Introduction to Galaxies and Cosmology
Exercises I

In this exercise session we will go through some fundamental astrophysical concepts relating to both astrophysical theory and observational astronomy. The exercises will touch upon parts from chapter 1 and 2 in J&L, so make sure that you have read through those chapters before working with the exercises.

- Basic galaxy dynamics
- Stellar physics
- The magnitude system and colours of stars/galaxies (the extra compendium about the magnitude system may be useful for these tasks)

Some units:
1 Ångström (Å) = 0.1 nm for wavelengths
1 $M_\odot$=1.9889 $\times 10^{30}$ kg, one solar mass (mass of the sun)
1 $L_\odot$=3.839 $\times 10^{26}$ W, one solar luminosity

Basic galaxy dynamics

1. The Sun orbits the centre of the Galaxy at a speed of 220 km/s, 7.9 kpc from the centre.
   a) Calculate the circumference of the Sun's orbit, assuming a circular orbit.
   b) Calculate the orbital period for the Sun around the Galactic centre.
   c) Use Newton's formulation of Kepler's third law (see equation 1) to calculate the mass of the Galaxy inside the orbit of the Sun.

2. The Newtonian formulation of Kepler's third law is given by:

$$m_1 + m_2 = \frac{a^3}{P^2} \cdot \frac{4\pi^2}{G}, \tag{1}$$

   with m the masses of the two celestial bodies, a the semimajor axis (orbital radius for a circular orbit) and P the period (all in SI units).
   a) Derive the rotation curve for objects in orbit around the Sun. You may assume that the orbits are circular.
   b) How does the solar system rotation curve differ from the rotation curve for the Galaxy?

3. An elliptical galaxy has an observed line of sight velocity dispersion of $\sigma_{\text{los}} = FWHM/2.35 = 100$ km/s, and an effective radius of $R_e = 1$kpc. Estimate the mass assuming the virial theorem to apply.

4. At the centre of our galaxy, the Milky Way (or just the Galaxy), astronomers have detected a radio source, Sgr A*. Based on these observations (and many others) it has been possible to find a limit to the physical size for the object at the centre, $r_c \leq 1.2 \cdot 10^{12}$m $= 0.8$ AU. In 2003 a team of observers (Schödel et al., 2003) presented their results on mapping the orbits of stars very close to the galactic centre. The orbit of one of the stars is shown in figure 1. The orbital period for this star was measured to be 15.7 years. Assume that the inclination angle of the orbital plane to the plane of the sky is 40° and that the distance to the galactic centre is 7.9 kpc.
a) Based on the orbit of the star, calculate the mass of the central object (assuming a keplerian orbit).
b) Calculate a lower limit for the density inside the central object.
c) Compare the density derived in b) to the density of the M3 globular cluster, the mass of M3 is $2.45 \cdot 10^5 M_\odot$ and the radius approximately 28 pc.

5. (**Challenging**) Observations of the rotational velocities in disk galaxies show that the velocity stay roughly constant out to very large radii. For the galaxy NGC 7331 the visible mass (composed of stellar mass in the bulge and the disk and gas mass in the disk) inside a radius of 20 kpc is estimated to be $1.55 \cdot 10^{11} M_\odot$.
a) Calculate the expected rotational velocity of an object at a distance 20 kpc from the centre of the galaxy based on the visible mass only using the newtonian formulation of Kepler's third law.
b) Observations show that the true rotational velocity of NGC 7331 at that radius is 240 km/s (which should be higher than the answer you got in a)). To explain this apparent discrepancy of theory and observations we need to include another mass component that is not detectable (so called *dark matter*). We will not go into detail on the properties of this matter but assume that it is distributed in a halo structure, giving a mass distribution function:
$$M(r) = \frac{V_H^2 r}{G}\left(1 - \frac{a_H}{r}\arctan\frac{r}{a_H}\right), \qquad (2)$$
with V_H and a_H parameters of the halo mass distribution model. Now assume that these parameters have been found for NGC 7331,

$V_H = 200$ km/s and $a_H = 7.1$ kpc. Calculate the mass of the dark halo inside the radius 20 kpc.
c) Combine the visible and dark matter mass estimates and compute the rotational velocity for an object at a radius 20 kpc from the galactic centre.

6. (**Challenging**) A disk galaxy is observed to have a neutral hydrogen, λ =21 cm, profile given by figure 2. Its inclination is estimated to be 45 degrees. The observed K-band flux is $m_{\rm K} = 15.0$, and the optical major axis diameter is 10 arcminutes.
a) Estimate the distance using the empirical relation (where V_{max} is the maximum rotational velocity):
$$\frac{L_K}{3 \cdot 10^{10} L_{K,\odot}} \approx (\frac{V_{max}}{200 \text{ km/s}})^{3.8}$$
b) What is this relation called?
c) What is the benefit of using the K-band in such a relation rather than the B-band?
d) What is the corresponding distance derived from Hubble's law?
e) Explain why this, and many other disk galaxies, show a characteristic "double-horn" profile (see the figure)?
f) Estimate the mass by assuming circular motions.

Stellar properties and physics

7. The black body spectrum of stars is directly determined by their effective temperature. In figure 4 the blackbody spectrum of Betelgeuse is given.
a) What is the effective temperature of Betelgeuse?
b) Look at the blackbody spectrum again, if you consider the colour index $B - V$, will the star be red or blue?
c) Consider table 1 in the magnitude compendium. The cooler stars are also the reddest ones. Why is this?
d) The bolometric correction is smallest for stars of type ~F0, why?

8. Show how the the measured angular diameters and observed energy fluxes of stars can be used to measure their effective temperatures.

9. (**Challenging**) The effective temperature of Betelguese was found in exercise 7. The angular radius of the star is $0.045''(arcsec)$ and the distance from earth is 140 pc. Calculate the the radius and luminosity of the star. Express your result in solar units ($R_\odot, L_\odot,$). How would you classify this star?

10. Assume that the fusion of Hydrogen into Helium that takes place in the centre of the sun converts 0.7% of the Hydrogen rest mass into light.
a) At what rate would it have to convert H into He in order to supply the luminosity that we observe?
b) Assuming that only the central 10% (by mass) is available for fusion, how long would the Hydrogen last?

11. The stability of a star is maintained by the so called *hydrostatic equilibrium*. The equation for this equilibrium is given by:

$$dP = -\frac{GM(r)}{r^2}\rho dr, \tag{3}$$

where dP is differential pressure outwards, G the gravitational constant, $M(r)$ the mass contained inside the radius r and ρ the density inside the star.

Assume that the interior of a star of total radius R has a constant density ρ_0.
a) Find the expression for $M(r)$.
b) Insert this expression into equation 3 and integrate over radius to find an expression for the pressure at radius r. The density is zero for $r > R$ (i.e. outside the star) which then also means that there is no pressure outside the star.
c) What is the central pressure of the Sun using this expression?

Magnitudes and colours

12. A star of type G5V is observed in two photometric filters (B and V). The measured magnitudes are: $m_B = 18.3$ and $m_V = 15.8$. A G5V star has a temperature of around 5500 K (also see figure 3).
a) What is the observed colour index of the star?
b) Calculate the interstellar extinction in V (A_V) towards the star. What is the the extinction-corrected V magnitude of the star?

13. The Sun is observed from a distant star to have an apparent blue magnitude (B) of 14.4 and an apparent visual magnitude (V) of 12.8. The Sun has a temperature of 5800 K and its absolute visual magnitude is +4.8. How far away is the star?

14. A galaxy is observed to have an apparent V magnitude of 11.0. Observations of the Hα spectral line shows that it is redshifted from 6563 Åto 6575 Å. If you assume that the extinction is negligible and that $H_0 = 72$km s^{-1} Mpc^{-1}, what is the V absolute magnitude of the galaxy?

15. (**Challenging**) Two galaxies in the process of merging together have $m_{\mathrm{B}} = 18.2$ and $m_{\mathrm{B}} = 19.6$, respectively. The first galaxy has a colour $B - V = -0.2$ and the second $B - V = 0.5$. If this system is observed in a telescope which cannot resolve the two components, what would the integrated m_{B} and $(B - V)$ of this object be?

16. Galaxies with luminosities above or around $L_\star$ ($\sim 2 \cdot 10^{10} L_\odot$) are characterized as bright galaxies. This corresponds to a blue absolute magnitude of $M_{B,\star} \approx -20$. If the detection limit for a galaxy survey is given by $m_B = 25.0$, what is the furthest distance an $L_\star$ galaxy can be detected from (assume that the extinction is negligible)?

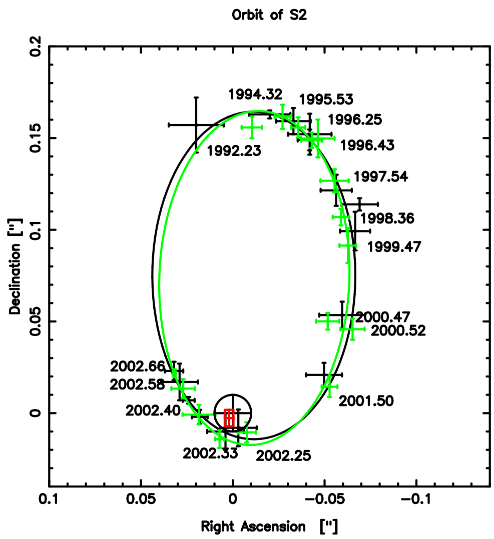

Figure 1: Orbit for the star S2, orbiting the source that is believed to be in the centre of the Milky Way, Sgr A* (figure from Schödel et al., 2003)

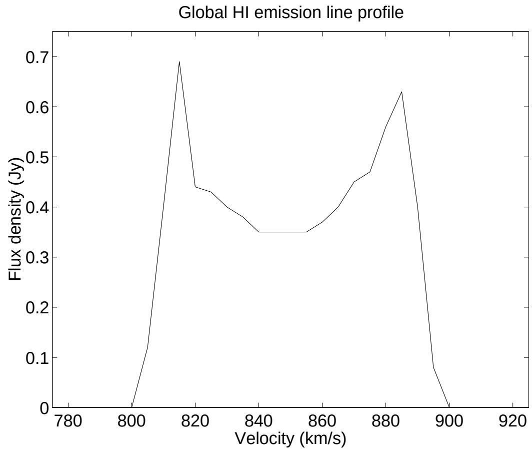

Figure 2: Neutral hydrogen 21 cm line profile for a disk galaxy

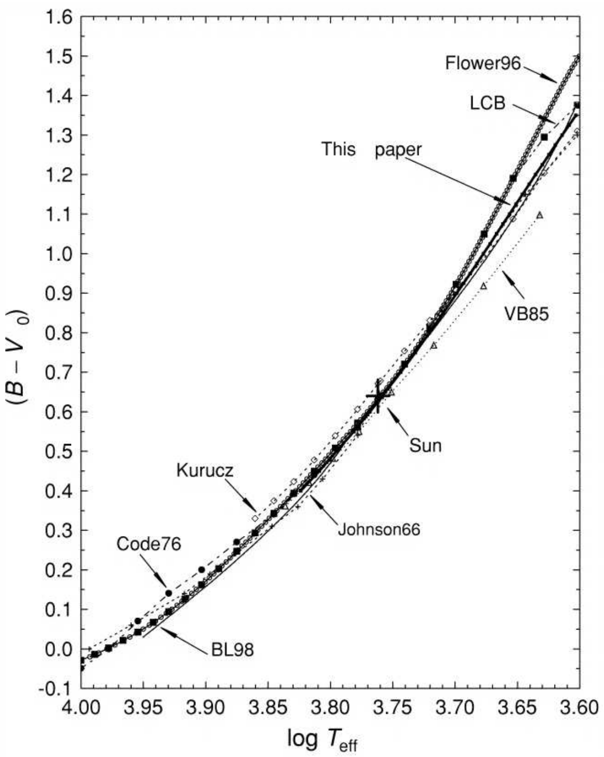

Figure 3: Theoretical $(B-V)$ colour index vs. stellar effective temperature (figure from Sekiguchi & Fukugita, 2000).

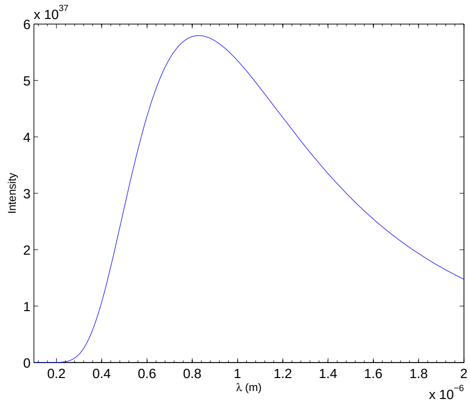

Figure 4: A blackbody curve with the same temperature as Betelgeuse.

Answers, Exercise I

1. a) The circumference of the Sun's galactic orbit is $1.5 \cdot 10^{18}$ km.
   b) $P = 7.0 \cdot 10^{15}$ s
   c) $M_{Galaxy}(r < r_\odot) = 8.9 \cdot 10^{10}\, M_\odot$

2. a) $v_{rot}(r) = \sqrt{GM_\odot/r}$
   b) –

3. $M = 1.7 \cdot 10^{10} M_\odot$

4. a) $M = 3.3 \cdot 10^{6} M_\odot$
   b) $\rho \geq 0.91$ kg/m^3
   c) The density of M3, $1.9 \cdot 10^{-19}$ kg/m^3, is much lower than that of the central object in the galaxy. It's very unlikely that the central object is some sort of cluster of stars.

5. a) $v_{rot} = 187$ km/s
   b) $M_{DM} = 1.05 \cdot 10^{11} M_\odot$
   c) $v_{rot,total} = 242$ km/s

6. a) $D = 27$ Mpc
   b) The Tully-Fisher relation
   c) The K band is less sensitive to star formation, hence gives better estimates for the total stellar mass.
   d) $D = 11.8$ Mpc
   e) The double-horn profile is a signature of a flat rotation curve, most gas is situated where the rotation curve is flat. Since the velocity is constant in this region doppler shift of the emission

from the gas causes the profile to be symmetric and double peaked (one peak for each side of the rotating galaxy).
f) $M = 2.3 \cdot 10^{10} M_\odot$

7. a) $T_{eff} = 3500$ K
b) –
c) –
d) –

8. –

9. $R = 676 R_\odot$, $L = 6.1 \cdot 10^4 L_\odot$, K/M supergiant

10. a) $\dot{M} = 6.127 \cdot 10^{11}$ kg/s
b) $\tau = 7.3 \cdot 10^9$ yrs

11. a) $M(r) = \frac{4\pi r^3}{3}\rho_0$
b) $P(r) = \frac{2\pi G \rho_0 R^2}{3}\left(1 - \left(\frac{r}{R}\right)^2\right)$
c) $P_c = 1.34 \cdot 10^{14}$ Pa

12. a) $(B-V)_{obs} = 2.5$
b) $A_V = 5.4$ (using $(B-V)_0 = 0.70$ and $m_{V corr} = 10.4$

13. $d = 100$ pc

14. $M_V = -18.4$

15. $m_{B,tot} = 17.94$, $(B-V)_{tot} = 0$

16. 10 Gpc