$	164.86
(Pluto)	$1.3 \cdot 10^{22}$	248.60

Formulae

SI units should be used in all formulae, unless otherwise specified.

The hour angle, t (θ is the siderial time and α the right ascension):

$$\theta = \alpha + t. \tag{1}$$

Approximate relation between the siderial time and the mean solar time (τ is the mean solar time and d the number of days passed since the autumnal equinox). The change is in minutes (i.e. the factor 4 is minutes per day).

$$\theta \approx \tau + 4 \cdot d \tag{2}$$

Theoretical limit to the resolution of a telescope (the diffraction limit), D is the diameter of the telescope aperture (NB, the resolution is given in radians):

$$\theta_d = 1.22 \frac{\lambda}{D} \tag{3}$$

The small angle approximation, the angular size of an object is given by (d is distance to the object, D is the size of the object):

$$\tan(\phi/2) = \frac{D/2}{d} \Rightarrow [\phi \text{ small}] \Rightarrow \phi \approx \frac{D}{d} \tag{4}$$

Definition of parsec (also how to calculate distance from a parallax). NB the parallax θ_p is here given in arcseconds and the distance d in parsecs.

$$d = \frac{1}{\theta_p} \tag{5}$$

Gravity and celestial mechanics

Newton's law of gravity: $F_g = G\frac{m_1 m_2}{r^2}$ (6)

Centripetal force: $F_c = \frac{mv^2}{r}$ (7)

Gravitational Potential energy: $E_p = -G\frac{m_1 m_2}{r}$ (8)

Kinetic energy: $E_k = \frac{mv^2}{2}$ (9)

Kepler's third law for orbits around the <u>sun</u>: $P[\text{years}]^2 = a[\text{AU}]^3$ (10)

Keplers's general third law (SI units): $P^2 = \frac{4\pi^2}{G(m_1 + m_2)}a^3$ (11)

Energy equation: $v^2 = G(m_1 + m_2)\left(\frac{2}{r} - \frac{1}{a}\right)$ (12)

Eccentricity of an elliptical orbit: $e = \frac{c}{a}$ (13)

Volume of a sphere: $V = \frac{4\pi R^3}{3}$ (14)

Surface area of a sphere: $A = 4\pi R^2$ (15)

Moment of inertia for a homogenous sphere: $I = \frac{2MR^2}{5}$ (16)

Radiation and energy

Energy of a photon: $E = h\nu = \frac{hc}{\lambda}$ (17)

Bohr formula for the hydrogen atom: $\frac{1}{\lambda} = R\left(\frac{1}{n_{lower}^2} - \frac{1}{n_{upper}^2}\right)$ (18)

Planck's law: $I(\lambda, T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$ (19)

Stefan-Boltzmann's law: $F = \sigma T^4$ (20)

Wien's law: $\lambda_{max} = \frac{2.898 \cdot 10^{-3}}{T}$ (21)

Mass-energy relation: $E = mc^2$ (22)

Redshift, definition: $z = \frac{\lambda - \lambda_0}{\lambda_0}$ (23)

Doppler effect ($z \ll 1$): $z = \frac{v}{c}$ (24)

(25)

The magnitude system

Flux from an object (L is the luminosity of the object): $F = \frac{L}{4\pi d^2}$ (26)

Definition of magnitude (magnitude difference): $m_1 - m_2 = 2.5 \log \frac{F_2}{F_1}$ (27)

Corrected magnitude: (A is extinction) $m_{korr} = m_{obs} - A$ (28)

Distance modulus (d in pc): $DM = m_{korr} - M = 5 \log d - 5$ (29)

Bolometric correction: $BC = M_V - M_{bol}$ (30)

Bol. magnitude-luminosity relation: $M_{bol,1} - M_{bol,2} = 2.5 \log \frac{L_2}{L_1}$ (31)

Colour excess: $E_{(B-V)} = (B-V)_{obs} - (B-V)_{true}$ (32)

Extinction in the V band, A_V: $A_V = 3 \cdot E_{(B-V)}$ (33)

Virial mass of a gravitationally bound system (V_{pec} refers to individual velocities of bodies in the system and R_{eff} to the effective radius of the system):

$$M = \frac{3\overline{V_{pec}^2} R_{eff}}{G} \quad (34)$$

The Hubble law:

$$v = H_0 \cdot d \quad (35)$$