Late Stages in the Evolution of Low and Intermediate Mass Stars

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1 Introduction

This part of the course treats the post-Main Sequence (post-MS) evolution of stars. The most massive stars (above \(\sim 9 \, M_\odot\)) will end their lives as supernovas, and are treated in another part of this course. Here we will consider the stars with an initial mass (Zero Age Main Sequence or ZAMS mass) of less than 9 \(M_\odot\), who will end their lives as White Dwarfs.

Stars evolve due to irreversible processes. The two main irreversible processes in stars are

1. nuclear burning (in the central regions)
2. mass loss (from their surface)

In the classical theory of stellar evolution (before the 1980s) only the first of these processes was taken into account. In absence of mass loss, the division between supernova- and white dwarf-producing stars would lie at \(M_{\text{ZAMS}} \approx 4 \, M_\odot\), when the core reaches the Chandrasekhar mass (1.4 \(M_\odot\)). The fact that this line in reality lies at a twice higher mass already indicates the central role mass loss plays in stellar evolution. Because of uncertainties in the description of the mass loss processes, the actual theoretical dividing line remains somewhat uncertain, but lies somewhere in the range 8–10 \(M_\odot\).

Supernovae are spectacular events, but why are lower mass stars important?

- Most stars are low mass stars; so they form an essential part of stellar populations.
- Source of material processed in nuclear burning (chemical enrichment), as well as source of dust.
- Our own Sun is a low mass star, so we are studying its future.
- Interesting (astro)physics and chemistry (complex molecules, including organic molecules).

The Main Sequence (MS) evolution time of a star depends on the stellar mass according to \(t_{\text{evol}} \propto M^{-2.5}\). This means in practice that a 8 \(M_\odot\) star will reach the end of its evolution in 50 Myr, and a 0.8 \(M_\odot\) star in 14 Gyr. The latter have therefore not had the time to evolve off the MS within the life of the Universe (13.6 Gyr). This means that in principle one can still find stars from the very first generations of stars among these lowest mass stars.

The empirical Salpeter Initial Mass Function (IMF) for stellar populations gives

\[
\frac{dN}{dM} \propto M^{-2.35}
\]  

which means that of a coeval stellar population, 96% of the stars have a mass between 0.8 and 8 \(M_\odot\).

The post-Main Sequence evolution of stars with \(M_{\text{ZAMS}} < 9 \, M_\odot\) goes through the following stages
1. Red Giant Branch (RGB) phase
2. Horizontal Branch (HB) phase
3. Asymptotic Giant Branch (AGB) phase
4. Planetary Nebula (PN) phase
5. White Dwarf phase

The emphasis of these lectures will be on the AGB phase, which together with the PN phase is the area of most active research worldwide, and also in Sweden (Stockholm, Onsala, Uppsala). Areas that are being actively investigated are for example:

- Mass loss and circumstellar material (CSM)
- Pulsations and variability
- Metal production
- Transition of AGB stars into PNe.
2 Basic Stellar Evolution

Essentially all detailed information we have about stellar evolution comes from numerical simulations. In their simplest form these simulations apply a slow evolution to the 1D, radial equations of (static) stellar structure:

\[ \frac{dM_r}{dr} = 4\pi r^2 \rho \] (2)

Hydrostatic equilibrium:

\[ \frac{dp}{dr} = -\frac{GM_r \rho}{r^2} \] (3)

Energy production:

\[ \frac{dL_r}{dr} = 4\pi r^2 \rho \epsilon \] (4)

Energy transport:

\[ \frac{dT_r}{dr} = \begin{cases} \frac{3}{4} \frac{\epsilon}{\kappa} \frac{L_r}{k_B \rho} & \text{if } \frac{d\ln p}{d\ln T} > \frac{\gamma}{\gamma - 1} \text{ (radiative diffusion)} \\ \left(1 - \frac{1}{\gamma}\right) \frac{\mu_{\text{H}} M_r}{k_B} \frac{GM_r}{r^2} & \text{if } \frac{d\ln p}{d\ln T} < \frac{\gamma}{\gamma - 1} \text{ (adiabatic convection)} \end{cases} \] (5)

combined with

Equation of state: \[ p = p(\rho, T, \text{composition}) \] (6)

Opacity: \[ \kappa = \kappa(\rho, T, \text{composition}) \] (7)

Nuclear reactions: \[ \epsilon = \epsilon(\rho, T, \text{composition}) \] (8)

where \( M_r \) is the interior mass \( (M(< r)) \), \( L_r \) the interior luminosity, \( \mu \) the mean molecular weight and \( \gamma \) the polytropic index (for example 5/3 for a monatomic gas and 4/3 for a relativistic gas).

The equations for hydrostatic equilibrium and mass play a rather passive role in this set of equations. For stellar evolution the interesting things happen in the processes of energy production \( (\epsilon) \) and transport \( (dT/dr) \).

We will now look at the results of two calculations by John Lattanzio, one for \( M_{ZAMS} = 1 \ M_\odot \) (low mass star, Fig. 1) and one for \( M_{ZAMS} = 5 \ M_\odot \) (intermediate mass star, Fig. 3).

2.1 A 1 \ M_\odot star

On the Main Sequence, the energy production in stars less massive than 1.3 \( M_\odot \) is dominated by the proton-proton chain (Fig. 2), which has a relatively mild dependence on temperature \( (\epsilon_{\text{pp}} \propto T^4) \), and therefore the core will use radiative energy transport.

This has the effect of decreasing the H abundance and increasing the He abundance increases ‘inside-out’ (see inset a), giving a smooth transition from nuclear H burning in the core to H shell burning around the He-core (at point 4).

As the inactive He-core grows through the addition of He from the H burning shell, it contracts, and the envelope expands. The densities in the He-core become high enough...
Figure 1: Schematic evolution of a $1 M_\odot$ star across the HR diagram. From Lattanzio & Wood (2004).
the electrons to become degenerate, after which the pressure of the material is only a function of density, no longer of temperature (points 4–7). As the star approaches the Hayashi limit of fully convective stars (point 7), its envelope becomes fully convective and it evolves at nearly constant $T_{\text{eff}}$, while increasing its luminosity $L$ (and hence its radius $R$). This phase is the Red Giant Branch (RGB). During this phase the convection may reach the H burning shell and mix up processed material into the stellar envelope. This process is known as the First Dredge Up (point 8). Its chemical signature is as follows: ($^4\text{He} \uparrow$, $^{14}\text{N} \uparrow$, $^{12}\text{C} \downarrow$, $^{12}\text{C}/^{13}\text{C} \downarrow$).

The RGB ends when He-fusion starts in the core (point 9). He-fusion occurs through a process known as the 3$\alpha$ process, which has a strong temperature dependence ($\propto T^{40}$). If the core is (electron) degenerate ($M_{ZAMS} < 2.3 \text{ M}_\odot$), He-fusion starts with a runaway process known as the Helium Flash. However, nearly all of the energy generated in the Helium Flash is used to rearrange the internal structure of the star, and so the flash itself is not observable.

In the next phase known as the Horizontal Branch phase (points 10–14), the star has He-core burning and H-shell burning. The He-core burning produces $^{12}\text{C}$ with the 3$\alpha$ process. The $^{12}\text{C}$ can react further via $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ to form a $^{12}\text{C}/^{16}\text{O}$ core. The rate of this last reaction is uncertain, so the final ratio between $^{12}\text{C}$ and $^{16}\text{O}$ in the core is also uncertain.

During this phase, the inner core is convective, but the outer core is semi-convective, leading to a smooth transition from He-core burning to He-shell burning, which is the start of the AGB (point 14).
Figure 3: Schematic evolution of a 5 $M_\odot$ star across the HR diagram. From Lattanzio & Wood (2004).
2.2 A \(5 \, M_\odot\) star

On the Main Sequence stars with masses above \(\sim 1.3 \, M_\odot\) have H-core burning through the CNO-cycle (Fig. 4). This process has a stronger temperature dependence than the pp-chain, which leads to a convective core. Because of this the H-concentration drops homogeneously through the core region (Fig. 3, inset a), and the end of H-core burning occurs suddenly (point 4).

At this point the core is heavier than the so-called Schönberg-Chandrasekhar limit

\[
\frac{M_{\text{core}}}{M} > 0.4 \left( \frac{\mu_{\text{env}}}{\mu_{\text{core}}} \right)^2 \sim 0.1
\]

(13)

and therefore contracts rapidly (points 4-5). This leads to the ignition of H-shell burning (point 5), and further rapid core contraction ('Herschprung gap', points 5-7). Upon reaching the RGB, also in these stars the first dredge-up takes place (point 8).

He-core burning starts without a flash, since the core is not electron-degenerate (point 9).

During the HB both the He-core and H-shell are active. Their relative strengths determine how high \(T_{\text{eff}}\) becomes. There is no semi-convective outer core for this type of star. When He-shell burning starts (point 14), the star enters the AGB phase.

2.3 Mass limits

There are a number of useful mass limits to keep in mind.
Main Sequence: $M_{\text{ZAMS}} \begin{cases} < 1.3M_\odot & \text{pp-chain dominates} \\ > 1.3M_\odot & \text{CNO-cycle dominates} \end{cases}$

RGB: $M_{\text{ZAMS}} \begin{cases} < 2.3M_\odot & \text{electron-degenerate He-core} \\ > 2.3M_\odot & \text{non-degenerate He-core} \end{cases}$

$M_{\text{ZAMS}} < 0.6M_\odot$ He-burning never starts

$M_{\text{ZAMS}} \begin{cases} < 9M_\odot & \text{electron-degenerate C/O core} \\ > 9M_\odot & \text{non-degenerate C/O core} \end{cases}$

The last division is interesting. In principle the start of C-burning in a degenerate core will lead to a C-flash, which would be powerful enough to disrupt the core and star, leading to a thermonuclear supernova. Because mass loss reduces the mass during the RGB/AGB phases, these type of supernovae may not exist. Thermonuclear supernova do of course occur when White Dwarfs due to external processes reach the Chandrasekhar mass, leading to the famous type Ia supernovae.

### 2.4 Shell sources

We end this introductory section with a review of some of the properties of shell sources, which are important both during the early and late post-MS evolution. When nuclear burning occurs in a shell, the physics imposes certain restrictions.

**The shell source is associated with a density jump.** The reason for this is that the pressure and the temperature at the core–shell interface should be constant (to have hydrostatic and thermal equilibrium), but the composition and the thus the mean molecular weight of the core and shell material differ. Since the pressure, density and temperature are related through the ideal gas law

$$ p = \frac{\rho k_B T}{\mu m_H}, \quad (14) $$

a jump in $\mu$ implies a jump in $\rho$. For example, if the core is 100% He, it will have $Y = 1$, giving $\mu_{\text{core}} = 4/3$. If the shell has 30% He, it will have $X = 0.7$, $Y = 0.3$, and $\rho_{\text{shell}} = 0.62$. This then implies $\rho_{\text{core}} = 2.2 \times \rho_{\text{shell}}$.

**The shell source position remains approximately fixed in space.** The cause is a negative feedback loop involving the strongly non-linear temperature dependence of the nuclear fusion processes. If the shell acquires a smaller radius, its temperature will go up. Therefore its energy production will go up enormously, increasing the pressure of the shell, making it expand, thus increasing its radius, and lowering the temperature again. For example, for a 1 $M_\odot$, $r_{\text{shell}} = 0.03 R_\odot$ throughout the RGB phase.
Shell sources reverse the contractive/expansive behaviour of the layers below them. When a core contracts, it in fact does not homologously contract, but instead becomes more highly concentrated towards its centre, thus increasing the density gradient (since $r_{\text{shell}}$ is roughly constant). This causes the stellar envelope to expand. Note that this is a real expansion, since the stellar radius can be changed. The actual physical cause for this reversal behaviour is still being debated, but the effect can clearly be seen in numerical stellar evolution models. There are two easy rules of thumb for this:

- Core contraction [active shell] Envelope expansion
- Core expansion [active shell] Envelope contraction

So the active shell source can be said to mirror the behaviour of the inner regions.
3 Stellar Evolution during the AGB Phase

Both low- and intermediate mass type stars actually reach the AGB with similar internal structures:

- C/O core (degenerate)
- He-shell
- Intershell region
- H-shell
- H-envelope

As we will see in more detail later, the He-shell is thermally unstable, leading to periodic He-shell flashes, also known as thermal pulses. The phase before the first thermal pulse is known as the Early-AGB (E-AGB), the phase thereafter (starting at point 15 in Fig. 1 and 3) is known as the thermally pulsing AGB (TP-AGB).

During the E-AGB stars of initial mass $M_{ZAMS} > 4M_{\odot}$ temporarily lose their H-burning shell. This allows envelope convection to reach into the region of partially burnt H, and mix up processed material in what is known as the second dredge-up ($^4\text{He} \uparrow, ^{12}\text{C} \downarrow, ^{13}\text{C} \uparrow, ^{14}\text{N} \uparrow$).

On the TP-AGB the evolution of low and intermediate mass stars is qualitatively similar, being characterized by

- A sequence of He-shell flashes, with longer interpulse periods during which the H-shell dominates the energy production.
- Extensive mass loss from the stellar surface ($> 10^{-7} M_{\odot} \text{ yr}^{-1}$...).

Since the typical luminosity of an AGB star is in the range $10^3$–$10^4 L_{\odot}$, one can estimate that the star needs to burn $\sim 10^{-8} M_{\odot} \text{ yr}^{-1}$ to produce this luminosity. This means that during the AGB phase, the stellar evolution is dominated by mass loss rather than by nucleosynthesis!

The internal structure of AGB stars is extremely inhomogeneous (see Fig. 5). For example for a 1 $M_{\odot}$ star

<table>
<thead>
<tr>
<th>Structure</th>
<th>$q = m_r/M$</th>
<th>$R(R_{\odot})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>He-shell</td>
<td>0.516</td>
<td>0.017</td>
</tr>
<tr>
<td>H-shell</td>
<td>0.560</td>
<td>0.035</td>
</tr>
<tr>
<td>Surface</td>
<td>1.0</td>
<td>225 (~ 1 AU)</td>
</tr>
</tbody>
</table>

This implies that

$$\rho_{\text{core}} \sim 10^5 \text{ g cm}^{-3}$$
$$\rho_{\text{envelope}} \sim 10^{-7} \text{ g cm}^{-3}$$

To appreciate these numbers we can compare them to the density of the Earth atmosphere ($\sim 10^{-3} \text{ g cm}^{-3}$).

In what follows we will look closer at the following aspects
Figure 5: A schematic view of a $1 \, M_\odot$ AGB star interior. On the left, various regions are plotted against mass fraction, while on the right the regions are plotted against radius. $M_{\text{base}}$ is the mass at the base of the convective envelope, while $M_{\text{H-shell}}$ and $M_{\text{He-shell}}$ are the masses at the middle of the hydrogen- and helium-burning shells, respectively. From Lattanzio & Wood (2004).
• Thermal pulses, nuclear synthesis and dredge-up (Sect. 4)
• Stellar pulsations (Sect. 5)
• Mass loss and the structure of the circumstellar envelope (Sects. 6 and 7)
• Post-AGB evolution (Sect. 8)
4 Thermal Pulses, Nuclear Synthesis, and Dredge-Up

Thermal pulses are brief periods of runaway He burning in the He-shell of an AGB star. In the interpulse periods the active H-shell produces He which is added to the inactive He-shell. Above a certain shell mass the He-shell ignites and makes a He-shell flash. The first question to ask is why this is a runaway process.

4.1 Gravothermal specific heats

The thermal evolution of a pocket of gas is given by

$$\frac{dT}{dt} = \frac{1}{C} \frac{dq}{dt}, \quad (15)$$

where $C$ is the specific heat, and $q$ is the specific energy added or removed. If $C$ includes the gravitational effects (work done by the pocket of gas), it is called the gravothermal specific heat, $C_\ast$. From hydrostatic equilibrium and basic thermodynamics it can be shown that

$$C_\ast = C_p \left( 1 - \frac{\alpha}{\delta} \frac{4\delta}{4\epsilon - 3} \right), \quad (16)$$

where $\alpha$ and $\delta$ are given by the equation of state (EOS) of the gas: $\rho \propto p^{\alpha T^{-\delta}}$ and $\nabla_{\text{ad}}$ is the adiabatic temperature gradient

$$\nabla_{\text{ad}} = \left( \frac{d \ln p}{d \ln T} \right)_S. \quad (17)$$

The derivation of this expression can be found in Ciardullo’s lecture notes (lecture 20). For a monatomic gas $\nabla_{\text{ad}} = 2/5$ and $\alpha = 1, \delta = 1$. This means that $C_\ast < 0$ for such a gas. The effect is that adding heat to a region causes a drop in temperature. This may seem counterintuitive, but is due to the fact that adding heat will make the region expand, requiring work since the surrounding gas has to be pushed away, which thus lowers the temperature. This also means that if such a pocket of gas has nuclear reactions, it will be stable against small perturbations: adding small amounts of heat will reduce the temperature, and hence reduce the energy production by nuclear reactions (which are strongly temperature-dependent). In other words there is a negative feedback loop.

For a (non-relativistically) electron-degenerate gas $\alpha = 3/5$ and $\delta = 0$, making $C_\ast$ positive: adding heat will make the temperature go up. Consequently, the pocket will be unstable when nuclear processes are active, since the rise in temperature will increase the energy production, which will increase the temperature even more, a positive feedback loop.

However, in a star energy can also be carried away by radiation, so for a more complete analysis we also need to include the effect of energy transport by radiation. As can be seen from the equations of stellar structure, the efficiency of energy transport by radiation is inversely proportional to the opacity $\kappa$ of the gas. The opacity will be a function of the gas density and temperature

$$\kappa \propto \rho^{\eta T^q} \quad (18)$$
We also need to consider the energy production rate $\epsilon$ which also depends on the density and temperature

$$\epsilon \propto \rho^{\lambda} T^{\nu} \quad (19)$$

Some algebra (Ciardullo, lecture 20) shows that when including the effect of opacity, the rate of change for the temperature can be written as

$$\frac{dT}{dt} = \frac{K}{C_\star} \frac{dT}{T} \quad (20)$$

with

$$K = \frac{L}{M} \left[ (\nu + q - 4) + \frac{\delta}{4\alpha - 3} (3\lambda + 3p + 4) \right] \quad (21)$$

where $L$ and $M$ are the luminosity and mass of the region under consideration (the stellar core or a shell source). For the pp-chain $\nu = 4$ and $\lambda = 1$, for Kramer’s opacity $p = 1$ and $q = -3.5$, so using an ideal monatomic gas gives a positive $K$ together with a negative $C_\star$, resulting in a negative feedback loop, and a stable nuclear burning region.

For the He-burning $3\alpha$ process, $\nu = 40$ and $\lambda = 2$, so for an electron degenerate gas starting He-fusion both $K$ and $C_\star$ are positive, leading to positive feedback and the He-core flash.

Now how does this work for a shell source? First of all it is important to realize that the derivation for $C_\star$ that led to Eq. 16 assumed a spherical region which could expand in all directions. If $r$ is the size of the region, its volume is $\frac{4}{3} \pi r^3$ and a change $dr$ in size leads to a change in volume

$$dV = \frac{3}{r} V dr \quad (22)$$

However, a shell cannot expand in all directions. If we have a thin shell of thickness $D$ at radial position $r$, its volume is $4\pi r^2 D$, and its change of volume when increasing its thickness by $dD$ is

$$dV = \frac{1}{D} V dD = \frac{r}{D} V dD \quad (23)$$

This means that the factor 3 in Eq. 16 has to be replaced by $r/D$

$$C_\star^{\text{shell}} = C_p \left( 1 - \nabla_{\text{ad}} \frac{4\delta}{4\alpha - r/D} \right) \quad (24)$$

For a monatomic gas $\alpha = 1$, so if $D < r/4$ then $C_\star > 0$ and the shell will be unstable! So even when the gas is non-degenerate (which it will be outside of the core), nuclear burning in shells can be unstable. The physical reason is that for a thin shell, a small change in $D$ will cause a large change in the density $\rho$, but only a very small change in $r$, so not much energy is used in the expansion of the shell, leading to insufficient energy loss due to expansion, and thus a positive gravothermal specific heat. The instability is quenched when by expanding the shell becomes thick enough to break the $D < r/4$ condition.
The conclusion is that shells with active nuclear burning have to be relatively thick, but inactive shells can of course be thin. However, upon starting nuclear fusion, the process will be unstable, leading to a shell flash. This is exactly what happens to the He-shell in AGB stars.

### 4.2 Interpulse times

Numerical modelling of the thermal pulse process with stellar evolution codes shows that the time between these events is

$$\log \tau_p = 4.5(1.678 - \frac{M_{\text{core}}}{M_\odot}) \text{ (years)} \quad (25)$$

For a 1 $M_\odot$ star with a core mass of 0.6 $M_\odot$ this means an interpulse time of 70,000 years, whereas for a 5 $M_\odot$ star with a core mass of 1 $M_\odot$, $\tau_p = 1100$ years. Since during a He-shell flash a lot of energy is released in a short time, the region around the C/O core is modified. The peak luminosity can be as high as $10^9 L_\odot$. This leads to a number of interesting and important effects, caused by convective mixing and nucleosynthesis, effects that due to convective dredge-up even affect the surface layers of the star.

### 4.3 Anatomy of a thermal pulse

A thermal pulse cycle consists of a number of phases, which we will now describe qualitatively. See also Fig. 6.

**Off** the He-shell is dormant and produces almost no energy. The main source of energy is the H-shell. This state is the one which takes most of the time in a cycle. For stars with $M \gtrsim 3 M_\odot$ the convective envelope reaches down to the H-shell (‘Hot Bottom Burning’, see later). For lower mass stars the envelope is radiative just above the H-shell (which leads to the luminosity-core mass relation, see later).

**On** the He-shell ignites, $L \sim 10^8$–$10^9 L_\odot$. A convective zone develops above the He-burning zone (‘intershell convective zone’ or ‘intershell region’, ISR). This convection transports up $^4$He and He-fusion products ($^{12}$C). This phase lasts 10–100 years (depending on core mass).

**Power-down** The energy production in the He-shell declines, but the energy it produced drives an expansion of the ISR, pushing the H-shell outward, so that it also extinguishes. In this phase there is no, or very little energy production, and it also lasts some 10–100 years.

**Dredge-up** The transport of the energy produced in the shell flash starts convection in the envelope, reaching all the way down into the ISR, mixing up material that was involved in He-burning (e.g., $^{12}$C). This phase also lasts typically some 10–100 years.
Figure 6: Some details of the interior evolution during one thermal pulse (top) and for two consecutive pulses (bottom). Convective regions are shaded. The line toward the bottom of the middle panel shows the position of the He-burning shell ($Y = 0.5$). The position of the H-burning shell is indistinguishable from the position of the base of the convective envelope in this panel. The "On", "Power-down", "Dredge-Up" and "Off" phases of the thermal pulse cycle are marked on the top panel.
Figure 7: Time dependence of various quantities during the TP-AGB phase of stars with $(Y, Z) = (0.25, 0.008)$ and masses 1 (top left), 1.5 (top right), 2 (bottom left), 2.5 $M_\odot$ (bottom right). The abscissa represents the time after the first major thermal pulse. The dotted vertical line at the right represents the end of the AGB phase. $M_\odot$ is the mass loss rate in units of $10^{-6} M_\odot$ yr$^{-1}$. From Vassiliadis & Wood (1993).
4.4 Evolution on the TP-AGB

During the TP-AGB phase the star will experience a series of thermal pulses of the type described above. The total number of thermal pulses depends on the duration of the entire AGB phase, which as we have seen above depends on the mass loss from the surface. Numerical simulations show typical numbers of 10 (for a $1\, M_\odot$ star) to 40 ($5\, M_\odot$). See Figs. 7 and 8 showing the evolution of a number of stellar parameters during the TP-AGB for six different initial masses.

Just as in the case of the He-core flash, the luminosity produced in the He-shell flash does not reach the surface of the star, at least not initially. But as the envelope mass is reduced by mass loss, for later pulses a short luminosity increase does become visible (see Fig. 12). However, the drop in luminosity during the “Power down” phase always becomes visible at the stellar surface.

As can be seen in the Figs. 7 and 8 other stellar parameters such as the effective temperature and pulsation period also vary during the thermal pulse cycle.

4.5 Core mass-Luminosity Relation

During the TP-AGB the luminosity in the “off” phase keeps going up. This can be expressed as the core mass-luminosity relation

$$L_{\text{AGB}} = 5.9 \times 10^4 (M_c - 0.52) \, L_\odot$$

(Paczynski 1970). This relation is due to the fact that the envelope essentially has zero pressure compared to the electron-degenerate C/O core, making its presence irrelevant for the energy producing core region. Various more or less equivalent CMLRs have been found in stellar evolution models (see Fig. 9).
Figure 9: Core mass-Luminosity Relations from various authors, each plotted in the range of $M_c$ considered to obtain the analytical fit to the model results. When a composition dependence is present, the solar case is considered, i.e. ($Z = 0.02, Y = 0.28$). The relation by Iben & Truran (1978) refers to a 7 $M_\odot$ model.
Figure 10: Luminosity evolution of a $5.0 M_\odot$, $Z = 0.008$ star experiencing HBB. The open circles correspond to the maximum quiescent luminosity before each thermal pulse. The numbers along the curve indicate the current stellar mass in solar units. The dashed and dotted lines represent the reference CMLR for $Z = 0.008$ and $Z = 0.016$ respectively. From Marigo (1998).

$L-M_c$ relations exist for all giant phases, but for the RGB phase are of the type $L \propto M_c^8$. For AGB stars this becomes the linear relation $L \propto M_c$ due to the domination of radiation pressure (see Ciardullo, lectures 21 and 24).

Because of the $L-M_c$ relation one can find limits for the luminosity during the AGB:

\[
\begin{align*}
\text{AGB: } L_{\text{min}} &\approx 2800 L_\odot \\
L_{\text{max}} &\approx 51000 L_\odot \\
\text{RGB: } L_{\text{max}} &\approx 2900 L_\odot
\end{align*}
\]

The lowest luminosity AGB stars are thus almost indistinguishable from the highest luminosity RGB stars. The position of the tip of the RGB is sometimes used to estimate the distance to nearby galaxies. Contamination by AGB stars is thus a worry. It is harder to use the tip of the AGB for this since this phase lasts so much shorter, and thus is less populated in a Herzsprung-Russell diagram.

The $L-M_c$ relation on the AGB breaks down for the more massive stars, as in those the base of the envelope gets involved in the energy production. This is known as Hot Bottom Burning (HBB). Figure 10 shows how the luminosity evolution of a $5 M_\odot$ deviates from the CMLR, until HBB stops due to the reduced mass of the stellar envelope and the star joins the CMLR.
4.6 Hot Bottom Burning

This is the process of H-burning at the bottom of the convective envelope. This only occurs above a certain stellar mass (∼ 5 M⊙ for Z = Z⊙, lower at lower metallicity). The process stops when the envelope has lost too much mass.

HBB burning breaks the $L-M_c$ relation, and so massive AGB stars do not follow this relation, as can be seen in Fig. 10.

HBB also leads to interesting nucleosynthesis products since convective motions carry away intermediate products of H-burning (CNO and pp-chain). In steady regions of nuclear processing these products are processed further, and never contribute to chemical enrichment, but as HBB happens at the base of the convective envelope these products may be mixed up into the envelope and change the surface abundances.

Take for example $^7$Li:

$$^{3}\text{He} + ^4\text{He} \rightarrow ^7\text{Be} + \gamma$$

$$^7\text{Be} + e^- \rightarrow ^7\text{Li} + \nu$$

Both $^7$Be and $^7$Li can react with $^1$H, but outside the area of nucleosynthesis this will not happen. So, if one produces $^7$Be in HBB and mix it up, it has a large chance of capturing an electron and producing $^7$Li. The stellar sources and sinks of $^7$Li are important to understand, as this is one of the elements that was formed in small quantities during the Big Bang. Measurements of the $^7$Li can thus constrain cosmological models, but only if one can estimate what stellar processes produce and destroy it.

4.7 Dredge Up

During thermal pulses, material from deep down the shells can be dredged up into the stellar convective envelope, and thus cause abundance changes even at the stellar surface. The dredge up during thermal pulses is also known as the 3rd dredge up (3DUP), as similar events happen on the RGB and during the E-AGB (the latter only for massive enough stars).

The efficiency of dredge-up is usually expressed by a parameter

$$\lambda = \frac{\Delta M_{\text{dredge-up}}}{\Delta M_c}$$

(27)

where

$\Delta M_{\text{dredge-up}}$ Mass dredged up into the envelope.

$\Delta M_c$ Growth of core mass between two thermal pulses.

For low mass stars $\lambda \lesssim 0.3$, and for stars initially more massive than ∼ 5 M⊙, $\lambda \lesssim 1$. However, these numbers are very uncertain.

4.7.1 Carbon Stars

The most dramatic effect of the 3rd dredge up is the transformation of AGB stars from M-type stars into C-type stars (Carbon stars). The cosmic and solar abundance ratio
Figure 11: Plots of (from top to bottom) the core masses, He-luminosity and total luminosity as a function of time for a $1 \, M_\odot$. The core masses shown are the mass of He-exhausted core, the H-exhausted core, and the base of the convective envelope.
Figure 12: The surface luminosity evolution of thermally pulsing AGB stars of mass 1, 2.5 and 5.0 $M_\odot$, and initial composition $Y = 0.25$ and $Z = 0.016$. The plots start on the early AGB when the H- and He-burning luminosities are equal and end when the stars have left the AGB and moved to the post-AGB part of the HR-diagram. From Vassiliadis & Wood (1993).
Figure 13: Schematic showing internal evolution between two pulses. Partial mixing during the third dredge-up phase produces a $^{13}$C pocket, which provides the neutrons for the s-processing. Note that this figure is not to scale: the protons are mixed downward a depth of only a few $\times 10^{-4} M_\odot$, whereas the intershell convective zone is up to a few $\times 10^{-2} M_\odot$ in extent.

Figure 14: Evolution of CNO surface abundances (by number, mole gr$^{-1}$) and ratios from the first thermal pulse until the complete ejection of the stellar envelope for a $Z = 0.008$ TP-AGB model with initial mass 2.5 $M_\odot$ (left) and 5 $M_\odot$ (right). The efficiency factor for the third dredge-up is assumed to be $\lambda = 0.5$; the mixing length parameter is $\alpha = 0.24$. Based on synthetic calculation by Marigo (1998).
between carbon and oxygen is $\text{C/O} \sim 0.4-0.5$. Carbon stars have $\text{C/O} > 1$. This is mostly $^{12}\text{C}$ from the He-burning shell that was dredged up during thermal pulses. The spectrum of Carbon stars differs dramatically from that of M-type stars, mostly because of a number of carbon containing molecules, such as CO and CN.

Careful modelling of this process suggests a minimum mass to form Carbon stars of about $1.5 \, M_\odot$. Below this, the stars experience too few thermal pulses and is the dredge up process too weak to turn the star into a Carbon star. There is also a maximum mass, since HBB can remove $^{12}\text{C}$ through the CNO-cycle, by turning it into $^{14}\text{N}$. This would thus mean that the maximum mass for Carbon stars is $\sim 5 \, M_\odot$. Note however that the HBB process switches off at the end of the AGB, so for initially more massive stars there is a chance of becoming a Carbon star quite late in the process. See Fig. 14 for two examples of how the surface CNO abundances change during the AGB.

### 4.7.2 Other abundance changes

Dredge up not only changes the surface abundances, it also facilitates unusual nuclear reactions, as it mixes primordial, H-burnt and He-burnt material. Convection will carry the products from these reactions to the surface, and as AGB stars lose mass, this makes the main cosmic producers of some elements/isotopes.

- Dredge up increases $^{12}\text{C}$, $^{22}\text{Ne}$, $^{25}\text{Mg}$, $^{26}\text{Mg}$
- H-shell and HBB
  - burn $^{12}\text{C}$ and $^{18}\text{O}$ into $^{14}\text{N} \Rightarrow ^{14}\text{N} \uparrow$, $^{18}\text{O} \downarrow$
  - burn $^{22}\text{Ne}$ into $^{23}\text{Na} \Rightarrow ^{23}\text{Na} \uparrow$
  - produce $^{25}\text{Mg}$, $^{26}\text{Mg} \Rightarrow ^{25,26}\text{Mg} \uparrow$
- More massive stars ($M_{\text{ZAMS}} \gtrsim 6 \, M_\odot$?): $^{24}\text{Mg} \rightarrow ^{27}\text{Al} \Rightarrow ^{24}\text{Mg} \downarrow$, $^{27}\text{Al} \uparrow$

### 4.7.3 s-process

In addition, AGB stars produce so-called s-process elements. The s-process is the process of slow neutron capture. Neutron capture is the only way to make elements beyond the Fe peak, since the binding energy per nucleon peaks around Fe. For the s-process, the (negative) $\beta$ decay should occur more rapidly than the next neutron capture. This way the most stable isotopes at each atomic weight are formed. The s-process does not produce elements beyond Pb since

$$
^{209}\text{Bi} + n \rightarrow^{210}\text{Bi} + \gamma \\
^{210}\text{Bi} \rightarrow^{210}\text{Po} + e^- + \bar{\nu}_e \\
^{210}\text{Po} \rightarrow^{206}\text{Pb} + ^4\text{He} \\
^{206}\text{Pb} + 3\alpha \rightarrow^{209}\text{Pb} \\
^{209}\text{Pb} \rightarrow^{209}\text{Bi} + e^- + \bar{\nu}_e
$$

All the heavier elements require the r-process, which operates in supernova explosions.
A special spectral class has been defined for stars with s-process elements in their spectra, the S-type stars. They show for instance lines of ZrO. There are also intermediate types such as MS-stars, which in addition show TiO, and CS-stars which also show C\textsubscript{2} and CN lines. Evolutionarily it is inferred that the transition from M-type stars to C-type stars happens as $\text{M}\rightarrow \text{MS}\rightarrow \text{S}\rightarrow \text{CS}\rightarrow \text{C}$, as the s-process elements also are produced in thermal pulses and are being dredged up to the surface. A star with s-process elements in its spectrum is thus on its way to become a C-star (but may not necessarily get there). Some AGB stars show lines of Technitium (Tc), the lightest element without any stable isotopes. The most stable isotope of Tc, $^{99}\text{Tc}$, has a half life of only $2 \times 10^5$ years, thus giving direct evidence for the occurrence of the s-process and dredge up.

Although there is good observational evidence for the occurrence of the s-process, how it proceeds inside AGB is stars is not well understood. The problem is that one needs an appropriate source of neutrons. This source is thought to be one (or both) of the following nuclear reactions:

\[
\begin{align*}
^{13}\text{C} + ^4\text{He} &\rightarrow ^{16}\text{O} + n \\
^{22}\text{Ne} + ^4\text{He} &\rightarrow ^{25}\text{Mg} + n
\end{align*}
\]

The second one of these can only operate in higher mass stars since it requires temperature in excess of $3 \times 10^8$ K.

Formation of $^{13}\text{C}$ is however, a sensitive process proceeding like

\[
^{12}\text{C} + p \rightarrow ^{13}\text{N} + \gamma \rightarrow ^{13}\text{C} + e^+ + \nu_e
\]

but if many protons are present it will quickly react with them according to

\[
^{13}\text{C} + p \rightarrow ^{14}\text{N} + \gamma
\]

So, to produce enough $^{13}\text{C}$ a limited supply of protons, mixed down from the envelope into the $^{12}\text{C}$ region left behind by the thermal pulse, is necessary. This then so-called $^{13}\text{C}$ pocket would produce the s-process elements, which will be dredged up during the next thermal pulse.
5 Pulsations

Most AGB stars are variable stars, due to the fact that they pulsate. The periods are long, from 10 to 1000s of days, and they are therefore called Long Period Variables, or LPVs. These long periods make it time-consuming to collect accurate light curves. In recent years a lot of progress has been made in this area due to several large area surveys programs (aimed at the more fashionable subject of micro-lensing) such as MACHO, OGLE, EROS and MOA.

Note however that the first ever variable star discovered in modern astronomy was an AGB star, o Ceti (Fabricius, 1596), from then on known as Mira. When later stars with similar variability characteristics were discovered these were called Mira variables. The LPVs are observationally divided into several classes (see Fig. 15)

**Mira variables** They have very regular light curves, with amplitude $\Delta V > 2.5^m$, and periods $\Pi > 100$ days.

**Semi-regular variables (SRVs)** These have fairly regular light curves with $\Delta V < 2.5^m$, and $\Pi > 20$ days.

**Irregular variables (Irr)** These have irregular light curves, with small $\Delta V$ and no clear period.

Note that these definitions are based on the optical light curves. The bolometric variations in luminosity are often much less dramatic, see Fig. 16. It is also true that many Irr’s just suffer from a poor sampling of their light curves, and on closer inspection do show signs of specific period(s).

Some AGB stars are so heavily obscured by their circumstellar material that they have no observable optical emission. The so-called OH-IR stars are only observable in the infrared (and from their OH maser emission), and are variable with very long periods, more than 500 days.

5.1 Period-Luminosity Relations

As expected from the theory of pulsations, relations exist between period and luminosity. A major breakthrough was made after the MACHO results allowed the determination of $\Pi$ and $L$ for a large sample of AGB stars in the Large Magellanic Cloud (LMC), see Fig. 17. A series of relations can be discerned, denoted by A, B, C, D, E in the figure. The ratios between the A, B, and C periods suggest that

- **C** fundamental mode radial pulsations (Miras)
- **B** first overtone mode radial pulsations (SRVs)
- **A** second overtone mode radial pulsations (SRVs)

Relation D is a mystery, either it represents another type of pulsation (non-radial), or it is connected to binarity. Relation E is definitely associated with binaries (Derekas et al. 2006).
Figure 15: Examples of light curves of Mira variables (top four panels) and semiregular variables (bottom six panels) in the LMC from the Macho database. The bandpass of the lightcurves is MACHO blue, which is centered near 0.53 µm, similar to the visual bandpass. All red giants in the MACHO database seem to show distinct periodicities at some times, but without high quality light curves such as these, they would possibly be classified as irregular.
Figure 16: Lightcurve for the Mira variable RR Sco in the bandpasses UBVRIJHKL. The UBVRI light curve come from Eggen (1975), while the JKHL light curves are from Catchpole et al. (1979).
Figure 17: The period-luminosity relations for a sample of MACHO observations. Several sequences can be discerned, showing that AGB stars pulsate in different modes. The stars indicated with plus signs are eclipsing binaries. From Derekas et al. (2006)
Before the MACHO results, there was very little data to work with, and one tried to derive stellar radii using interferometry, so as to establish the mode of the pulsations. Those measurements showed Miras to be first overtone pulsators. Since the MACHO results show them to be fundamental mode pulsators, the conclusion is that there is something wrong with the interferometric measurements of stellar radii. Recent results show that some AGB stars may be aspherical (Ragland et al. 2006).

The story does however not end here. More recent data from the OGLE survey has revealed a large group of variable red giant stars that have been called OSARGs (OGLE Small Amplitude Red Giants). Some of these are on the RGB (showing that also those stars are variable), some on the AGB. These stars occupy most of the A and part of the B sequence from the MACHO data (Soszyński et al. 2007, see Fig. 18). The proper Mira and SRVs lie on two sequences indicated by C and C’ in Fig. 18, corresponding to fundamental mode and first overtone pulsators. Differences between C- and M-stars can also be seen. Clearly the interpretation of all this variability data is still variable itself.

5.2 Theory of Pulsations

There is a large body of work on understanding stellar pulsation, the foundations of which were laid by Eddington. Much of this work concentrates on hotter stars with radiative stellar envelopes, which simplifies the theory compared to the convective envelopes of the cool AGB stars. The hotter stars are for example the Cepheid variables (important for the cosmological distance scale) and RR Lyrae stars.
The theory of pulsation is basically a stability analysis for stars, i.e. studying how a star reacts to perturbations. From such an analysis one obtains

- frequencies of modes
- whether or not the star is stable against these modes.

Stability analysis usually starts with linear perturbations: small variations around an equilibrium solution, neglecting the higher order (non-linear, quadratic and higher) terms in the equations. For AGB stars matters are complicated by the fact that one tries to calculate the stability of a convective stellar envelope, but no fully self-consistent theory for convection is available.

### 5.2.1 Estimate of luminosity-period relations

The fundamental mode is a radial pulsation with its wavelength equal to the stellar diameter

$$\Pi = 2R_*/c_s$$  \hspace{1cm} (28)

where $c_s$ is the (adiabatic) sound speed, $c_s^2 = \gamma_{ad} P/\rho$.

By assuming that the variations are around an equilibrium solution, one can use the Virial Theorem

$$-\Omega_{grav} = 2K_{\text{thermal}} = 3 \int_0^{R_*} 4\pi r^2 \rho \, dr.$$  \hspace{1cm} (29)

Since $\frac{dm}{dr} = 4\pi r^2 \rho$, this can be rewritten as

$$2K_{\text{thermal}} = 3 \int_0^{R_*} \frac{c_s^2}{\gamma_{ad}} \, dm \approx 3 \frac{c_s^2}{\gamma_{ad}} M_*$$  \hspace{1cm} (30)

The gravitational energy of the star is

$$-\Omega_{grav} = \alpha \frac{G M_*^2}{R_*}$$  \hspace{1cm} (31)

where $\alpha$ is a dimensionless number which depends on the internal density structure of the star. Combining the expression for the gravitational and thermal energies through the Virial Theorem gives

$$\Pi = 2 \sqrt{\frac{3}{\gamma_{ad} GM_*^2}} R_* \approx Q R_*^3 M_*^{-\frac{1}{2}} \approx (\langle \rho \rangle)^{-\frac{1}{2}}$$  \hspace{1cm} (32)

which means that the pulsation period for the fundamental mode will depend on the average density of the star. Using typical parameters for a Mira, $M = 1 M_\odot$, $R = 200 R_\odot$, $\alpha = 2$, one obtains $\Pi \approx 0.3$ years.

The above analysis also leads immediately to a period-luminosity relation

$$L = 4\pi R^2 \sigma T_{\text{eff}}^4, \quad \Rightarrow \quad L \propto \left( \frac{\Pi}{Q} T_{\text{eff}}^3 M^{1/2} \right)^{4/3}$$  \hspace{1cm} (33)
A more elaborate perturbation calculation, assuming adiabatic conditions, but using

\[ p(m, t) = p_0(m) [1 + \delta p(m) \exp i\omega t] \]  

\[ \rho(m, t) = \rho_0(m) [1 + \delta \rho(m) \exp i\omega t] \]  

\[ r(m, t) = r_0(m) [1 + \delta r(m) \exp i\omega t] \]

leads to a series of frequencies \( \omega_0, \omega_1, \omega_2 \), etc., which are the fundamental mode, the first overtone, second overtone, etc. Expressed as a period, the relation between the \( \Pi \) values of these modes is

\[ \Pi_n \propto \sqrt{n + 1} \langle \rho \rangle^{-\frac{1}{2}} \]  

5.2.2 The \( \kappa \) mechanism

For a star to be actually pulsating in one or more of these modes, energy needs to be fed in at the right time during a pulse cycle. As first realized by Eddington, the key is to block energy transport during the compression phase. This raises the pressure after compression, leading to expansion and restoration of the state at the start of the cycle. If energy transport is radiative, it is inversely proportional to the opacity \( \kappa \), so in order to achieve the effect described above, we need to increase \( \kappa \) upon compression. A reasonable approximation for the opacity inside a star is Kramer’s opacity

\[ \kappa \propto \rho T^{-3.5} \]  

For this opacity law, the density increase in a compression goes in the right direction of increasing the opacity, but this effect will be more than offset by the decrease of the opacity caused by the temperature increase during compression. So, standard stellar opacity will not trigger pulsations.

However, in zones of partial ionization (of either He or H), the heat added by compression will be mostly used to increase the ionization fraction, and will hardly raise the temperature. Then as the density goes up, and the temperature remains constant, the opacity will go up. This is known as the \( \kappa \) mechanism for stellar pulsations, and it explains the pulsational instabilities in Cepheids and RR Lyrae stars. In fact, the requirement of having these zones while still keeping a radiative stellar envelope, defines a region in the HR-diagram known as the instability strip.

5.2.3 Pulsation models for AGB stars

For AGB stars energy transport is dominantly convective, and thus independent of the opacity, so in the simplest interpretation the \( \kappa \) mechanism should not work. The observations show this conclusion to be wrong. However, to explain the pulsational instability in AGB stars one needs a dynamic theory of convection, for which currently only approximations exist (‘mixing length theory’). Models based on this approximation of convective energy transport are the only ones that are available, and should hopefully give at least a qualitative understanding of the pulsational behaviour of AGB stars.

Figure 19 shows some results from such a model. The top panel shows that most energy is transported convectively, and also indicates the the H and He ionization zones, which are still important for the model. The bottom panel shows how the radial excursion is
roughly proportional for the fundamental mode, while the first overtone is concentrated towards the photosphere. The third panel shows the so-called partial work integral \( W_r \) which is the sum of all the work done interior of \( r \) during one pulsation cycle. The regions where \( W_r \) increases with radius contribute to exciting the pulsation, the regions of negative \( W_r \) gradient are damping pulsations. Clearly the H and He ionization zones contribute to exciting the pulsation. The second panel shows the quantity \( J \Sigma_2^r \), the absolute gradient of which shows which regions contribute most to the period. For the fundamental mode it are the top layers of the H ionization zone which contributes most to determining the pulsation period. For the first overtone no regions clearly dominates.

5.3 Non-radial pulsations

Stars may also pulsate non-radially: angular patterns of pulsation in addition to the radial patterns. Not much is known about non-radial pulsations in AGB stars, but they are sometimes invoked to explain observed asymmetries (for example in the mass loss).
Figure 19: Various properties of a model Mira variable with $L = 5000 \ L_\odot$ and $M = 1 \ M_\odot$ and solar metallicity plotted against radius within the star. The fundamental mode period of the model is 333 d, similar to that of the prototypical Mira o Ceti. Top panel: log $T$ (dotted line) and the fraction of the energy flux carried by convection (solid line); second panel: the partial integral $\Sigma_{2}^{J}$ for the fundamental mode (solid line) and the first overtone (dotted line); third panel: the partial work integral $W_r$ for the fundamental mode (solid line) and the first overtone (dotted line); bottom panel: the real part $\delta R$ of the eigenfunction for the fundamental mode (solid line) and the first overtone (dotted line), where the eigenfunction is defined to have complex amplitude (1.0,0.0) at the stellar surface. Based on Fox & Wood (1982).
Mass Loss

As was pointed out earlier, mass loss dominates the stellar evolution on the AGB.

Nuclear fusion $\sim 10^{-8} \, M_\odot \, \text{yr}^{-1}$

Mass loss $> 10^{-7} \, M_\odot \, \text{yr}^{-1}$

Historically it took a long time to appreciate the full magnitude of mass loss on the AGB since the circumstellar material (CSM) only emits appreciably at IR- and short radio-wavelengths, which only became observable in the 1970s. After the data from space infrared observatories such as IRAS (1984) became available, as well as submm observations of CO lines, mass loss from AGB stars could be studied systematically.

6.1 Initial-Final Mass Relation

From the point of view of stellar evolution, it is the total, integrated mass loss $\int \dot{M} \, dt$ that is most important. This can be observationally constrained using the Initial-Final Mass Relation, derived from White Dwarf masses in open clusters of known age and distance, see Fig. 20. Due to the paucity of data, this IFMR is not extremely well established and for example its metallicity dependence is disputed. However, it seems clear that stars with mass $1—6 \, M_\odot$ end up as WDs with masses $\sim 0.5—1 \, M_\odot$.

6.2 Mass loss measurements

The estimated mass loss rates range from $10^{-8}$ to $10^{-4} \, M_\odot \, \text{yr}^{-1}$ (see Fig. 22). The highest values are often referred to as the super wind. Typical velocities for the mass loss are in the range 5—30 km s$^{-1}$ (see Fig. 21), so these are relatively slow winds. However, since the temperatures in the winds are also low, they are still supersonic ($c_s(100 \, \text{K}) \sim 1 \, \text{km s}^{-1}$).

There exists a clear relation between mass loss rate and the pulsational period. However, this relation is not a simple one, probably because other factors (e.g., metallicity) come in. The relation is shown in Fig. 23 with data for a sample of AGB stars (separated into M- and C-stars).

Measuring mass loss rates accurately is notoriously difficult. For a steady, spherically symmetric mass loss we have

$$\dot{M} = 4\pi r^2 \rho v,$$

so if the velocity is constant, $\rho \propto r^{-2}$. However, both the mass loss rate and the velocity may vary in time.

To measure $\rho(r)$ we need a ‘probe’ (atom, molecule, dust grains) emitting radiation. This probe may have a position dependent abundance $A(r)$, depending on the chemistry in the gas. Furthermore, the emissivity of the probe may also vary with radius, depending on the local temperature and radiative transfer effects. So, typically we measure only a small part of the circumstellar medium, and the above effects (abundance, temperature, radiative transfer) are not always easily quantified. Examples of probes for measuring mass loss are: CO thermal emission, dust thermal emission, OH maser emission. We will come back to these later.
Figure 20: The initial-final mass relation. Filled circles are binned points from open clusters with four or more WDs; crosses are from clusters or binary systems with three or fewer WDs. The solid line is a least-squares linear fit to these points. The dashed line is the linear fit from Ferrario et al. (2005); the dotted line is the inversion of the field WD mass distribution presented in that work. Open squares, which were not included in the fits, are from Dobbie et al. (2006) for GD 50 and PG 0136+251. The agreement between these points and the extrapolation of the linear fit is encouraging. From Williams (2006).
Figure 21: Gas expansion velocity distributions for selected samples of M- and C-stars. From Olofsson (2004).
A sceptical view would be that observationally derived mass loss rates are uncertain by \( \sim 1 \) order of magnitude.

### 6.3 Mass loss recipes

To include mass loss in evolutionary calculations requires relating it to the basic stellar parameters. However, as we will see, we lack a detailed understanding of the physical processes behind the mass loss from AGB stars, so mass loss cannot be self-consistently included in evolutionary calculations. However, a number of physical-empirical relations have been suggested and are used in actual calculations. The first
Figure 23: Mass loss rate versus period for three samples of M-stars [SRVs (diamonds), Miras (circles), and Galactic Center OH/IR stars (squares)], and a sample of optically bright C-stars. From Olofsson (2004).

Of these relations was proposed by Reimers (1975)

$$\dot{M}_{\text{Reimers}} = 4 \times 10^{-13} \eta \frac{L}{L_\odot} \frac{R}{R_\odot} \frac{M}{M_\odot} \text{ yr}^{-1}$$  \hspace{1cm} (40)

where $\eta$ is a ‘fudge factor’. This relation is really only valid for RGB stars, but since it was simple to use it also became popular for use on the AGB (but with a higher value for $\eta$).

From trying to reproduce the empirical IFMR, Blöcker (1995) proposed a modified form for use during the AGB:

$$\dot{M}_{\text{Bl}} = 4.83 \times 10^{-9} \left( \frac{M}{M_\odot} \right)^{-2.1} \left( \frac{L}{L_\odot} \right)^{2.7} \dot{M}_{\text{Reimers}}$$  \hspace{1cm} (41)

Vassiliadis & Wood (1993) proposed another relation, which was derived from fits to observed mass loss rates and the relation with the pulsational period $\Pi$ (see Fig. 24), imposing a maximum mass loss due to a radiation pressure limit (single scattering, see Eq. 56).

$$\dot{M}_{\text{VW}} = \min(\dot{M}_{\text{radpres}}, \dot{M}_{\text{superwind}})$$  \hspace{1cm} (42)

with

$$\dot{M}_{\text{radpres}} = \frac{L}{c \eta_{\text{exp}}}$$  \hspace{1cm} (43)

and

$$\eta_{\text{exp}} = \min(15, \max(3, -13.5 + 0.056\Pi(\text{days}))) \text{ km s}^{-1}$$  \hspace{1cm} (44)
Figure 24: Mass loss rate $\dot{M}$ ($M_\odot$ yr$^{-1}$) plotted against period for Galactic Mira variables of spectral type M and S (filled circles) and C (open circles) and for pulsating OH/IR stars in the Galaxy (triangles) and the LMC (squares). The solid line is the analytic fit used for low-mass stars ($M < 2.5 M_\odot$) with the mass loss rates less than the radiation-pressure driven limit. The dashed line is the equivalent relation for a 5 $M_\odot$ star, while the dotted line corresponds to mass loss at the radiation-pressure-driven limit for a typical intermediate mass (5 $M_\odot$) LPV in the LMC with $M_{bol} = -6.5$ and $v_{exp} = 12$ km s$^{-1}$.

and

$$\log_{10} \dot{M}_{superwind} = -11.4 + 0.012321 \Pi \text{ (days)} \quad M_\odot \text{ yr}^{-1}. \quad (45)$$

All these are popular recipes, but they lack any profound base in the fundamental physics of the mass loss process.

### 6.4 Theory of pulsation/dust-driven mass loss

AGB winds are slow and have high mass loss rates, a combination that has been difficult to achieve with stellar wind models. The observations suggest a connection with the pulsational period (Figs. 23 and 24). The only successful models are based on pulsating stellar atmospheres and radiation pressure on dust, see Fig. 28 for a cartoon impression. We will now describe this in some more detail. For a more complete treatment of stellar winds, and radiation pressure driven winds, see the book by Lamers & Cassinelli.

#### 6.4.1 Stellar Wind Equation

Stellar winds are radial flows in the star’s gravitational field, which means that they need to fulfill certain strict criteria. The momentum equation for the gas can be written as

$$\frac{dv}{dr} = -\frac{1}{\rho} \frac{dp}{dr} - \frac{GM}{r^2} + f(r) \quad (46)$$

( steady flow), where $v$ is the velocity, $p$ is the pressure, $\rho$ the mass density, and $f(r)$ some external force (responsible for driving the mass loss). Combining Eq. 46 with
Figure 25: Mathematical solutions for the stellar wind equation. The only solution that represents a stellar wind is the one accelerating through the critical point.

39 and using the isothermal sound speed $\bar{c}_s^2 = p/\rho$ (different from the adiabatic sound speed $c_s^2 = \gamma p/\rho$), we get

$$\frac{dv}{dr} = \frac{v}{v^2 - \bar{c}_s^2} \left( \frac{2\bar{c}_s^2}{r} - \frac{d\bar{c}_s^2}{dr} \frac{GM}{r^2} + f(r) \right)$$

(47)

For $v = \bar{c}_s$ this equation has a singularity. In Fig. 25 this point is indicated by (1,1). The lines in that figure represent solutions to Eq. 47. The solution families II and III are multivalued, and therefore unphysical. Stellar wind solutions need to start at low velocity, and keep increasing their velocity until they reach some equilibrium velocity ($v_\infty$). Figure 25 shows that there is only one solution that does this, and this is the one that goes through the point (1,1), or $v = \bar{c}_s$, the so-called critical point. This also implies that a successful stellar wind will always become supersonic.

Without an external force $f(r)$ the mass loss is typically weak. This is for example the case for the Sun ($\dot{M}_\odot \sim 10^{-14} M_\odot$ yr$^{-1}$). However, just applying any radial force will not necessarily increase the mass loss rate. The radial dependence of the external force is crucial for its effect on the wind.

First note that for a constant $\bar{c}_s$ (i.e. isothermal conditions), the position of the critical point is

$$r_c = \frac{GM}{2\bar{c}_s^2} - \frac{f(r_c)r_c^2}{2\bar{c}_s^2}$$

(48)
so if \( f(r_c) > 0 \), \( r_c \) moves inwards. Since the density is typically higher at smaller radii, the effect is that \( \dot{M} \) increases. However, if \( f(r_c) = 0 \), and \( f(r_c) > 0 \) for \( r > r_c \), the mass loss will not increase. Instead the wind will experience an extra acceleration beyond \( r_c \) and achieve a higher velocity. The conclusion is thus that high mass loss rates can only be achieved if the extra force operates effectively at or below \( r_c \).

When the external force is due to continuum radiation it typically has a \( r^{-2} \) dependence, just like gravity. It is therefore useful to write

\[
-\frac{GM}{r^2} + f(r) \equiv -(1 - \Gamma)\frac{GM}{r^2}
\]

and with this definition Eq.48 becomes

\[
r_c(\Gamma) = \frac{GM(1 - \Gamma)}{2c_s^2}
\]

Analysing the wind equation (Eq. 47) we can distinguish three regions

1. \( r < r_c \): the \( \partial p/\partial r \) term dominates and the solution is ‘atmosphere-like’.
2. \( r \approx r_c \): apply the external force in this region to boost the mass loss rate
3. \( r > r_c \): the \( \partial p/\partial r \) term becomes unimportant and the solution is ‘wind-like’; apply external force here to boost the wind velocity.

A hydrostatic, stationary atmosphere has an exponential radial density law

\[
\rho(r) = \rho_0 \exp\left\{ -\frac{r - r_0}{H} \right\} = \frac{c_s^2}{GM}r_0^2
\]

where \( H \) is the so-called scale height and for \( r < r_c \) the solution to Eq. 47 is indeed very close to this solution.

### 6.4.2 Scale height problem

For \( M = 1M_\odot, r_0 = R_\ast = 300R_\odot, T = 2500 \text{ K} \), we get that \( H/R_\ast = 0.05 \), so this means that the density drops rapidly as we move away from the stellar surface. The effect is that even when the force \( f \) pushes the critical point \( R_c \) inward, the mass loss is found to be too low. For example, the parameters \( M = 1M_\odot, r_0 = R_\ast = 100R_\odot, T = 2500 \text{ K, } \Gamma = 0.5 \) give \( \dot{M} \sim 10^{-16} M_\odot \text{ yr}^{-1} \).

However, for a pulsating atmosphere, the effective scale height will be substantially larger. The reason is that the pulsations create sound waves which travel outward, effectively ‘lifting up’ the atmosphere. In fact the sound waves will often steepen into shock waves, causing substantial compression, thus increasing the density even more. A simple energetic argument can be used to estimate the magnitude of this effect.
If a shell of mass $M_s$ is accelerated by a shock of velocity $v_0$, it will acquire a kinetic energy $\frac{1}{2} M_s v_0^2$. This will allow it to travel to a radius $r_{\text{max}}$ given by

$$\frac{1}{2} M_s v_0^2 = -GM_s \left( \frac{1}{r_{\text{max}}} - \frac{1}{r_0} \right) \Rightarrow$$

$$\frac{r_{\text{max}}}{r_0} = \left( 1 - \left( \frac{v_0}{v_{\text{esc}}} \right)^2 \right)^{-1} \quad (54)$$

For $M = 1 M_\odot$, $r_0 = R_\odot = 300 R_\odot$, we get $v_{\text{esc}} = 35 \, \text{km s}^{-1}$. If we take $v_0 = 15 \, \text{km s}^{-1}$ (from observations), we get that $r_{\text{max}}/r_0 \approx 1.3$, so the shock waves will lift the atmosphere by a substantial factor. Detailed numerical hydrodynamic models give roughly the same number (see Fig. 26).

In fact, the increased time-averaged scale height of such a pulsating atmosphere (see Fig. 27) can become so high that this process alone can set up a mass outflow. However, also this wind has too low a mass loss rate to explain the winds from AGB stars.

### 6.4.3 Radiation pressure

The force thought to be responsible for driving the high mass loss rate winds from AGB stars is radiation pressure on dust. For the the process the $\Gamma$ factor can be written as

$$\Gamma_d = \frac{\kappa_{\text{TP}} L_\ast}{4\pi c GM} \quad (55)$$
Figure 27: The density versus $r$ as influenced by shocks in a Mira variable. The density falls roughly as $r^{-5}$ just above the hydrostatic region and falls roughly as $r^{-3}$ in the outermost region. The curves are labeled with the velocity amplitude (in km s$^{-1}$) of the pulsations. Notice that the scale of the density for the region beyond the static base is set by the amplitude of the pulsation, $\Delta \nu$. From Bowen (1988).

where $\kappa_{\text{rp}}$ is the mean opacity for radiation pressure. This $\kappa_{\text{rp}}$ is much higher for dust particles than for gas particles, making them a particularly efficient agent for accelerating the gas.

Let us look at what mass loss we can get from radiation pressure. Each photon carries a momentum $h\nu/c$. If it transfers this to one dust particle one can obtain an estimate for the mass loss rate. The total momentum rate in the photons is $L_* / c$, and the total momentum rate in the wind is $\dot{M}v_\infty$, implying that

$$\dot{M} = L_*/(cv_\infty),$$

an estimate that is known as the single scattering limit. For $L_* = 10^4 L_\odot$, $v_\infty = 10$ km s$^{-1}$, we get from this limit $\dot{M} = 2 \times 10^{-5} M_\odot$ yr$^{-1}$, a very appreciable mass loss rate for an AGB star. We saw above that the mass loss recipe of Vassiliadis & Wood (1993) was using this limit.

However, the single scattering limit is a very conservative estimate. Note that if one considers the energy budget instead of the momentum one, the energy rate in the wind is $L_{\text{wind}} = \frac{1}{2} M v_\infty^2$, which is much less than the available energy rate in the photons $L_*$, since in the single scattering limit $L_{\text{wind}} / L_* = \frac{1}{2} v_\infty / c \ll 1$. This suggests that a higher mass loss should be possible. This can be achieved through multiple scatterings.

If we take the wind momentum equation Eq. 46 and integrate it over $dm = 4\pi r^2 dr$
from the stellar photosphere to infinity, we get
\[
\int_{R_\ast}^{\infty} 4\pi r^2 \rho v^2 \frac{dv}{dr} dr + \int_{R_\ast}^{r_c} \left[ \frac{1}{\rho} \frac{dp}{dr} + \frac{GM}{r^2} \right] dm \\
+ \int_{r_c}^{\infty} \frac{1}{\rho} \frac{dp}{dr} dm + \int_{r_c}^{\infty} \frac{GM}{r^2} (1 - \Gamma_d) \rho 4\pi r^2 dr = 0 \tag{57}
\]

The first integral is actually the momentum rate in the wind \(\dot{M}v_\infty\), since the velocity at \(R_\ast\) will be much less than \(v_\infty\). The second integral considers the pressure and gravity force below the critical point. Since the flow is close to hydrostatic equilibrium in that region, this term can be taken zero. The third integral does not contribute much as the gas pressure gradient is no longer important beyond the critical point (in the supersonic part of the wind). We thus get from Eq. 57
\[
\dot{M}v_\infty = 4\pi GM_\ast (\Gamma_d - 1) \int_{r_c}^{\infty} \rho dr \tag{58}
\]
but since the optical depth in the wind is defined as
\[
\tau_w = \int_{r_c}^{\infty} \kappa_{vp} \rho dr \tag{59}
\]
we find that
\[
\dot{M}v_\infty = \frac{L_\ast}{c} \left( \frac{\Gamma_d - 1}{\Gamma_d} \right) \tau_w \approx \frac{L_\ast}{c} \tau_w \quad \text{for} \quad \Gamma_d \gg 1 \tag{60}
\]
where we have used Eq. 55. So for a high optical depth, \(\dot{M}\) can be substantially higher than the single scattering value. However, the value for \(\tau_w\) cannot be arbitrarily high. We already saw that we get an absolute maximum due to the available energy. This gives
\[
\tau_w < \frac{2c}{v_\infty} \tag{61}
\]
Taking into account the fact that stellar luminosity is reduced due to its use as wind accelerator, one in fact find a more strict
\[
\tau_w < \frac{c}{v_\infty} = 27 \sqrt{\frac{\dot{M}/10^{-5} M_\odot yr^{-1}}{L/10^5 L_\odot}} \tag{62}
\]

### 6.4.4 Velocity distribution

To find the velocity at infinity we can use the momentum equation beyond \(r_c\) where the pressure terms can be neglected
\[
\frac{v}{dr} = \frac{1}{2} \frac{dv}{dr} = \frac{GM}{r^2} (\Gamma_d - 1) \tag{63}
\]
Integrating this from \( r_c \) until \( \infty \) gives
\[
\begin{align*}
\nu^2(r) &= \nu^2(r_c) + \nu^2_\infty \left(1 - \frac{r_c}{r}\right) \\
\nu^2_\infty &= \frac{2GM_\star}{r_c} (\Gamma_d - 1) = \frac{v_{\text{esc}} R_\star}{r_c} (\Gamma_d - 1)
\end{align*}
\] (64) (65)

The last expression shows that \( \nu_\infty \) is typically of order the escape velocity from the stellar surface (since \( \Gamma > 1 \) and \( r_c > R_\star \)). If we substitute \( M = 1M_\odot, L_\star = 10^4 L_\odot \), \( r_c = 2 \times 10^{14} \) cm, we get \( \nu_\infty = 16 \) km s\(^{-1}\) and at \( r = 5r_c \) already 90\% of this value has been reached.

### 6.5 The role of dust

For a complete picture of the mass loss process we need two more ingredients

- dust formation
- gas-dust coupling

#### 6.5.1 Dust formation

Dust formation is a complicated issue, of which we have only limited understanding. Observationally it is clear that AGB stars form two different types of dust grains

**Carbon stars** amorphous carbon grains

**M-type stars** ‘dirty’ silicates
Figure 29: As Fig. 28, but with detailed labelling of the various regions and the relevant physical processes, and showing the differences between O- and C-rich stars. From Hron.
Although the silicates are more common, we have a better understanding of carbon grain formation (partly due to research carried out for non-astrophysical reasons). Carbon as an element can easily form long chain-like molecules, and from those chains and sheets (so-called Polyaromatic Hydrocarbons, PAHs) may form. These may then aggregate into amorphous carbon grains.

Dynamical models including carbon grain formation (using an approximate approach called nucleation theory) are able to produce typical AGB winds, suggesting that we largely understand the mass loss physics in C-stars, at least for the simplified case of a steady, spherical wind.

The dust around M-type stars is Si-based. Si behaves chemically different from C, it does not form large chains and sheets. Therefore these dust particles cannot grow gradually, but rather have to condense out from the gas phase into the solid phase. This happens when the partial pressure is larger than the vapor pressure: $P_p > P_v$, where $P_p = n_{\text{molecule}} k_B T$ and $P_v = P_M \exp(-T_M/T)$. $P_M$ and $T_M$ are properties of the molecule under consideration.

The silicon oxides SiO and SiO$_2$ actually do not condense out at high temperatures, but other molecules like TiO, TiO$_2$, Al$_2$O$_3$ (corundum) do. It is therefore thought that these form the seed nuclei for Si-grain growth. The full grown silicate grains are made ‘dirty’ by both Mg and Fe silicates. The dirt is necessary for these grains to absorb radiation from the star, as pure silicates would be too glassy and transparent. Still, detailed modelling of winds driven by silicate grains show that it is difficult to condense enough dirty silicates around the critical point, which is where they are needed in order to produce a high mass loss wind. We have therefore currently no working model for explaining the mass loss from M-type AGB stars.

### 6.5.2 Dust-gas coupling

The radiation pressure works only on the dust particles, which only form a small fraction of the material. So in order to set up a wind, the motion of the dust has to be transferred to the gas particles. This happens via collisions. The collective effect of the gas on the dust is known as the ‘drag’, and can be expressed as a force, which can be approximated as

$$ f_{\text{drag}} = \sigma_d n_g m_d |v_{\text{coll}}| v_{\text{drag}} $$

with

$$ v_{\text{drag}} \equiv |v_g - v_d| $$

and

$$ v_{\text{coll}} = v_{\text{thermal}} \sqrt{\frac{64}{9\pi} + \left( \frac{v_{\text{drag}}}{v_{\text{thermal}}} \right)^2} $$

where $\sigma_d$ and $m_g$ are the dust cross section and the gas particle mass, $n_g$ and $n_d$ are the gas and dust number densities, $v_{\text{coll}}$ is the collision velocity, and $v_{\text{thermal}}$ the typical thermal velocity of the gas (as obtained from the Maxwell-Boltzmann distribution).

The drag force tries to minimize the drag velocity $v_{\text{drag}}$, but is also proportional to $v_{\text{drag}}$. This means that there will be an equilibrium value for $v_{\text{drag}}$. Usual approximations assume either $v_{\text{drag}} = 0$ (‘perfect coupling’) or $v_{\text{drag}} = v_{\text{eq}}$ (‘momentum coupling’).
Figure 30: Positions of selected mass shells as a function of time for a model of a carbon dust driven wind (Höfner & Dorfi 1997). During each new pulsation cycle a new dust layer is formed, triggered by enhanced density behind the shock waves. Below about $2 R_*$ the dustfree atmosphere is periodically passed by strong shocks (marked by sharp bends in the lines). The formation of dust layers and their subsequent acceleration due to radiation pressure (indicated by the steepening of the lines) takes place between 2 and 3 $R_*$. Time in pulsation periods, radius in units of the stellar radius $R_*$ of the corresponding hydrostatic initial model.
The extensive mass loss of AGB stars makes that they are surrounded by an extended circumstellar envelope (CSE), the maximum radius of which can be estimated as
\[ r_{\text{CSE}} = t_{\text{AGB}} v_\infty \simeq 10^6 \text{yr} \times 10 \text{km/s}^{-1} \sim 10^{19} \text{cm} \] (69)
This is a maximum value since the wind material will start to mix with the ISM at very large radii, and it also assumes that there is no velocity difference between the star and the interstellar gas. In addition the density drops with radius \( r \) in the expanding CSE (as \( r^{-2} \) for a constant mass loss rate \( \dot{M} \) and velocity \( v_\infty \), so the outermost parts are too low density to be observable. Using some typical numbers we can estimate the density as
\[ n(r) = \frac{\dot{M}}{4\pi \mu m_H v_\infty r^2} \simeq 10^6 \left[ \frac{\dot{M}}{10^{-5} M_\odot \text{yr}^{-1}} \right] \left[ \frac{15 \text{km/s}^{-1}}{v_\infty} \right] \left[ \frac{10^{15} \text{cm}}{r} \right]^2 \text{cm}^{-3} \] (70)
For comparison, the canonical value for the density of the ISM is 1 cm\(^{-1}\). As we saw above, dynamically all of the interesting action is quite close to the star, where the wind in launched and the mass loss rate is determined. However, the outer layers of the CSE show chemically interesting processes, and are of because of their size and lower optical depth, easier to observe.

### 7.1 Temperatures
The temperature of the CSE is a function of radius, and is set by the heating and cooling rates. Since the heating and cooling processes are different for the gas and the dust, their temperatures will generally be different.

Even in the absence of heating and cooling the gas temperature will drop as the CSE material is flowing away from the star. The reason is sometimes called ‘adiabatic cooling’, and is simply caused by the fact that the same energy density has to fill a larger volume as the CSE is expanding. Assuming that the density drops as \( r^{-2} \), as is the case for a stellar wind of constant mass loss rate and in which the velocity is close to \( v_\infty \), and taking the adiabatic relation between pressure and density, \( p \propto \rho^{\gamma} \), together with the ideal gas law \( p \propto \rho T \), immediately shows that for adiabatic expansion, \( T \propto r^{-2(\gamma^{-1})} \), or \( T \propto r^{-4/3} \) for \( \gamma = 5/3 \).

However, heating and cooling does occur. The main cooling process is radiative cooling from molecular lines, dominated by H\(_2\)O and CO around M-stars, and by CO and HCN around C-stars. The main heating process is collisions with dust grains. As we saw, there is generally a velocity difference \( v_{\text{drag}} \) between the gas and the dust, so upon a collision a certain amount of kinetic energy is transferred to the gas particle. Detailed calculations of these processes show that a typical radial temperature profile goes as
\[ T_{\text{gas}}(r) \approx 400 \left( \frac{10^{15} \text{cm}}{r} \right)^{0.9} \] (71)
The dust temperature is set by the absorption and emission properties of the dust grain. The heating process is the absorption of radiation from the star (at short wavelengths),
Figure 31: Schematic chemical structure in an AGB CSE. From Marwick (2000)
the cooling process is the emission of thermal radiation from the warm grain (at longer wavelengths). To some extent the problem is not dissimilar to that of deriving the temperature of a planetary surface. It is important to realize that the dust grains neither absorb as a perfect black body (i.e. their absorption coefficient is frequency-dependent), nor emit as one. Balancing the heating and cooling rates gives a radial dependence of the dust temperature

\[
T_{\text{dust}}(r) \approx T_{\text{eff}} \left( \frac{R_*}{2r} \right)^{\frac{1}{2+s}}
\]

where an black body spectrum for the star was assumed. The parameter \( s \) comes from the frequency dependence of the absorption/emission coefficient \( (\propto \nu^s) \). Observations suggest that \( s \approx 1 \).

From the expression above we see that the dust is generally warmer than the gas, but that the temperatures in the CSE are low. The fact that the dust temperature drops for larger distances means that the thermal emission from dust will peak at higher frequencies for the inner parts of the CSE. Observations of the dust continuum at different frequencies will therefore show different parts of the CSE.

### 7.2 Chemistry

A chemical picture of an AGB CSE looks more or less like this (see Fig. 31):

1. Photosphere: LTE chemistry
2. Pulsating stellar envelope: shock chemistry
3. Dust formation zone
4. Chemically quiet
5. Interstellar UV radiation: photo-dissociation chemistry

Starting at the photosphere, the local high densities produce an equilibrium chemistry with the dichotomy between the case of O-rich and C-rich stars

\[
\begin{align*}
C > O & \quad \text{C-chemistry (C}_2\text{H}_2, \text{ Polycyclic Aromatic Hydrocarbons (PAHs), ...)} \\
C < O & \quad \text{O-chemistry (H}_2\text{O, SiO, ...)}
\end{align*}
\]

The reason for this is the stability of the CO molecule, which forms abundantly in both cases, and thus captures all of the C in the O-rich case, and all the O in the C-rich case. The photospheric temperature (\( \sim 2000 \) K) is already low enough for molecules to form.

The shocks driven into the outer stellar atmosphere by the stellar pulsations push the chemistry away from LTE. This is the explanation for the ‘unusual’ molecular abundances found from observations (e.g. increased abundances for SiO, SiS, CS, decreased abundances for NH\(_3\) and CH\(_4\)).

In the dust formation zone the so-called refractory elements are removed from the gas phase (Fe, Si, Mg, ...) and end up in dust grains. At the same time the newly formed
dust will also become the site of specific chemical reactions which occur much more efficiently on dust surfaces (‘surficide reactions’). As the temperature drops, some molecules may also start to condense out on grain surfaces, for example H$_2$O, leading to the formation of ice mantles around dust grains.

Beyond radii of $\sim 10^{16}$ cm the gas density has dropped so much that interstellar UV photons can penetrate and start to destroy molecules, a process known as photodissociation. This release chemically active radicals and ionized molecules in the gas phase, starting a range of interesting chemical reactions. One of the most abundant radicals is OH, which is the photo-dissociation product of H$_2$O. The most abundant molecules, H$_2$ and CO are self-shielding and survive to large radii.

Detailed modelling with chemical networks can give a good idea of the chemical richness of AGB CSEs, see Table 1.

### 7.3 Observational aspects of CSEs

Many molecules have transitions at millimeter and sub-millimeter wavelengths, see Table 2, and this wavelength range is therefore one of the most interesting ones for studying the CSEs. This type of high frequency radio emission is observed with dishes (SEST, APEX, IRAM). With single dish telescopes it is difficult to achieve high spatial resolution. Still, because these envelopes are expanding, good spectral resolution can partly compensate for the lack of spatial resolution, and single dish observations are commonly used. Figure 32 shows how line shapes can be used to derive information about the optical thickness of the line, the expansion velocity of the CSE, and the size of the emitting region.

The use of an interferometer considerably improves the spatial resolution, and allows one to obtain more detailed images. Actually, since one retains also the velocity information, one obtains ‘image cubes’: images at various frequencies. Figures 33 and 34 show examples of interferometer observations of the C-stars IRC+10216 and TT Cyg. The observations of IRC+10216 show how different molecular lines trace different parts of the CSE, whereas the circular ring of CO emission around TT Cyg highlights the fact that there are mass loss variations. These observations were both made with the IRAM interferometer. The construction of the ALMA interferometer facility in Chile will considerably improve our imaging capabilities for AGB CSEs, both in spatial and velocity resolution, as well as sensitivity. This is expected to have a large impact on the field.

At infrared wavelengths (best observed from space, for example with IRAS, ISO and Spitzer) both molecules (Table 3) and dust features (Table 4) dominate. The most pronounced dust feature is the 10 $\mu$m (or more accurately 9.7 $\mu$m) Si-feature. Note that not all features have been identified with absolute certainty. The fact that they are rather broad complicates accurate identification.

### 7.4 Maser emission

The CSEs of AGB stars also often show the phenomenon of natural maser emission. This arises through a combination of a non-LTE distribution of the electrons over the energy levels, combined with stimulated emission. To achieve a non-LTE distribution...
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| 1 | H_2 | 1 | H_2 | 1 |
| 2 | CO  | 1.1(-3) | CO  | 1.6(-3) |
| 3 | H_2O| 2.9(-4) | C_2H_2 | 2.2(-4) |
| 4 | N_2 | 1.3(-4) | C_2H | 1.1(-4) |
| 5 | SO  | 6.9(-5) | N_2 | 9.5(-5) |
| 6 | OH  | 9.6(-6) | HCN | 8.5(-5) |
| 7 | SH  | 7.7(-6) | CS  | 2.3(-5) |
| 8 | H_2S| 7.2(-7) | SiS | 9.8(-6) |
| 9 | HCl | 3.4(-7) | C_3H | 9.5(-6) |
| 10 | SiS | 2.0(-7) | CN  | 1.6(-6) |
| 11 | HF  | 1.7(-7) | SH  | 7.0(-7) |
| 12 | TiO | 1.6(-7) | SiH | 3.7(-7) |
| 13 | PO  | 9.7(-8) | SiC_2| 3.7(-7) |
| 14 | NP  | 8.2(-8) | HCl | 3.4(-7) |
| 15 | CO_2| 6.3(-8) | CR_3 | 2.6(-7) |
| 16 | SO  | 4.0(-8) | CH  | 1.6(-7) |
| 17 | MgH | 3.9(-8) | O_2 | 6.2(-8) |
| 18 | AlH | 3.2(-8) | NP  | 5.5(-8) |
| 19 | AlOH| 1.6(-8) | SiO | 4.8(-8) |
| 20 | CrH | 1.5(-8) | H_2S| 4.4(-8) |

Table 1: The atoms and top 20 molecules produced in LTE calculations for an O-rich star (R Cas with C/O=0.75 and $T_{\text{eff}} = 2215$ K) and a C-rich star (IRC+10216 with C/O=1.5 and $T_{\text{eff}} = 2300$ K). In both cases the pressure was chosen to be $1.033 \times 10^{-3}$ atm, corresponding to a total hydrogen density of $3 \times 10^{15}$ cm$^{-3}$. $f(X)$ is the fractional abundance of species $X$ relative to H$_2$ ($k(l) = k \times 10^l$). From Millar (2004)
Table 2: Molecules detected in AGB CSEs at radio wavelengths. The (rough) number of sources detected in each species is given ($\Sigma$), as well as abundances with respect to $\text{H}_2$ ($\text{O}:\text{C}/\text{O}<1; \text{C}/\text{C}/\text{O}>1; k(l) = k \times 10^l$). From Olofsson (2004).

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From Olofsson (2004).
Figure 32: Observed line profiles toward AGB CSEs: (a) optically thin, spatially unresolved emission, (b) optically thin, spatially unresolved emission, (c) optically thin, spatially resolved emission, (d) optically thick, spatially resolved emission, (e) emission from geometrically thin, spatially resolved shell, (f) double-component line profile, (g) and (h) maser emission-line profiles. From Olofsson (2004).
Table 3: Molecules detected in AGB CSEs at infrared and optical wavelengths. The (rough) number of sources detected in each species is given (Σ), as well as abundances with respect to H₂ (O: C/O<1; C:C/O>1; k(l) = k × 10^l). From Olofsson (2004).

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<th>Molecule</th>
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<th>Σ</th>
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<td>CH₄</td>
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Table 4: Dust spectral signatures in AGB CSEs. From Olofsson (2004).

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<tr>
<th>Feature [μm]</th>
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<td>3.1</td>
<td>Stretching of O-H bond in amorphous H₂O ice, O-CSEs</td>
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<td>9.7</td>
<td>Stretching of Si-O bond in amorphous silicate, C-CSEs</td>
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<td>Amorphous Mg₂SiO₄(?) O-CSEs</td>
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<tr>
<td>11.3</td>
<td>Phosphor in silicate, C-CSEs</td>
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<td>13</td>
<td>Spinell (?) O-CSEs</td>
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<td>15-50</td>
<td>&gt;40 features in crystalline silicates, such as olivines (e.g., forsterite) and pyroxenes (e.g., enstatite), O-CSEs</td>
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<tr>
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<td>19.5</td>
<td>Magnesiowustite (Mg₅SiO₄(?) O-CSEs</td>
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<td>20</td>
<td>TiC(?) “21 μm feature” post-AGB C-CSEs</td>
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<td>39</td>
<td>MgS(?) peaks in the range 26-33 μm, C-CSEs</td>
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<td>Spinel(?) O-CSEs</td>
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<td>Dolomite, PN</td>
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<td>93</td>
<td>Calcite, PN</td>
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Figure 33: Molecular line brightness, in a narrow velocity interval centered on the systemic velocity, observed towards IRC+10216 (IRAM PdB interferometer; arcsec scale). The observed lines all lie around 100 GHz. From Guélin et al. (1996).
Figure 34: A CO(J=1→0) map of the carbon star TT Cyg in a narrow velocity interval centered at the systemic velocity (IRAM PdB interferometer). The brightness is well fitted by a circular ring of diameter 68″. The emission near to star is from current mass loss, the ring points to an earlier episode of increased mass loss.
Figure 35: The maser principle. The circles are the electron populations of the two energy levels drawn. Normally, radiation is absorbed by molecules (upper panel) and as a result, the upper population temporarily increases. The absorbed radiation is later emitted at the same frequency. In the maser, a masing molecule is excited into a higher energy level by a pump. Instead of absorbing radiation, the electrons at the upper level are now stimulated by incident radiation to decay to the lower energy level, which results in the emission of photons at the same frequency as the radiation travelling through the maser (lower panel). Consequently, the maser amplifies the radiation of one specific frequency exponentially while the radiation propagates through the masing medium.

Figure 36: A schematic overview of the masers in a circumstellar envelope and their corresponding spectra. The two peaks in the OH spectrum correspond to the front- and back-side of the expanding envelope. The H$_2$O masers are found closer to the star and show much more irregular spectra. The SiO masers are found closest to the star and have a velocity closely centered on the stellar velocity.
one needs a so-called ‘pump’, which can be either radiative or collisional. The stimulated emission process then produces a cascade of photons. Since stimulated emission is proportional to the local intensity, it enters the equation of radiative transfer as a negative form of absorption. This directly translates into a negative optical depth

\[ I_\nu = I_{bg} + S_\nu \left(e^{-\tau_\nu} - 1\right) \]  

which means that the intensity grows exponentially and can reach very large values. This makes masers interesting observationally since they can be observed from large distances.

The intensity is limited by the rate at which the level population can be pumped. If the stimulated emission de-excites faster than the pump can restore the non-LTE distribution, the maser is said to ‘saturate’. A small velocity gradient is also needed.

The types of masering molecules found around AGB stars are

**SiO masers**  Inside the dust formation zone \( \sim 10^{14} \) cm

**H$_2$O masers**  Outside the dust formation zone \( \sim 10^{15} \) cm

**OH masers**  In the photo-dissociation zone \( \sim 10^{17} \) cm

These three types thus provide us with a good sampling of the most important regions of the CSE.

Figure 37: Images of H$_2$O masers around four different AGB stars. The maser action occurs in spots, but around a clearly defined radius. From ???
The SiO and H$_2$O masers mostly show individual ‘spots’ or lumps, whereas the OH maser emission often takes the shape of a thin round shell. In spectra the OH maser line shows a very characteristic two peak shape (coming from the front- and the backside of the maser shell) at a frequency of 1612 MHz. This frequency makes it accessible to most ‘classical’ radio telescopes (such as the VLA and the WSRT), and also to the VLBI networks (giving spatial resolution in the μas range). The stars showing this OH maser emission very often are completely obscured in the optical, and are therefore known as OH-IR stars. They represent a class of O-rich AGB stars with the highest mass loss rates, and the large amount of dust formed inhibits the escape of optical emission.

The pump for the OH maser is infrared emission from the star. Since these stars are variable (with periods of several years), the maser emission also varies, but with a slight delay because of the light travel time across the CSE. The typical radius for OH maser emission is $10^{17}$ cm, corresponding to a light travel time of a month or so. If we can observe both the front- and backside of the shell, there will be a phase difference between their variability, corresponding to $2R_{OH}/c$. If the angular size of the OH maser shell can be measured, this gives us a direct measurement of the distance to us. Because of the very non-linear emission mechanism it is hard to use maser data for determining the density structure of CSEs of AGB stars (including their geometry), but they can be excellently used for several other things

- Distance determinations
- High precision astrometry (proper motions and parallaxes).
- Magnetic field estimates.
8 Evolution after the AGB

As stated earlier, the evolutionary speed of AGB stars is set by their mass loss, not by their nuclear burning. As AGB stars evolve both their pulsation periods and luminosity increase, and so their mass loss rates also increase (although the exact dependence of \( \dot{M} \) on the stellar parameters is not known). The highest known mass loss rates are of the order \( 10^{-4} M_\odot \text{ yr}^{-1} \), but these stars are difficult to study since they are completely obscured by dust at optical and possibly even at near-IR wavelengths. This final phase of extreme mass loss on the AGB is often called the superwind phase.

When the stellar envelope mass drops below a certain, low value (\( \sim 10^{-2} - 10^{-3} M_\odot \)), it can no longer maintain itself in hydrostatic equilibrium, and starts to contract. This is the end of the AGB phase.

The core region does not care about the envelope and keeps the same luminosity as before, and since

\[
L = 4\pi R^2 \sigma T_{\text{eff}}^4
\]

this means that as the star shrinks, the effective temperature goes up. Plotted in the HR-diagram this phase of the evolution is characterized by a horizontal (constant luminosity) movement towards the blue side of the diagram, see Fig. 38. This blueward evolution continues to very high values of \( T_{\text{eff}} \), more than \( 10^5 \text{ K} \), making these stars among the hottest known stars. It stops when the stellar envelope can no longer supply the nuclear burning zone with enough fuel, and nuclear burning stops. At that point the stellar luminosity starts to drop, and the effective temperature slowly starts to go down. This process continues turning the star into a White Dwarf on the so-called White Dwarf cooling track.

This is the general picture of post-AGB evolution. The details depend on various stellar properties

1. Stellar mass
3. Nature of the energy source at the end of the AGB (active H-shell or active He-shell).

which we will look at in the following sections.

8.1 Stellar mass and post-AGB evolution

Just as on the AGB the post-AGB phase has a core mass-luminosity relation, as to a large extent the core is decoupled from the rest of the star. The form of this \( L - M_{\text{core}} \) relation is

\[
L = 5.9 \times 10^5 (M_{\text{core}} - 0.522) \quad L_\odot
\]

(Paczynski 1970). Obviously, lower mass stars have lower luminosity. The core mass also determines the evolutionary speed: the lowest mass (\( \sim 0.5 M_\odot \)) stars take 10,000s of years to reach their highest \( T_{\text{eff}} \), the highest mass stars (\( \sim 1.2 M_\odot \)) do it in \( \sim 100 \) years, see Fig. 38.
Figure 38: Stellar evolution calculations for a range of (AGB) masses (indicated at the start of the evolutionary tracks. The lines crossing the tracks connect points of equal post-AGB age in units of 1000 years. The dots are observed post-AGB stars. The pile up of points show that the evolution is slowing down in that part of the diagram. From Blöcker (1995)
8.2 Mass loss during the post-AGB phase

Any mass loss during the post-AGB phase speeds up the evolution, since it will remove additional material from the already thin stellar envelope. Unfortunately for those who want to precisely model post-AGB evolution, the post-AGB mass loss rates are not well known.

Initially, the mass loss process is identical to that on the AGB, but as the stellar photosphere gets hotter, dust production becomes impossible and other mechanisms have to take over. For $T_{\text{eff}} > 3 \times 10^4$ K a stellar wind can be driven using radiation pressure on (resonance) lines. This is the same process that drives the mass loss in massive O stars. The theoretically calculated mass loss rates and wind velocities match the observed ones quite closely with typical values of $\dot{M} \sim 10^{-9} - 10^{-7} M_{\odot}$ yr$^{-1}$ and wind velocities of 1000—2000 km s$^{-1}$ (Pauldrach et al. 1988). These winds are therefore called ‘fast winds’.

Below effective temperatures of $3 \times 10^4$ K the situation is unclear. There are observational indications for outflows of typical speeds $\sim 100$ km s$^{-1}$, but the mechanism is not understood. The observed outflows are often well collimated into two or more narrow beams. Estimates of the momentum in the outflows indicates that they exceed the limits of radiation-driven winds, even the multiple scattering limit. This has triggered discussions about jet- or bullet-like outflows being launched through magnetic forces from some sort of disc-like structure close to the star. All of this remains very uncertain, see also Sect. 8.4.3.

8.3 Thermal pulse phase and post-AGB evolution

Depending on during which part of a TP cycle a star leaves the AGB, its main energy source will be the H-shell or the He-shell.

0 < $\phi$ < 0.15 He-shell

0.15 < $\phi$ < 0.3 H + He-shell (50/50)

0.3 < $\phi$ < 1 H-shell ($L_{\text{He}} \sim 0.1 L_{\text{H}}$)

The “He-burners” evolve at a somewhat lower luminosity, which could in principle be used to estimate their fraction in samples at the same distance (Galactic Bulge, Magellanic Clouds).

If the TP phase $\phi$ is close to 1 at the end of the AGB, the star may suffer a late thermal pulse during the post-AGB phase. Evolutionary calculations show that this can bring back the star to the location of the AGB in the HR-diagram, and start a new phase of post-AGB evolution (see Fig. 39). These are known as “born-again AGB stars”. That this is not just a theoretical construction is shown by a small number of objects, V605 Aql, FG Sge, V4334 Sgr (“Sakurai’s Object”), which show rapid changes in their effective temperature, as well as in their atmospheric abundances.

These kind of late thermal pulses have also been proposed as the cause of H-deficient post-AGB stars. These stars have spectra similar to massive, C-rich Wolf-Rayet stars, and are designated as [WC]-stars\(^1\). These stars have almost no H, and very high He, C

\(^1\)The square brackets indicate that their spectra show forbidden lines; these are produced in the CSE.

66
Figure 39: Stellar evolution calculations of a $M_{\text{ZAMS}} = 3 \, M_\odot$ star which undergoes a late thermal pulse ($\phi = 0.87$). Its final mass is 0.625 $M_\odot$. The times along the track are in units of 1000 years. From Blücker (1995).
and O abundances. They also show much larger mass loss rates than normal post-AGB stars, which can be understood since line-driven mass loss mostly relies on metal lines. Once they have evolved in WDs, they are probably the so-called PG 1159 stars.

### 8.4 The circumstellar envelope during the post-AGB

The post-AGB evolution is rapid enough for the mass lost during the AGB to still be present around the star. All post-AGB stars are thus surrounded by circumstellar material (CSM), and it is often this CSM that makes these stars noticeable. Two stellar effects modify the CSE during the post-AGB phase:

1. The changing stellar spectrum (as the star evolves from stellar type M via K, etc. to O).
2. The increase of the velocity of the stellar mass loss

As the star evolves through the various spectral types, its $T_{\text{eff}}$ increases, and the spectrum hardens. UV photons start to destroy first the molecules, and then ionize the atoms. Traditionally the division line is placed at $T_{\text{eff}} = 3 \times 10^4$ K beyond which the star produces so much UV radiation that a substantial part of the CSE becomes ionized. The CSE then gets called a Planetary Nebula. At lower $T_{\text{eff}}$ there is almost no, or very little ionization and then the objects are called Pre-Planetary Nebulae, or proto-planetary nebulae\(^2\), or transition objects.

#### 8.4.1 Pre-Planetary Nebulae

Pre-Planetary Nebulae (PPNe) are a separate class of objects. Often the star itself is too deeply embedded to be directly observable. If the star is observable, its spectrum resembles that of supergiants. The reason for this is that the stellar atmosphere is still quite extended (so the surface gravity is low), and the stellar photosphere has temperatures that place it in the range of spectral types G, F, A. In fact the group of “high (galactic) latitude supergiants” are most likely all post-AGB stars, since it is unlikely that massive stars would form at high galactic latitudes.

The CSE in this phase is a strong IR source, requiring satellite data to study it. In some cases spectacular reflection nebulae are seen (Red Rectangle, Egg Nebula, Frosty Leo, etc., see Fig. 40). All of these are strongly aspherical, and some show signs of fast ($\sim 100 \, \text{km s}^{-1}$) outflows.

The youngest PPNe are probably hard to separate observationally from AGB stars. For example, PPNe also sometimes show OH-maser emission, although not with the same thin shell-like morphology. Infrared spectroscopy (which first made possible with sufficient spectral resolution by the ISO mission) shows how the chemical composition of the CSE slowly starts to be modified because of the hardening radiation from the star. Certain molecules disappear, others form. The interpretation of these data require careful modelling using so-called photo-dissociation models.

\(^2\)Confusion with proto-planetary discs is possible when using this name.
Figure 40: Three well known preplanetary nebulae objects, shown in optical scattered light: Egg Nebula (CRL 2688), Red Rectangle (HD 44179), Cotton Candy Nebula (IRAS 17150-3224).
Figure 41: Overview of PNe detected in the galaxy NGC 7457 with the PN-spectrograph on the William Herschel Telescope.

8.4.2 Planetary Nebulae

Planetary Nebulae (PN, plural PNe) are characterized by emission from ions, and are normally found through their Hα and [OIII]5007 Å emission. PNe are the only galactic objects that produce large quantities of O^{2+}, due to the very hard spectra of their central stars. Because of the typical excitation properties a large fraction of the stellar light comes out in the line. This even makes it possible to detect PNe in other galaxies (see Fig. 41), using narrowband surveys. The Doppler shift of the line then allows one to study the galactic dynamics of these galaxies. In a similar way, PNe have been used as distance indicators by comparing luminosity function.

Due to the continuing expansion of the CSE, PNe are rather large objects: $10^{17} - 10^{18}$ cm, and therefore can even be imaged in the Magellanic Clouds (with the Hubble Space Telescope). Since the nebula emits mostly line emission, narrowband filters are often used for this (Fig. 42). Images in different lines are not always identical, indicating that the photon field changes as the radiation moves out through the nebula (see Fig. 43). PNe are traditionally seen as part of the Interstellar Medium (ISM), and textbooks on the ISM often include sections on PNe. The ISM course (AS7001) will address in detail the physics of ionized gasses.

The image data show very few PNe to be spherical ($\lesssim 10\%$), and a wide range of morphologies exist. One could claim that when imaged in detail, every PN is unique. Still, general patterns show up: elliptical shells, bipolar nebulae, point-symmetry (see
Figure 42: A collection of HST narrowband images of PNe, illustrating the wide range of morphologies found in PNe. The different colours correspond to different narrowband filters (but the colour coding and filters differ between images).
Figure 43: HST image of IC 418. Red is the [NII] filter, yellow the Hα filter, green the [OIII] filter. The outer bright edge may be an ionization front, whereas the inner green structure may be the windblown bubble.
for example Fig. 42). Spectroscopy of high enough resolution can measure the expansion velocity of PNe. These fall in the range 20—100 km s\(^{-1}\), depending on the object or the position inside the object. These velocities are higher than those around AGB star, pointing to the process of interacting stellar winds (ISW): the faster post-AGB wind (100s to 1000s of km s\(^{-1}\)) runs into the \(\sim 10\) km s\(^{-1}\) AGB material and pushes it outward at a higher velocity.

A simple model of such a fast wind interacting with a slow wind (AGB mass loss) leads to a bubble structure (Fig. 44) with three discontinuities

\(R_{\text{inner}}\): inner shock

\(R_{\text{contact}}\): contact discontinuity

\(R_{\text{outer}}\): outer shock

and four regions

\(I\): unshocked fast wind

\(II\): shocked fast wind, low density, high (\(\sim 10^7\) K) temperature; also known as the 'hot bubble'.

\(III\): shocked slow wind: high density, nebular (\(\sim 10^4\) K) temperature (mostly due to the stellar radiation); the PN shell.

Figure 44: Schematic overview of a stellar wind bubble, such as found in PNe. See the text for a description.
IV: unshocked slow wind: high density (but lower than in region III), nebular temperature (due to the stellar radiation)

If region IV contains density variations, the shell (III) will become aspherical. This model can be used to explain the various morphologies seen in PNe.

The hot gas in region II emits X-ray radiation which has been observed in a number of PNe, showing that the ISW picture is valid (see Fig. 45). As the PN expands it will merge with the ISM, carrying back stellar material to that may again be used for star formation. Due to the convective mixing and mass loss on the AGB this material will have become more enriched in elements. Also some of the dust particles formed around the AGB star survive the exposure to UV radiation and form the basis of the interstellar dust population.

Not all post-AGB stars will make a PN though. The post-AGB evolution of low mass stars ($\lesssim 0.5 \, M_\odot$) is so slow (see Fig 38) that the CSE will have dispersed before the star becomes hot enough to ionize it.

8.4.3 Asphericity in the post-AGB phase

Observations of AGB stars do not show strong signs of aspherical mass loss, and yet asphericity is commonly seen in PPNe and PNe. Some of these objects show an aspherical shape inside more spherical structures (see Fig. 46). This suggests that the introduction of asphericity takes place at the very end of the AGB, or during the earliest post-AGB phases. However, the physical cause of this remains under debate. Probably some, or all of the following processes play a role:

- rotation
- magnetic fields
- binarity / planetary systems
- accretion discs

The cause of aspherical mass loss around the end of the AGB is perhaps the single most important unsolved problem in the evolution of low mass stars. The short time scales on which it becomes active, as well as the fact that it happens during the most embedded/obscured phase of AGB evolution, make it an extremely tough problem to solve.
Figure 45: An overview of PNe in which extended X-ray emission has been detected (apart from NGC 246). The X-ray emission is shown in blue, the other colours indicate optical emission. From Kastner (2008).
Figure 46: A collection of PNe showing (low surface brightness) spherical rings around their interior bright nebula. From Corradi et al. (2004).