THE YOUNG PULSAR
PSR B0540-69.3 AND ITS SURROUNDINGS

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Abstract:

Massive stars live short lives and die violently. Their end products are energetic and/or represent extreme physical conditions. The most massive ones can produce black holes, γ-ray bursts, pulsars, magnetars etc. The death of massive stars therefore provide a laboratory to test physics that we can not do on Earth. In this thesis I discuss one object in the Large Magellanic Cloud that exploded \( \sim 1000 \) years ago (or actually in reality \( \sim 160000 \) years ago). The shock wave from this explosion can now be seen to interact with surrounding gas, and in the center there lurks a rapidly spinning pulsar. Here I mainly concentrate on the pulsar and its wind nebula, and some comparisons are made with its sibling, the Crab pulsar. The first part of the thesis provides a more general outline of the subject.
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Chapter 1

Introduction

1.1 Supernovae and supernova remnants

All stars are born, live by means of thermonuclear reactions, and die. During their lives heavy elements are formed in their interiors. This process is known as nuclear fusion and releases of a lot of energy that allows a star to shine.

Low-mass stars die and form white dwarfs when the outer parts of the progenitor star are ejected forming a beautiful planetary nebulae. There is no explosion in these cases, although the white dwarf may explode at a much later stage if it is part of a tight binary system (see below).

On the contrary, massive stars normally die in powerful stellar explosions, so-called supernovae (SNe). The mass ejected in these explosions expands out into the surroundings for tens of thousands years, forming supernova remnants (SNRs). The structures of SNRs are highly inhomogeneous due to e.g. clumpiness of the ejecta and interaction with inhomogeneous circumstellar and/or interstellar

Figure 1.1: An X-ray composite image of a beautiful supernova remnant in the Large Magellanic Clouds (LMC) SNR 0540-69.3. The colors represent different energy bands, red: 0.3-0.8 keV, green: 0.8-2.1 keV, blue: 2.1-4.0 keV. Note that the pulsar wind nebula (PWN) and the pulsar itself are in the white colored area in the center of the remnant. The image is 1.7 × 1.7 arcmin (Courtesy of NASA/CXC/SAO).
media. As an example, Figure 1.1 shows an X-ray composite image of the beautiful supernova remnant SNR 0540-69.3 in the Large Magellanic Clouds (LMC), which is the main object of this thesis.

In our galaxy there have been approximately 100 million supernova explosions so far. They have enriched the Galaxy with heavy elements, like oxygen in the air, and calcium, and iron in our bodies. These elements, and many other, can be observed via the study of emission lines in spectra of SNe and their remnants.

Astronomers classify SNe into two main classes, thermonuclear (Type Ia) supernovae and core-collapse supernovae (CC SNe).

Type Ia supernovae originate from binary systems of less massive stars. When a carbon-oxygen white dwarf accretes matter from a nearby companion star, typically a red giant, and exceeds the Chandrasekhar mass limit ($\sim 1.4M_\odot$), the process is going out of hand. A so-called thermonuclear runaway occurs and the star is blown to pieces. The thermonuclear supernovae leave no compact object after the explosion. Type Ia SNe may also be a result of two merging white dwarfs.

The CC SNe stem from massive stars ($\gtrsim 8 M_\odot$). When nuclear fusion has produced iron in the core no more energy can be generated there, because iron has the highest binding energy per nucleon among all stable elements, and an iron core grows. The SNe explosion starts when the size of the iron core reaches the Chandrasekhar limit. The central iron core collapses and the collapse is followed by a bounce and rebound, and finally an explosion. The core-collapse supernovae may form neutron stars (NS) or black holes (BH) at their centers. A complete disruption may occur for very massive cases.

1.2 Neutron stars

In this thesis I will focus on CC SNe, and in particular the inner parts of them. The center harbors the compact object, in most cases a neutron star, formed in the explosion.

A neutron star (NS), created by the collapse of the iron core, is actually a giant atomic nucleus made almost entirely of neutrons and held together by gravity. The neutrons are closely packed, so that neutron degeneracy pressure supports them against gravitational collapse. A pulsar is a rotating neutron star that beams radiation along its magnetic axis and across Earth.

Observationally, neutron stars (NSs) were discovered (Bell & Hewish 1967) four decades ago. Since then NSs have fascinated astronomers due to their extreme properties in almost all respects. A neutron star is one of the densest objects in the Universe, the most rapidly rotating, and the best astronomical laboratory for condensed-matter physics.

Here I will mainly concentrate on young pulsars and their properties. I will discuss pulsar PSR B0540-69.3, its pulsar wind nebula (PWN) and touch on the properties of its supernova remnant SNR 0540-69.3.
I will present two papers. My contribution to them is data reduction and analysis. The analysis in the section “X-ray spectrum. Interstellar absorption” in **Paper I** was mainly made using models of Peter Lundqvist. Also these models were used in **Paper II** for the density and temperatures estimate in SNR 0540-69.3 and nearby H II regions. Note that my surname was Serafimovich before March 2006.


The outline of the thesis is that I will first discuss properties of pulsars in great details in Chapter 2, and in Chapter 3 I will discuss **Paper I** and **Paper II**, and put them in a broader context. My discussion in Chapter 2 will be an “inside-out” discussion where I will start with the neutron star at the center, and then continue with the supernova ejecta.
Chapter 2
Theory and observations of pulsars

2.1 Types of pulsars

Figure 2.1: A composite image of the Crab Nebula showing the X-ray (blue) and optical (red) images superimposed on each other. The extension of the X-ray image of the PWN is smaller than the optical because the higher energy X-ray emitting electrons radiate away their energy faster than the lower energy optically emitting electrons as they move. The bright star in the center of the X-ray emission is the Crab pulsar itself (Courtesy of NASA).

The possibility that neutron stars might exist was first proposed in the 1930s by Walter Baade and Fritz Zwicky (Baade & Zwicky 1934), but many astronomers did not believe that nature could really make anything so strange. Nevertheless, direct observations now make it quite clear that neutron stars do exist. Even though the Crab pulsar (Staelin & Reifenstein 1968; Cocke, Disney & Taylor 1969 and see Fig. 2.1) is perhaps the most well-known, it was not the first to be discovered. That was instead the PSR J1921+2153 with a period of 1.337 seconds, which was discovered by Jocelyn Bell Burnell and Antony Hewish (Hewish et al. 1968), and later rendered Antony Hewish the Nobel Prize in Physics.

When NSs are born in the supernova explosion they are hot ($\sim 10^{11}$ K). The main energy sources are rotation and release of internal energy. The typical cooling time scale is $10^7$ years. After a few tens of millions of years the NS is a cold, dead object. According to modern theory, the life of a NS defines
the period when the star efficiently produces electron-positron pairs in a strong curved magnetic field, and death — when this mechanism is switched off.

Observationally, NSs revealed themselves as pulsars, i.e. pulsating radio sources, and the brightest radio pulsars show pulsed emission at all wavelengths, from radio to γ-rays. Two of these, the Crab pulsar and PSR B0540-69.3 are important for this thesis.

Figure 2.2: (left:) Distribution of periods for 1139 pulsars. The data for 744 pulsars are from M. Bailes’ (University of Swinburne, Australia) catalogue and data for 395 pulsars are from the Parkes Multibeam Survey catalogue. The plot is courtesy of Glenedenning N. K. (right:) $P - \dot{P}$ diagram in logarithmic scale for about 1500 radio pulsars (dots). The millisecond pulsars are located in the lower-left corner of the diagram. The pulsars from Zavlin (2006) are shown with red dots. Straight lines correspond to constant values of pulsar characteristic age $\tau_c = 10^3, 10^6$ and $10^9$ yr, surface magnetic field $B_{\text{surf}} = 10^8, 10^{10}, 10^{12}$, and spin-down energy $= 10^{30}, 10^{33}$ and $10^{36}$ erg s$^{-1}$. Courtesy of V. Zavlin.

There are two main populations of pulsars, the main differences being their pulse period and the derivative of the pulse period (see Fig. 2.2 and Appendix I for basic formulae). The two classes can be grouped as millisecond pulsars (MSPs) and normal pulsars. The observational properties of MSPs are significantly different from those of ordinary radio pulsars. First, MSPs have short (usually less than 30 ms, Lyne & Graham-Smith 2006) and stable spin periods. Second, they have relatively low values of the dipole magnetic field strength, typically $10^8$ G, compared to ordinary pulsars which have of order $10^{12}$ G. Third, they have very small period derivatives, less then $10^{-18}$ s s$^{-1}$. The separation between MSPs and ordinary radio pulsars can be seen clearly in the $P - \dot{P}$ diagram shown in Fig. 2.2 (right panel). Most of the MSPs (65 out of 80) are old pulsars born in binary systems and have been spun up due to accretion from the companion (Lorimer 2005). The MSPs will not be discussed further in this thesis.

Pulsars like the Crab pulsar and PSR B0540-69.3, with periods 33.5 and 50.2 ms, respectively, belong to the normal pulsar population despite their short periods. They are both young and have magnetic fields on the order of $10^{12}$ G. Older pulsars in this category can have pulse periods of several seconds.

Age is probably not the only reason for the difference in pulse period of normal pulsars. They are actually most likely born with different spin periods. The birth spin period distribution probably follows a Gaussian distribution with a mean period of several hundred milliseconds (Faucher-Giguere & Kaspi 2006). Recent models by Blondin & Mezzacappa (2007) show that shorter birth spin periods can be achieved by an instability in the accretion shock during the supernova explosion even for non-rotating progenitor stars. However, no pulsar younger
2.2 Distances and velocities of pulsars

Recent investigations by Faucher-Giguere & Kaspi (2006) show that isolated radio pulsars are born in the spiral arms of the Galaxy. This is quite natural because neutron stars are formed in core collapse supernovae which are, in turn, dying massive short-lived population I stars that form in spiral arms of galaxies. Models (Faucher-Giguere & Kaspi 2006) and observations (Manchester et al. 2001; Edwards et al. 2001) agree in this case.

The distances to the nearest pulsars are usually obtained from parallax measurements. For the ones that are at distances greater than ~ 1 kiloparsec, more accurate distances come from measuring the dispersion measure (DM). This stems
from the important effect that the ionized interstellar medium (for ex. an H II region) causes radio waves on the way from a source to the observer to travel faster (e.g., higher phase velocity, see Appendix I for more details) for waves with low frequency than with high frequency. The value of the delay between low- and high-frequency waves is proportional to the integrated column density of electrons along the line of sight, i.e, the DM (see Appendix I for details). The derived column density is model dependent since the main uncertainty of the DM is caused by scattering processes but for radio this blurring occurs mostly in the interstellar medium (ISM). The column density in the ISM is often not so well established and actually constructed from measured values of DM for pulsars with known distances. For example, the Crab pulsar has DM equal to $57 \, \text{cm}^{-3} \, \text{pc}$ and PSR B0540-69.3 has DM $\approx 146 \, \text{cm}^{-3} \, \text{pc}$. Both pulsars are associated with supernova remnants with well known distances and provides constraints on the electron distribution along the directions to these pulsars.

Soon after their discovery, pulsars were called runaway stars (Gunn & Ostriker 1970). This was because many appear to have moved far away from their place of birth, based on a correlation between age and distance from the Galactic plane. There is, however, one big uncertainty in this, and that is the age. The most popular way to estimate the age of a pulsar is to use the so-called “characteristic age”, $\tau_c$ (see Appendix I for the details). However, the physics of pulsars is not fully understood and Faucher-Giguere & Kaspi (2006) point out that “characteristic age” should rather be used as an upper limit.

![Figure 2.4: The 3-D velocity distribution obtained for the observed 1-D (left) 2-D (right) distributions. The dotted curve shows the 3-D distribution from Arzoumanian et al. (2002). The solid curve is the best-fitting Maxwellian distribution to the histogram from the 2-D distribution with $\sigma = 265 \, \text{km} \, \text{s}^{-1}$ (cf. Hobbs et al. 2005).](image)

The uncertainty in age and distances also makes the study of pulsar initial or birth periods quite complicated. First, the bimodality of birth velocity distribution has been theoretically predicted in 2002 by Arzoumanian et al. (2002). The authors tried to account for everything including selection effects. They simulated NSs using standard theory and compared the sample with real surveys taking into account limitations of different surveys. This work inspired statistical study of real pulsars known up to date.

The latest detailed study of pulsar birth spin periods shows that they can be shorter than observed now. For example, the Crab pulsar was the first for which the birth spin period was calculated with a value about 19 ms. Also for PSR B0540-69.3 the estimated birth spin period is about 30 ms. While for some pulsars the birth spin period is about the same as is observed now (see Faucher-Giguere & Kaspi 2006 and references therein).
Fig. 2.4 shows the 3−D birth velocity histograms. Most of these pulsar velocities come from measurements of their proper motions and for the nearby ones via parallax measurements. This pure statistical work by Hobbs et al. (2005) does not support the bimodality distribution. These authors obtain the mean 3−D birth velocity $400 \text{ km s}^{-1}$, which was confirmed later by Faucher-Giguere & Kaspi (2006). Their three-dimensional space velocity distribution is well described by a Gaussian function with mean velocity $380^{+40}_{-60} \text{ km s}^{-1}$ (Faucher-Giguere & Kaspi 2006). A single-component Gaussian model combined with exponential or Lorentzian functions describe the data better than two-component Gaussian models. On the other hand, Faucher-Giguere & Kaspi (2006) note that parameters of two-component Gaussian models vary quite much depending on what weight is given to the high velocity pulsars.

Up to now only 10% of all pulsars with measured proper motion are high velocity pulsars, i.e. $v \gtrsim 1000 \text{ km s}^{-1}$. An interesting question is whether these pulsars represent pulsars with different properties than low-velocity pulsars. This is not unlikely since at least the momentum input to these pulsars is different at birth than their low-velocity siblings.

Many models for high velocity pulsars use asymmetry in the supernova explosion to give “kicks” to the newly born NS with high speed. For example, PSR B0540-69.3 has an asymmetric remnant (see Fig. 1.1) and shows an indication of having high space velocity (see results and detailed discussion in Paper I). There is also evidence that there exists a correlation between the spin axis and the proper motion of a pulsar such that the pulsar seems to move in the direction of the spin axis, which also supports the theories about “kicks” at the birth of NS (Kargaltsev & Pavlov 2008). This was first shown by Helfand et al. (2001) for the Vela pulsar and its surrounding nebula. Later Ng & Romani (2004) used asymmetric neutrino emission simulations to explain the correlation and compared with Chandra observations for other young pulsars (PSRs J0538+2817, B1951+32, B1706-44, Crab and Vela).

If a pulsar has a surrounding nebula, then the geometry of that nebula can be used to guide our understanding. In the case where there is no nebula, polarization observations can be used to determine the position angle of the pulsar rotation axis (Johnston et al. 2005). An interesting fact is that the youngest pulsars in the sample of Johnston et al. (2005) show the best alignment of the spin axis and the proper motion direction. An indication of such an alignment is shown also for the pulsar PSR B0540-69.3 (see Paper I). In that paper we used direct measurement of the proper motion through comparison of images from a HST/WFPC2 archival data separated by 4 years. This work is now continuing by using our new HST data obtained at the end of 2005. The data have been reduced and the analysis is underway (see Section 3.4).

2.3 Brief theory of Neutron stars

According to modern theory (for details and a review see Page & Reddy 2006), a NS consists of a very thin atmosphere (mainly hydrogen and maybe with a mixture of heavy elements), an envelope (the matter there is not fully degenerated), a crust (a lattice of nuclei is immersed into super fluid of neutrons), an outer core (a mix of neutron super fluid and proton fluid). The envelope is thinnest, i.e. a few tens of meters, and the crust has a thickness of about 500 – 1000 meters. The NS has a few kilometers of outer and inner core. All different parts of a NS
are illustrated in Fig. 2.5. Inset A in Fig. 2.5 shows the transition region from inhomogeneous crust to the homogeneous core. Due to the increase in density, nuclei are getting squeezed and the core becomes a homogeneous neutron + proton liquid. Inset B in Fig. 2.5 shows the structure of the NS’s crust. Due to rotation of the star the neutron super fluid should form vortices (Page & Reddy 2006), which may be responsible for observable glitches in pulsars. Inset C in Fig. 2.5 shows the structure of the outer core, which consists of neutron super fluid + proton superconductor. Due to the high magnetic fields in NSs the proton superconductor confines the external magnetic field into so-called magnetic flux tubes or fluxoids/fluxons. In turn, a fluxon is the smallest magnetic flux (flux quantum) that exists in nature. Just as electrons are quantized charge, fluxons are a quantized magnetic flux. The nature of the inner core is still very uncertain and is in Fig. 2.5 marked by “?” correspondingly.

For the calculations of the structure of a NS one needs to know the equation of state (EOS) or, i.e., the pressure – density relation for degenerate matter. There are many possible EOSs depending on the assumption about the star composition. This is the most uncertain part in the theory of NSs due to their extreme physical conditions.

If an EOS is given, then the maximum mass of a NS can be estimated (Lattimer & Prakash 2004). The “stiffness” of the EOS depends on the amount of bosons in high density matter. Modern theories consider matter at this stage to contain two types of particles, i.e., bosons and fermions. The bosons have an in-
integer spin quantum number and obey the Bose-Einstein statistics. The fermions have half integer spin quantum number and obey the Fermi-Dirac statistics and Pauli exclusion principle which prevents two identical fermions from occupying the same quantum state. For example, photons, gluons (particles that mediate the strong nuclear forces, cause quarks to interact and are indirectly responsible for holding together the protons and neutrons), W and Z bosons (particles that mediate the weak force) are bosons. The bosons can be composite, i.e., particles composed of a number of other particles like protons, neutrons or nuclei. For example, Deuteron \(^{2}\text{H}^+\), \(^{4}\text{He}\) and so on. The bosons do not contribute to the fermi pressure, which stabilizes a NS, they just will try to “soften” the EOS. For a soft EOS, the maximum neutron star mass will be low, e.g. \(\lesssim 1.55 \text{ M}_\odot\) (van der Meer et al. 2007). Therefore, accurate measurements of the NSs are very important to constrain their EOS.

The easiest way to estimate masses of NSs is the observation of pulsars in binary systems. Van der Meer et al. (2007) estimated NS masses \(1.06 \pm 0.11, 1.25 \pm 0.11\) and \(1.34 \pm 0.15 \text{ M}_\odot\) for the eclipsing X-ray pulsars SMC X1, LMC X4 and Cen X3, respectively, studying their UV spectra. The variation of line profiles with orbital phase was used to derive orbital parameters and masses of the NSs. As pointed out by Van der Meer et al. (2007), up to now masses for only 32 NSs in binary systems have been estimated and are in the range of 1 to \(2\text{M}_\odot\) (see their Fig. 12).

The radius of some NSs can be derived from the fit of their thermal spectrum. For example, in case of black body emission and uniform surface temperature, \(L = 4\pi R^2 \sigma T_{\text{eff}}^4\), where \(\sigma\) is Stefan-Boltzmann constant, \(T_{\text{eff}}\) is the effective temperature, \(R\) the radius and \(L\) the luminosity. This method will be reliable only for a few NSs. First, the surface temperature can be non-uniform, because the heat is transported from the interior to the surface of the star (with strong magnetic fields) by electrons that flow along the magnetic field lines. Furthermore, Romani (1987) predicted that a thermal spectrum will have high deviation from a black body spectrum if a cooling NS has an atmosphere. In the case of hydrogen- or helium- dominated surfaces of NSs with low effective temperature the observed flux in soft X-rays would be higher than the black body value, very likely due to some contribution of non-thermal or multi component thermal emission, while in case of high-Z elements dominated surface, the observed spectrum comparable with black body and show prominent absorptions (Romani 1987). Finally, the thermal spectrum can not be observed from young pulsars like Crab or PSR B0540-69.3, because the magnetospheric synchrotron emission overwhelms any thermal emission. Better objects for measurements of mass and radius are Low Mass X-ray Binaries (LMXBs) in globular clusters which show radii around 13 km (see Gendre et al. 2003a,b).

### 2.4 The pulsar magnetosphere

It is now well known that pulsars are rotating neutron stars and that their emission is beamed. The main goal in studying them is to learn how they convert their rotation energy into radiation. From theoretical studies (see Harding 2007 for a review) it is generally agreed that this transformation occurs due to acceleration of charged particles to highly relativistic speeds using a magnetic field as an inductor to create a very strong electric field. The question is where this happens, close to the surface of the neutron star or further away? This is still
The magnetic field of a pulsar governs its characteristics and can be described by a magnetic dipole model. The main structure of a pulsar’s magnetosphere is shown in Fig. 2.6. The light cylinder is the distance, $r_c$, where the co-rotating magnetosphere with angular velocity $\Omega$, would have a velocity equal to the speed of light, $r_c = c/\Omega$. This distance can be written in the way, $r_c = 5 \times 10^9 P$ cm, where the $P$ is pulsar period measured from observations. This in turn shows that emission is different for longer period pulsars than for shorter period ones (Harding 2007). The polar caps are the regions at the North and South pole on the pulsar’s surface where magnetic field lines are open and cross the the light cylinder (see Fig. 2.6 inset “C”). The violet region in inset “A” represents a so-called polar cap accelerator.

The beam of electromagnetic radiation is emitted along the magnetic dipole axis within $\sim 10$ degrees angle in the pulsar’s magnetosphere. Highly relativistic particles flow out and form a plerion or pulsar wind nebula (PWN) around the pulsar (see Fig. 2.6 inset “B”). During this active phase, the pulsar loses most of its angular momentum due to radiation, and the rotation slows down. The slowdown rate can be used as a measure of the bipolar field (see Appendix I for details). On the other hand, Lyne & Graham-Smith (2006) point out that for some young pulsars the dipole field is increasing while the pulsars themselves slow down.
2.4. THE PULSAR MAGNETOSPHERE

2.4.1 Multiwavelength emission from pulsars

The rotation-powered pulsars are observable in a very broad wavelength range from radio to γ-rays. Studies of their multiwavelength spectra and pulse profiles will provide us with some clues about particle acceleration and emission geometry (Harding 2007).

The radiation mechanism of NSs is still not fully understood. The first theory that was proposed for the radiation mechanism by Goldreich & Julian (1969), suggested that the electric field induced by the NS ejects the plasma from the surface beyond the light cylinder and forms a relativistic wind. In this case the radio emission will be due to the acceleration of particles and avalanche production of ($e^\pm$) pairs (Ruderman & Sutherland 1975). However, since then theoreticians developed two other types of models, the polar-cap models and the outer-gap models. In the polar-cap model acceleration and radiation comes from the inner magnetosphere, i.e., near the magnetic poles. In the outer-gap model it comes from the outer magnetosphere (Kaspi, Roberts, & Harding 2006). These models also argue that charge-deficient regions where the electric field is parallel to the magnetic field lines ($E_\parallel \neq 0$) accelerates the ($e^\pm$) pairs to relativistic energies.

The radiation of a NS has two main components: thermal radiation from the surface of the star and non-thermal or synchrotron radiation from its magnetosphere. The synchrotron mechanism of radiation, which can be described by a power-law (PL), $F_\nu = F_\nu_0 (\nu/\nu_0)^{-\alpha}$, absolutely prevails at all energy ranges, i.e., it constitutes a multiwavelength spectrum, in particular for young pulsars like Crab and PSR B0540-69.3. Here $\alpha$ is the spectral index which typically has a value in the range $0 \lesssim \alpha \lesssim 1$. The nature of the synchrotron radiation in the case of PSR B0540-69.3 appears to be more complicated than for the Crab pulsar (see Fig. 3.3). The multiwavelength spectrum of PSR B0540-69.3 has at least two breaks and can be described with several PLs, while the spectrum of the Crab pulsar shows a smooth transition from X-rays to the optical range. The reason for this difference is unclear and certainly needs to be studied in more detail. It could originate from intrinsic differences between the pulsars, e.g. the PSR B0540-69.3 might be heavier because its progenitor star was a more massive star compared to the Crab progenitor (Lundqvist et al. 2008). Also it could be that PSR B0540-69.3 is just more efficient in terms of emitting in the optical and in X-rays (see Table 8 in Paper I).

When a pulsar is getting older (characteristic age, $\tau_c \gtrsim 10^4$ yrs, see Appendix I) the synchrotron radiation becomes fainter and the thermal radiation can be detected. The spectrum displays a “thermal bump” on top of the PL spectrum (see Fig. 2.7). This is a typical situation for middle-aged pulsars. The thermal component starts to show up in the near-UV and is then seen in the X-rays due to a black body spectrum with effective temperature a few times $10^6$ K which peaks around a few hundred eV. The nature of the thermal radiation emitted from the NS surface is also not trivial and sometimes can be fitted with several different temperature components. An origin of this multi-component radiation could be a non-uniformly heated surface of the star.

Pavlov et al. (2002); Zavlin & Pavlov (2004); De Luca et al. (2005) suggest a thermal soft (TS) and thermal hard (TH) component for middle-age pulsars like Vela, B0656+14 and Geminga. The TS component is cooler and very likely its source is emission from the entire surface, while the polar caps, which could be additionally heated by the magnetospheric particles, are responsible for the
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Figure 2.7: Multiwavelength spectrum of a typical middle-aged pulsar, the Vela pulsar (Shibanov et al. 2003). Note, that a thermal component is clearly seen on top of the non-thermal power law.

hotter TH component.

Multiwavelength spectra of middle-aged pulsars (10 $\lesssim \tau \lesssim$ 500 kyr) are especially interesting because they emit all the three emission components. The nearby middle-aged pulsars, Vela ($\tau \approx$ 11 kyrs), B0656+14 ($\tau \approx$ 110 kyrs) and Geminga ($\tau \approx$ 340 kyrs), are relatively bright and well studied in both the optical and X-rays. The multi-component emission decays with age as the NS cools down (Zavlin & Pavlov 2004).

2.4.2 Interstellar extinction

In order to establish the emitted multiwavelength spectrum of the pulsar and PWNe, we need to correct the observed flux for interstellar extinction.

The interstellar extinction is the absorption and scattering of light by dust grains on the way from the source to the observer. The extinction depends on the optical properties of the dust grains. Study of the extinction can shed light on the composition and size distribution of the grains along line of sight (see, e.g., the detailed study of Weingartner & Draine 2001). Extinction is wavelength dependent and is important to know in order to remove it from the observed flux of the source.

Extinction can be deduced by comparing observed and intrinsic colors for sources with known spectral distribution. Usually the ratio of total extinction and color excess is defined as $R_V = A_V/E_{B-V}$, where $B$ and $V$ denote the standard B and V photometric filters. The total extinction at any wavelength, $A_\lambda$, is defined as $m_\lambda = M_\lambda + 5 \log(d) - 5 + A_\lambda$, where $m_\lambda$ and $M_\lambda$ are apparent and absolute magnitudes of the star, respectively. The distance to the star in parsec is $d$. Cardelli et al. (1989) showed that this ratio, $R_V$, governs the extinction curve behavior. It is measured to be around 3.1 for the diffuse ISM in the Milky Way (MW).

Another way to determine extinction is to look at spectral lines with known theoretical ratio, for example, the recombination lines of hydrogen. The forbidden
lines give bigger errors due to uncertainties in temperature, $T_e$, and density, $n_e$, values (see discussion in section 2.6.4), except for a widely used combination of optical and near-infrared [S II] lines (e.g., Osterbrock 1989).

Figure 2.8: The “mean” normalized Galactic extinction curve. The dotted line is from Cardelli et al. (1989), the dashed curve is from Seaton (1979). Note the bump around 2200 Å (due to graphite) and increased uncertainty toward short wavelength (Courtesy of E. Fitzpatrick).

Figure 2.9: Average extinction curves for LMC, SMC and the Milky Way (MW). The dotted-dashed line is for MW with $R_v = 3.1$ from Cardelli et al. (1989). Adopted from Gordon et al. (2003)

Several estimates of the shape of the “mean” Galactic extinction curve is shown in Fig. 2.8. The extinction curve has several typical features in the shape, e.g., “power law” type behavior in the infrared (IR), “knee” in the optical, “bump” in the near-UV at $\lambda 2175$ Å and quite steep rise in the far-UV (Fitzpatrick 1999). The near-UV “bump” was discovered almost 40 years ago by Stecher (1969). Small graphite dust particles are probably responsible for that strong extinction (see Weingartner & Draine 2001 and references therein).

In the case of PSR B0540-69.3 we have to take into account that it is located in the LMC which has much lower ISM metallisity than the MW. This in turn is reflected in the density and composition of the dust grains. The extinction
curves are substantially different for both Magellanic Clouds (LMC and SMC) and for the MW, especially in the UV. Gordon et al. (2003) (see Fig. 2.9) showed the difference in extinction curves for different lines of sight in LMC, SMC and MW galaxies. See Paper I for a detailed discussion of the extinction towards the PSR B0540-69.3 which is located in the star-forming region 30 Doradus in the LMC.

2.5 Pulsar Wind Nebulae

Neutron stars are remarkable objects. During the life as working pulsars they spend their rotational energy on electromagnetic radiation and relativistic winds of particles ejected from the magnetic poles. The interaction between the outflow of relativistic particles from the NS and the strong magnetic fields produces pulsar wind nebulae (PWNe) (see inset B Fig. 2.6). Ultrarelativistic electrons and positrons in strong magnetic fields radiate synchrotron and inverse Compton emission which can be observed at all wavelengths from radio to $\gamma$-rays. If the pulsar wind interacts with the surrounding medium then we will observe optical emission lines. PWNe are important to study as they provide us with additional knowledge about particle acceleration, relativistic shocks and interaction with the supernova ejecta or interstellar medium (ISM).

Up to now, about 50 PWNe sources have been identified in the Milky Way and in the Magellanic Clouds with properties similar to the Crab Nebula, so-called Crab-like PWNe (Green 2004; Kaspi, Roberts & Harding 2006). The very beautiful and most well studied example, the Crab PWN itself, is shown in Fig. 2.1. Note, that only two of them, the Crab PWN and the PSR B0540-69.3 PWN can be clearly seen in the optical bands. Recently to this list one more object, 3c58, has been added (Shibanov et al. 2008). Shibanov et al. identified the optical counterpart of the 3c58 PWN with almost 3\(\sigma\) confidence level, and, Spitzer observations support this detection (Slane et al. 2008).

Since July 23, 1999 when the Chandra satellite started its operation, PWNe observers got unprecedented opportunities to study such unique objects as pulsars and PWNe. Kargaltsev & Pavlov (2008) tried to classify all PWNe detected in X-rays into 3 types, torus-jet PWNe like the Crab PWN; bow shock-tail PWNe, which shapes are affected by a high speed pulsar motion, and irregular type PWNe.

The torus-jet PWNe are the most well studied type of PWNe. They were quite successfully modeled by Komissarov & Lyubarsky (2004) and Del Zanna et al. (2006). They used relativistic Magneto-Hydro Dynamics (MHD) models for pulsars with anisotropic wind outflows.

The bow shock-type PWNe are produced by pulsars with high space velocities. The sample of supersonically moving pulsars with cometary or bow shock morphological PWNe is still small. High velocity pulsars, i.e., pulsars with space velocities $\gtrsim 500$ km s$^{-1}$, eventually escape from their original PWN and penetrates into the SNR ejecta forming a new cometary-like PWN (see Appendix I and Gaensler & Slane 2006). The irregular-type PWNe are still a complete mystery to scientists, as noted by Kargaltsev & Pavlov (2008).
2.5.1 Properties of young Pulsar Wind Nebulae

A pulsar-driven wind decelerates as it expands into cold, slowly expanding SN ejecta. As a result it will produce a wind termination shock, where electron/positron pairs will be accelerated to ultrarelativistic velocities. The relativistic electrons and the strong magnetic fields in the magnetospheres and winds around pulsars create an ideal site for generation of synchrotron emission. Accelerated particles at the termination shock form the toroidal structure of a young PWN and the outflow of particles from the polar regions of a pulsar forms jet-like structures. Due to the synchrotron cooling of the relativistic particles, the size of the PWN increases with decreasing frequency as observed in the Crab and PSR B0540-69.3 PWNe.

The expansion of the nebula into unshocked SNR ejecta gives rise to Rayleigh-Taylor instabilities, which form very complicated filamentary structures with SN ejecta “raining” down into the PWN. This effect has been intensely studied observationally and theoretically during the last decades in the Crab and other PWNe (e.g., Hester et al. 1996, Blondin et al. 2001 and references therein).

The radio and X-ray luminosities, $L_R$ and $L_X$, respectively, can be estimated if the distance to a pulsar/PWN source is known. The values of $L_R$ and $L_X$ can vary by orders of magnitude from source to source, but are typically $L_R \sim 10^{34}$ ergs s$^{-1}$ and $L_X \sim 10^{35}$ ergs s$^{-1}$ (Becker & Truemper 1997, Frail & Scharringhausen 1997). Another representative parameter which can be derived from observations is an efficiency factor, $\eta$, i.e., the efficiency of conversion of spin-down energy into synchrotron emission (see Appendix I for a definitions). Typical values for radio and X-rays are $\eta_R \sim 10^{-4}$ and $\eta_X \sim 10^{-3}$ (Becker & Truemper 1997; Frail & Scharringhausen 1997).
2.6 Supernovae and Supernova Remnants

As mentioned in the “Introduction”, the supernova explosion is the remarkable end point of stellar evolution. What do observers see when a star explodes and how do they classify these events?

2.6.1 Classification of Supernovae

First, when a supernova event occurs, the spectral and photometric monitoring of the object is starting. This allows the event to be classified (see Fig. 2.10 for the detailed classification) at the so-called “Early phase”, i.e., at a time of about one week (Filippenko 1997). The “Late phase” starts about 4 months after explosion and provides additional information for the classification.

Observers distinguish between two spectroscopically different types, Type I and Type II, by the absence or presence of hydrogen lines in the spectrum. Then each type divides into three subgroups. The Type I SNe branch out in the subclasses Type Ia, Ib and Ic subtypes, while the Type II SNe can be divided into the Type IIP, IIL and IIn subclasses.

The spectra of Type Ia SNe are characterized by strong Si II absorption, while Type Ib show no Si II lines but strong He I, and Type Ic have neither He I nor
Si II but have prominent Na I and O I lines (see Fig. 2.10).

The Type IIP (“plateau”) and IIL (“linear”) subclasses are photometrically distinguished at times about 100 days after explosion. SNe of Type IIP show a prominent “plateau” in the light curve, while SNe of Type IIL have “linear” decline of brightness (see Fig. 2.11). Just after the explosion, SNe spectra contain broad lines which provide evidence for fast moving ejecta. On the contrary, SNe of Type IIn stand out by showing narrow emission lines which originate from direct interaction of the ejecta with the circumstellar medium or by photoionization of not yet shocked circumstellar gas. These SNe could be of any Type of core collapse in terms of explosion mechanism and mass of the progenitor star (Filippenko 1997 and references therein).

Usually, the spectral behavior of a classified SNe evolves along the typical path for its subclass. However, there are some exceptions, when a SN changes its type. These kinds of objects are specified in separate class, Type IIb or Type IIpec (see Fig. 2.10). First, these objects show hydrogen lines, namely Hα, which makes them belong to the Type II group, and then later they resemble spectra of SNe Type Ib/Ic (see discussion about SN 1987K and SN 1993J in Filippenko 1997).

The SNe of Type II, Ib and Ic unite in core collapse SNe (CC SNe) while Type Ia are thermonuclear SNe (see middle part in Fig. 2.10). This is reflected by their location among different stellar populations. For example, Type Ib, Ic and II occur in spiral galaxies near spiral arms and H II regions, while Type Ia SNe seem to occur in all types of galaxies (Filippenko 1997).

### 2.6.2 From Supernova to Supernova Remnant

When does the supernova become a Supernova Remnant (SNR)? This is still not well defined. Fesen (2001a) suggested that the SN turns into a SNR when the UV/optical line and continuum emission from the SN is much less than the emission generated by interaction with CSM or ISM. In this classification SN 1987A has just begun its remnant stage and is the youngest observable supernova remnant right now.

McKee (2001) suggests the following time evolution classification for SNRs: A Young Supernova Remnant (YSNR) is a SNR in which the total ejected mass,
including circumstellar mass, ejected by the progenitor star, and mass ejected by the SN explosion, prevail over the mass of swept-up interstellar gas, $M_{\text{cir}}^{\text{prog}} + M_{\text{ej}}^{\text{SN}} > M_{\text{sw}}$.

At the ejecta-dominated stage, i.e., $M_{\text{ej}} > M_{\text{sw}}$, there are two shocks: 1) the blast-wave shock which reaches the ambient medium, 2) a reverse shock which goes back into the ejecta. So, between the shocks there will be shocked ejecta and shocked ambient medium separated by a contact discontinuity.

The last stage of the SNR evolution is the Sedov-Taylor stage when the mass of swept-up gas is larger than the mass of ejecta, i.e., $M_{\text{ej}} < M_{\text{sw}}$ and the total energy is conserved. At that point, after about a few thousand years, the total energy is equally divided between kinetic and thermal contributions (Gaensler & Slane 2006). Quite many historical remnants are in the progress to change from an ejecta-dominated stage to the Sedov-Taylor stage.

### 2.6.3 Oxygen-rich Supernova Remnants

The YSNRs can be further classified as O-rich SNRs (e.g., Cas A and SNR 0540-69.3), pulsar or plerion remnants (e.g., Crab) and collisionless shock remnants (e.g., Tycho’s SNR and SN 1006) (Fesen 2001b). In the last group we usually find objects which result from Type Ia SNe. With regard to the first two groups, the SNR 0540-69.3 is actually a link between them, i.e., it shows abundances typical for O-rich remnants and has a Crab-like pulsar with a PWN.

The O-rich SNRs class of objects got the name after detailed studies of their optical spectra. The common features in these spectra result from both narrow-line knots with interstellar abundances and broad-line knots with strong oxygen lines and limited amount of hydrogen lines. The narrow-line knots usually come from the slower, shocked circumstellar gas, while the broad-line knots are fast moving and represent the nuclear-processed matter from the inside of the progenitor star, i.e., the SN ejecta (see Eriksen et al. 2001 for more details). The ejecta are detectable for about a few thousand years, before they dilute into the ISM.

This class of O-rich SNRs is quite small and currently consists of 7 members (see Table 2.1). Sometimes the SNR G11.2-0.3 is included in the sample. This object is really interesting. It has a very high extinction (up to 13 magnitudes) in the optical. All other characteristics in the radio and X-ray bands, resemble those of Cas A. This makes it a probable candidate for being an O-rich SNR. Probably this is not a unique case, and underlines the selection effect in the completeness of the O-rich SNR sample.

As can be seen from Table 2.1, in most of the cases the Central Compact Object (CCO) is identified. The exceptions are for older or more distant objects. The theoretical studies for O-rich SNRs show that all remnants originate from the death of high mass stars, i.e., about 30 $M_\odot$ (see references in the Table 2.1 for particular objects), with a variety of mass loss at the presupernova stage.

Let us now discuss the prototype of the O-rich SNR class, namely the nearby and well-studied Cassiopeia A SNR (or simply Cas A) in more details. This is of particular interest because the main object of this thesis is SNR 0540-69.3 which belongs to the O-rich SNRs. The light from the explosion reached Earth in about 1667 but was never registered by observers since the explosion may have happened behind an interstellar dust cloud (see Encyclopedia Britannica). A similar age is obtained by studying the proper motion of numerous filaments and
Table 2.1: The population of Oxygen-rich SNRs.

<table>
<thead>
<tr>
<th>Object</th>
<th>Distance (kpc)</th>
<th>Age (years)</th>
<th>CCO</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>NGC 4449</td>
<td>3900.0</td>
<td>∼ 100</td>
<td>not detected</td>
<td>1</td>
</tr>
<tr>
<td>Cas A (MW)</td>
<td>3.4</td>
<td>∼ 327</td>
<td>AXP/SGR</td>
<td>2</td>
</tr>
<tr>
<td>SNR 0540-69.3 (LMC)</td>
<td>51.0</td>
<td>∼ 750</td>
<td>50.2 ms pulsar</td>
<td>3a, 3b (PWN)</td>
</tr>
<tr>
<td>1E 0102-7219 (SMC)</td>
<td>59</td>
<td>2050 ± 600</td>
<td>suspected NS</td>
<td>4a, 4b, 4c</td>
</tr>
<tr>
<td>N132D (LMC)</td>
<td>51.0</td>
<td>∼ 2500</td>
<td>not detected</td>
<td>5a (PWN)</td>
</tr>
<tr>
<td>G292.0+1.8 (MW)</td>
<td>∼ 6</td>
<td>∼ 2600</td>
<td>135 ms pulsar</td>
<td>6</td>
</tr>
<tr>
<td>Puppis A (MW)</td>
<td>2.2</td>
<td>∼ 3700</td>
<td>a neutron star</td>
<td>7a (RX J08224300)</td>
</tr>
<tr>
<td>SNR G11.2-0.3 (MW)</td>
<td>5.0</td>
<td>∼ 1620</td>
<td>65 ms pulsar</td>
<td>8a, 8b (PWN)</td>
</tr>
</tbody>
</table>


Knots by Fesen et al. (2006). The SNR is located in the Milky Way at a distance of 3.4 kpc. Cas A has a main shell of 4′ diameter (or radius of 1.5 pc).

An overview of a couple of different studies of the Cas A SNR is shown in Fig. 2.12. The Cas A shell contains numerous ejecta knots and CSM clumps. The optical/IR knots show emission lines of O, S, Ar (see left panel of Fig. 2.12) while the CSM clumps are rich in He and N (Fesen 2001b and references therein). However, in contrast to the slow-moving CSM clumps, there are observed high-velocity nitrogen-rich knots (Fesen 2001b) probably shock accelerated ejecta in the nitrogen layer. This may be the nitrogen-rich outer layers of the progenitor if it ended its life as a Wolf-Rayet star (Fesen 2001b, Young et al. 2006) in a SN Ib/c event (Fesen 2001b).

The remnant shows highly asymmetrical expansion which most likely originates from an asymmetrical explosion (see middle panel of Fig. 2.12). Recent models by Young et al. (2006) argue for a progenitor of 15–25 $M_{\odot}$, which lost its hydrogen envelope in a binary interaction and went off as an energetic explosion. In their models explosion asymmetries were treated in a parameterized way excluding jet-driven mechanisms of a SN explosion. The authors claim that their models can match all the observational constraints. Young et al. (2006) assumed that a companion of about 1 $M_{\odot}$ merged with the primary star during the common-envelope phase and removed its entire hydrogen envelope. The complete hydrogen-rich envelope can, however, not be fully lost since the echo of Cas A’s early evolution displays broad Hα emission (Krause et al. 2005 and Krause et al. 2008). This very recent finding shows that the Cas A explosion seems to have been very similar to SN 1993J, and hence a Type IIb event. In this case the H-rich envelope may not have been more than 0.2 – 0.9 $M_{\odot}$ as inferred for SN 1993J (Nomoto et al. 1993, Woosley et al. 1994). Interestingly enough, the outermost hydrogen-rich ejecta of SN 1993J were nitrogen-rich (Fransson et al. 1996), just like the fast-moving knots of Cas A. There is, however, no real clear evidence of asymmetry in SN 1993J (Fransson et al. 1996), although polarimetry
Figure 2.12: Left: Chandra 1 Ms image of Cas A with 1825 knots overplotted in outer ejecta (Fesen et al. 2006). Red open circles represent knots with strong [N II] line emission, green open circles knots with strong [O II] emission, and light blue open circles strong [S II]. The circle represents 14 000 km s$^{-1}$ at the assumed remnant distance of 3.4 kpc, which is the transverse high-velocity limit. Middle: The same 1825 knots (Fesen et al. 2006) with their expected motions away from the remnant’s known center of explosion (COE), showing an interesting asymmetric structure. The circle represents the radial distance of 200$''$ or 10 000 km s$^{-1}$ transverse velocity. The red arrow shows the apparent motion of Cas A’s X-ray point source (XPS) in the direction of the southern gap of high velocity, outer ejecta knots. Right: Cas A bremsstrahlung temperature map of the fitted bremsstrahlung continuum of the remnant by Stage et al. (2006).

The main mystery of the Cas A remnant is the X-ray point source (XPS) identified in the center of the remnant (Tananbaum 1999). Many different models, like classical pulsar, accreting neutron star, accreting black hole, cooling neutron star, or anomalous X-ray pulsar/soft gamma-ray repeater (AXP/SGR) for this XPS were tested. The X-ray luminosity of Cas A XPS is 3–10 times dimmer than the dimmest existing AXP. All classical pulsar models are ruled out due to absence of any pulsations being detected so far (Young et al. 2006 and references therein). The infrared echoes arising from a $2 \times 10^{46}$ erg aperiodic outburst from the compact remnant discovered by Krause et al. (2005) puts the Cas A XPS among AXP/SGR type of objects. The results of Krause et al. (2008) strengthens this.

Another interesting fact about Cas A XPS is that it has a transverse velocity of about 350 km s$^{-1}$ and that its proper motion is not aligned with the northeast-southwest jets axis (see middle panel of Fig. 2.12 and Fesen et al. 2006), contrary to what is common for young normal pulsars, including PSR B0540-69.3.

In addition to all this, the Cas A remnant is a remarkable laboratory to study cosmic ray acceleration in the forward shock (Stage et al. 2006). The Cas A bremsstrahlung temperature map by Stage et al. (2006) is shown in the right panel of Fig. 2.12. Note that the synchrotron radiation and the very high temperature regions are located at the shock front.

2.6.4 Spectral lines as a tool

A SNR is often characterized by filamentary structures radiating in emission lines. Quite a lot of information can be deduced from studying emission lines in spectra of SNRs, like abundances of various elements, which in turn provide us with some constraints on the progenitor star. The main sources powering the emission can be different shocks, photoionization and for young remnants also radioactive decay. Studies of the line profiles in different places of a remnant can
give a clue about asymmetry of the explosion and of the surrounding medium.

To estimate the temperature and density in the remnant we use a couple of different observed line ratios. For example, the pair of lines [O II] $\lambda\lambda$ 3729, 3726 Å and [S II] $\lambda\lambda$ 6716, 6731 Å in the optical are sensitive to electron density of the gas. The [O III] $\lambda\lambda$ (4959 + 5007)/4363 Å line ratio is the most commonly used for the temperature. This analysis for the SNR 0540-69.3 was done in Paper II.

For the measurements of abundances further nebular analysis can be invoked, i.e., some estimate or just assumptions, of the size of different ionization zones for different elements. For example, if we assume that zones of [O III] and [Ne III] have the same size, we can estimate the abundances of these elements. Then the observed abundances can be compared with nucleosynthesis models to give some clues about the mass of progenitor star. This work is planned for one of my future papers about SNR 0540-69.3.
Chapter 3

Summary of papers

3.1 The PSR B0540-69.3 and its surroundings

PSR B0540-69.3 in the Large Magellanic Cloud (LMC) is a fairly young (characteristic age 1660 yr) pulsar with short period (period 50.2 ms). It was discovered by Seward et al. (1984) as an X-ray source. The pulsar is located in the middle of a compact synchrotron nebula (see Fig. 3.1).

The similarities with the Crab pulsar and its nebula are so pronounced that PSR B0540-69.3 with its supernova remnant, SNR 0540-69.3, are sometimes referred to as the “Crab twin”. Even the structures of the PWNe appear to be similar. Both have a torus and jets (Gotthelf & Wang 2000).

However, there are differences between them. For example, SNR 0540-69.3 is oxygen-rich (e.g., Kirshner et al. 1989; Paper II), whereas the Crab Nebula has nearly normal solar abundances of metals (Blair et al. 1992 and references therein). It is therefore believed that the progenitor to PSR B0540-69.3 was a much more massive star than the Crab progenitor (Kirshner et al. 1989).

Figure 3.1: Chandra HRC image of SNR 0540-69.3. The inset shows the X-ray image of the Pulsar Wind Nebula (PWN). Note the features perpendicular to the torus. These are jets of high energy particles streaming away from the poles of the neutron star. The neutron star itself is an unresolved point-like source buried in the middle of the white region. The scale of the upper left image is 1.7 arcmin on the side (Courtesy of NASA/CXC/SAO).
3.2 Paper I

PSR B0540-69.3 is one of a few pulsars for which there is a near-UV spectrum. The main goal is to eventually establish the Spectral Energy Distribution (SED) or multiwavelength spectrum of the pulsar. This will in turn shed light on the radiation mechanism and geometry of the pulsar (see discussion in section 2.4 and 2.4.1). With a multiwavelength spectrum established, it is also possible to study interstellar absorption in the direction to the object using data from near-UV and soft X-rays. An attempt to piece these things together was made in Paper I.

3.2.1 Results of spectral and photometrical data analysis

PSR B0540-69.3

A near-UV spectrum was obtained for PSR B0540-69.3 by Hill et al. (1997). However, the absolute flux and spectral index of the HST/FOS spectrum are significantly higher than suggested by previous broad-band time-resolved ground-based UBVRI photometry (Middleditch et al. 1987). To investigate this difference, observations with ESO/VLT/FORS1 and analysis of HST/WFPC2 archival data were done. All reductions were done using standard procedures within the NOAO IRAF package.

![Optical spectrum of PSR B0540-69.3 obtained with different instruments. The uppermost spectrum is the ESO/VLT/FORS1 data set with bright nebular [O III] lines removed. The dashed line and associated hexagonal is the power law fit from Hill et al. (1997). Filled triangles show HST photometry with 10 pixels aperture to compare with the spectral data. The filled circles are our photometry using HST archival data. Open rectangles are the UBVRI photometrical data from Middleditch et al. (1987). All data were dereddened using E(B−V)=0.20. Adopted from Paper I.](image)

As can be seen in Fig. 3.2 all the spectral data and all the photometrical data...
are self consistent, but not between each other. This means that all pulsar spectral data have significant contribution in the absolute flux from the PWN. The overestimated flux is 2–4 times higher than measured using the photometrical data sets. Thanks to the unprecedented HST spatial resolution it is possible to separate the pulsar flux from the PWN contamination without having to resort to high time resolution.

The comparison of photometrical data from the HST archive (F791W, F547M and F336W bands were used) with Middleditch et al. (1987) shows a significantly steeper pulsar spectral energy distribution in our photometry of the archival data. If we define the spectrum as \( F_\nu = F_\nu(\nu/\nu_0)^{-\alpha_\nu} \), then our power-law index is \( \alpha_\nu = 1.07^{+0.20}_{-0.19} \) while that of Middleditch et al. is \( \alpha_\nu = 0.33 \pm 0.45 \) using updated dereddening corrections. The flatter spectrum of Middleditch et al. could be due to a systematic error in their U band flux. The reason for suspecting this is that the Crab-pulsar broadband spectrum by Middleditch et al. (1987) has a significant excess in the U band, which has not been confirmed by more recent spectral observations extending even further into the UV (Sollerman et al. 2000). However, possible variability of the pulsar emission in the UV range cannot be excluded as another cause of the difference between our results for PSR B0540-69.3 and the result by Middleditch et al. (1987).

The PWN of PSR B0540-69.3

Using the HST archive images (for the F791W, F547M and F336W bands) we have also made aperture photometry of the continuum emission from the whole PWN, as well as different parts of it, selected on the basis of the morphology of the nebula. As expected, the measured broadband spectra from the whole PWN and its different parts are well described by power-laws with negative spectral index, which confirm the non-thermal origin of the continuum nebular emission.

Compared with the pulsar, the whole PWN is more than an order of magnitude brighter, and its spectrum is significantly softer \( \langle \alpha_\nu \rangle = 1.48^{+0.09}_{-0.08} \). This shows in turn that the NS spin-down power is transformed to optical emission more efficiently in the PWN than in the pulsar magnetosphere.

There is a significant decrease of the surface brightness going from the brightest area toward the N-E edge of the torus (Fig. 8 in Paper I). The brightness difference exceeds the 6σ level of the uncertainty level of the dimmest area and shows an asymmetry of the flux distribution with respect to the pulsar position which is similar to what is also seen in X-rays (see Fig. 3.1). This asymmetry can be produced either by a breakdown of axial symmetry in the pulsar wind (e.g., due to plasma instabilities) or by inhomogeneity of the PWN environment, i.e., an asymmetry of the SN ejecta. The latter is indeed indicated by the asymmetric distribution of optical filaments projected on the PWN of PSR B0540-69.3 (Morse 2003; Morse et al. 2006), as well as the general redshift of the gas emitting optical lines (Kirshner et al. 1989; Paper II).

Also, there is an indication (with 1.4σ confidence level) that brighter structures of the PWN have a steeper spectral index (see Fig. 10 in Paper I). Deeper observations of the PWN are needed to probe a correlation between the brightness and the spectral index. Such a correlation, as well as a flat index versus flux distribution, would contradict simple expectations from synchrotron cooling of relativistic particles which suggest a softening of the underlying electron spectrum toward the PWN boundary.
3.2.2 Proper motion of PSR B0540-69.3.

Our analysis of HST archival data from two epochs separated by 4 years showed an indication of a proper motion $\mu = 4.9 \pm 2.3$ mas y$^{-1}$ in the South-East direction at a position angle of $108.7 \pm 32.9^\circ$ (along the southern jet). The alignment of proper motion with the symmetry axis is not surprising for pulsars (Johnston et al. 2005). However, while the Crab and Vela pulsars both have transverse velocities of $\sim 130$ km s$^{-1}$, our results for PSR B0540-69.3 indicate a higher transverse velocity $1190 \pm 560$ km s$^{-1}$, assuming a distance to the LMC of 51 kpc (Panagia 2005). This result shows $2\sigma$ confidence level. Further investigation with a third epoch of HST imaging is required, and we obtained HST data in 2005 to look at that (see also 3.4).

3.2.3 X-ray spectrum and interstellar absorption

In two recent independent analyses, Kaaret et al. (2001) analyzing time-resolved Chandra data, and Finley et al. (1993) analyzing ROSAT X-ray data, the same result for the X-ray absorption was obtained. Both groups used Milky Way (MW) abundances and derived a total column density of absorbing gas towards PSR B0540-69.3 of $N_{\text{HI}} \approx 4 \times 10^{21}$ cm$^{-2}$. However, the use of MW abundances is obviously a simplification for PSR B0540-69.3 since it sits in the LMC, and at least part of the absorption must arise in the LMC which has very different abundances than the MW.

The Parkes 21 cm multibeam survey of the LMC (Staveley-Smith et al. 2003) shows that the MW contribution to the column density in the direction of PSR B0540-69.3 is just $N_{\text{HI}} \approx 0.6 \times 10^{21}$ cm$^{-2}$. This survey also shows that the maximum value of $N_{\text{HI}}$ in the LMC is $\sim 5.6 \times 10^{21}$ cm$^{-2}$, and that this occurs close to the position of PSR B0540-69.3.

The interstellar X-ray absorption is a sum of absorptions from various elements whose abundances are different in the LMC and MW (LMC has lower metal content compared to the Galaxy). The role of dust is small in both galaxies (Morrison & McCammon 1983, see their Fig. 1), why we only have to concentrate on differences in the gas phase. In the energy range $0.6 - 10$ keV the dominating elements are C, N, O and Fe. Because each element contributes to the X-ray absorption at different energies, the different abundances of these elements in the two galaxies make the total cross section to become a function of energy different in the MW compared to the LMC (see Figs. 11-12 in Paper I).

We also checked another possible source of X-ray absorption, namely the supernova ejecta. Observations of SNR 0540-69.3 show that the remnant is oxygen-rich (Kirshner et al. 1989; Paper II), which means that the progenitor star probably was massive. Kirshner et al. (1989) suggest it to have had a mass similar to that of the SN 1987A progenitor, i.e., around 20 M$_\odot$. As a result (see Fig. 13 Paper I) it was shown that it is possible to obtain a de-absorbed power-law spectrum above 0.6 keV (because the high oxygen column density in the oxygen-rich ejecta compensates for the low metal abundance in the LMC) for a supernova ejecta composition similar to SN 1987A but at an age of 500 years. However, it is also clearly seen that a remnant with an age closer to the spin-down age of PSR B0540-69.3 will not contribute significantly to the X-ray absorption. Without contribution from SN ejecta the analysis in Paper I shows that a power-law spectrum may not be an accurate description at low energies, but instead shows a depression below $\sim 1$ keV.
Multiwavelength spectrum of PSR B0540-69.3

To connect the optical pulsar emission to the emission at other wavelengths, we have compiled results from different telescopes and instruments (see Fig. 3.3). The optical and X-ray parts of the spectrum can be fitted with power-laws, which would suggest a non-thermal nature of the emission in both domains, likely to be formed in the magnetosphere of the rotating neutron star. The data for PSR B0540-69.3 suggest at least two spectral breaks between the optical and X-ray spectral bands, while for the Crab (also compiled and shown in Fig. 3.3), it seems that a smooth turn-over can be possible between the X-ray band and the optical. This is in contrast to spectra of the middle-aged pulsars Vela and PSR B0656+14, whose optical fluxes are generally compatible with the low-frequency extrapolation of a power-law spectral tail for $E \gtrsim 1 - 2$ keV (Fig. 2.7; Koptsevich et al. 2001; Shibanov et al. 2003).

Based on the available data it is not yet clear whether these spectral peculiarities in the pulsar optical emission do indicate a spectral evolution with pulsar age (PSR B0540-69.3 has a spin-down age which is $\sim 400$ years higher than the Crab pulsar) or whether they are connected to the pulsar optical efficiency, or just reflects specific parameters (e.g., viewing angle and magnetic field geometry) of each pulsar.

The multiwavelength spectrum of PSR B0540-69.3 PWN suggests the same double knee connection between the optical and X-ray spectral parts as for the pulsar. However, the assumed knee-breaks appear to be less pronounced than for the pulsar because the order of magnitude higher ratio for the intrinsic optical to X-ray flux for the PWN than for the pulsar. This is also reflected in the PWN luminosities. The smoothness of the knee in the PWN spectrum as compared to the pulsar spectrum may be explained by propagation effects of the relativistic particles generated in the pulsar magnetosphere and moving through the PWN.
The optical efficiency of the PWN of PSR B0540-69.3 is a factor of $\sim 30$ higher than for the pulsar. This is markedly different from the situation in X-rays where the efficiency of the PWN is only $\sim 4$ times higher than for the pulsar (Kaaret et al. 2001).

The PWN of PSR B0540-69.3 has similar sizes in the optical (the Paper I) and in X-rays (Gotthelf & Wang 2000; Kaaret et al. 2001) and extends up to 4" away from the pulsar, which corresponds to $\approx 1$ pc at 51 kpc. This in turn suggests that the particle spectra in the bright, central parts of PWNs are not affected by synchrotron cooling.

### 3.3 Paper II

This paper is a conference paper which is a summary of Paper I and a detailed SNR 0540-69.3 paper (in preparation). Here I will only discuss the part that concerns the supernova remnant.

The supernova SNR 0540-69.3 in LMC is an extraordinary object. The SNR is classified as an “Oxygen−Rich SNR”, because it has strong optical emission dominated by forbidden oxygen lines. The remnant consists of an outer shell which is about 30" in radius and inner region, i.e. ‘central part of the SNR’ (SNRC) with a diameter about 8".

#### 3.3.1 Observations and results

The observations were performed on 1996 January 17, using the ESO NTT telescope with the ESO Multi-Mode Instrument (EMMI)\(^1\) in the 3850−8450 Å range. Subsequent observations were performed on 2002 January 9 and 10 with the ESO VLT telescope using the FOcal Reducer/low dispersion Spectrograph 1 (FORS1)\(^2\) in the 3600−6060 Å range.

The main finding is that spectroscopic observations of the remnant covering the range of 3600−7350 Å centered on the pulsar produced results consistent with those of Kirshner et al. (1989), but also revealed many new emission lines. The lines of [Ne III] $\lambda\lambda$ 3869, 3967 and Balmer lines of hydrogen were seen for the first time due to better spectral resolution and higher sensitivity of the VLT/FORS1. All these newly discovered lines have redshifts which correspond to velocities in the range of 500−800 km s\(^{-1}\) and are consistent with the velocities of the strong [O III] and [S II] lines. This ensure us that the emission in [Ne III] $\lambda\lambda$ 3869, 3967 and Balmer lines of hydrogen come from the SNR 0540-69.3 and not from the LMC. This has been confirmed by Morse et al. (2006).

In both the central part of the remnant, as well as in nearby H II regions, the [O III] temperature is higher than $\sim 2 \times 10^4$ K, but lower than previously estimated. The main reason for the latter is that [O III] $\lambda\lambda$ 4363 blends with H$\gamma$ due to velocity broadening which leads to overestimation of the O III temperature in the previous estimates. The lines [Ne III] $\lambda\lambda$ 3869, 3967 are in a similar situation, i.e., blended with H I lines. Paper II suggested to deblend lines using H$\beta$ as a template, for all H I lines, and then subtract of the H I lines from the original [O III] and [Ne III] lines. This method allowed us to measure more accurately the flux of the blended lines. The flux scaling of the Balmer

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\(^1\)http://www.ls.eso.org/lasilla/Telescopes/NEWNT

\(^2\)http://www.eso.org/instruments/fors1/
lines follows that expected from Case B theory which simplifies the subtraction procedure.

### 3.3.2 SNRC density from [S II] $\lambda\lambda 6716, 6731$ lines

The intensity ratio $R[S II] = \frac{I(\lambda 6716)}{I(\lambda 6731)}$ is sensitive to the electron number density, $N_e$ (e.g., Osterbrock, 1989). The lines are velocity broadened and therefore overlap, but there are some positions along the spatial axis of the slit where the deblending in the case of [S II] $\lambda\lambda 6716, 6731$ is possible. To obtain an electron number density the multilevel model for S II (see Lundqvist & Fransson 1996) was used. We obtained the electron number density for two assumed values of temperature, i.e. for $T_1 = 10,000 \text{ K}$ and $T_2 = 20,000 \text{ K}$ (The estimated density is not sensitive to the exact value of the temperature). A reasonable value of $N_e$ is in the range of $(1-5) \times 10^3 \text{ cm}^{-3}$ for SNR 0540-69.3 which is in good “agreement” with similar measurements for the Crab nebula (see Davidson & Fesen, 1985).

### 3.3.3 SNRC temperature from [O III]

For the temperature measurements we used the observed [O III] $\lambda\lambda 4363, 4959, 5007$ lines and compared them to a six-level model atom for O III also used by Maran et al. (2000). We adopted values for the density in the range $N_e = (1-5) \times 10^3 \text{ cm}^{-3}$ (In this case, temperature is not very sensitive to density). The line [O III] $\lambda 4363$ was deblended from $H\gamma$ using the method described above. The resulting temperature is a $2.4 \times 10^4 \text{ K}$ in the central part of SNR 0540-69.3. This value of the temperature is lower than previously estimated by Kirshner et al. (1989), who obtained $T = 3.4 \times 10^4 \text{ K}$. The reason for this is at least partly due to lack of deblending in their case.

The SNR 0540-69.3 is situated close to the DEM 269, LH 104 and 30 Doradus LMC H II regions. In our VLT/FORS1 spectra we detected several H II regions or filaments along the spatial axis of the slits, and we named them F1 – F5. Spectra of all the detected filaments show typical H II region lines, like Balmer lines of hydrogen up to H8, He I, [Fe II], [Fe III] and so on. The filaments F4 and F5, which are located close to the outer shock front of the remnant, show [Fe XIV] $\lambda 5303$ together with [Ca V] $\lambda 5309$.

The [O III] temperatures, estimated in the same way as described above, show some excess compare to a normal H II regions, i.e. above $10^4 \text{ K}$. Filament F1, which is the one closest (in projection) to center of the remnant, is very hot, $\sim 3.7 \times 10^4 \text{ K}$. Filaments F2, F4 and F5, which are close to the outer shock front in projection, also show quite high [O III] temperatures, $\sim 2.3 \times 10^4 \text{ K}$. Even a filament F3 located quite far from the remnant, $\sim 1'$ East of SNRC, has $\sim 1.7 \times 10^4 \text{ K}$, while the mean temperature in, for example, 30 Doradus is $10270 \pm 140 \text{ K}$ (Krabbe et al. 2002). This leads us to the conclusion that all F1 – F5 filaments are affected by the SNR 0540-69.3. The hypothesis of pure shock excitation mechanism is in this case ruled out because we don’t see any line broadening (except instrumental) and no line displacements were detected, like it is for SN 1987A (Pun et al. 2002). So, photoionization excitation by X-rays could be the clue to the high temperatures and ionization levels in the 0540 filaments, for example caused by the blast wave when it overtakes the filaments and creates X-rays. Time dependent effects can also be important. It was shown
by Lundqvist & Fransson (1996) that temperatures in this case can reach several times $10^4$ K before it relaxes to its steady-state value of closer to $\sim 10^4$ K.
3.4 Future work

After both papers were published we got new data from HST/WFPC2 in the fall of 2005. The main goal of this observation set was to improve the proper motion measurements published in Paper I for the PSR B0540-69.3. During preliminary analysis of the data we found that the new data set and archival data actually suffer from a Charge Transfer Efficiency (CTE) problem or, in other words, charge leakage. The CTE problem is getting worse with time. This was one reason for the delay of our own publication of the observation set from 2005. In 2007 De Luca et al. (2007) analyzed the data and published it. These authors set a 3σ upper limit on the PSR B0540-69.3 velocity of 290 km s$^{-1}$. They claim that it was a null displacement in the pulsar position during 10 years. That is a quite natural conclusion if one does not consider the CTE effect, because the orientation of the instrument during the observations at different epochs was rotated about 70° every time, which gives the preferred line for charge leakage oriented differently at different epochs. Since the expected shift due to pulsar proper motion is small, correctly taking care of the CTE problem plays a crucial role in estimating the proper motion of PSR B0540-69.3.

Recently, Kaplan et al. (2008) showed that all previous measurements of the proper motion for the Crab pulsar using HST archival data did not take into account saturation of the pulsar. This includes the measurements of the same team Caraveo & Mignani (1999) who published the PSR B0540-69.3 data. Previous measurements for both the Crab and PSR B0540-69.3 have substantial uncertainties. The most accurate measurement, by Kaplan et al. (2008), estimate a misalignment of $14° \pm 9°$ between projected spin-axis and proper motion, which might be considered as consistent with “good” alignment. The question about the alignment of the proper motion of the Crab pulsar and its spin-axis therefore also remains open. According to Ng & Romani (2007), young pulsars with short spin periods are expected to have perfect alignment.

For the PSR B0540-69.3, substantial progress in our data analysis has recently been made. We found a way to deal with the CTE problem by using wavelet filtering of the images. This will allow us to study the dynamics of the PWN in greater detail and to better constrain the pulsar proper motion. The work is planned to be finished during fall 2008.

Paper II is an extract of a paper on SNR 0540-69.3 that will also be completed in 2008. One of the goals is to make comparisons of obtained abundances of Ar, Ni, O and S to modeled values of Limongi & Chieffi (2005), Woosley et al. (1995) and Nomoto et al. (1997) to provide constraints on the progenitor of PSR B0540-69.3/SNR 0540-69.3.
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3.4. FUTURE WORK


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Chapter 4

Appendix I

4.1 Basic formulae

- Electromagnetic waves, phase and group velocity

The plane electromagnetic waves that carry energy are usually written in form below, where bold characters represent vectors, so that $\mathbf{E}$ and $\mathbf{B}$ are electric and magnetic field vectors; $\mathbf{a}_1$ and $\mathbf{a}_2$ are unit vectors; $\mathbf{k}$ is wave vector, $\omega$ is frequency, $t$ is time, $\mathbf{r}$ is direction vector; $E_0$ and $B_0$ are complex constants.

$$\mathbf{E} = a_1 E_0 e^{i(k \cdot \mathbf{r} - \omega t)}$$

$$\mathbf{B} = a_2 B_0 e^{i(k \cdot \mathbf{r} - \omega t)}$$

These equations are valid in vacuum. Phase velocity is defined as $v_{ph} = \omega/k$ and group velocity as, $v_g \equiv \partial \omega / \partial k$.

In case of electromagnetic waves not propagating in vacuum, refraction effects would be present. The group velocity would always be less than the speed of light, while phase velocity could be greater than the speed of light. The wave energy travels at the group velocity, as does any modulation of the wave (see Rybicki & Lightman 1979). “An important application of the formula for group velocity is to pulsars. Each individual pulse from the pulsar has a spectrum covering a wide band frequency. Therefore, the pulse will be dispersed by its interaction with the interstellar plasma, since each small range of frequencies travels at a slightly different group velocity and will reach earth at a slightly different times” (cf. Rybicki & Lightman 1979).

4.1.1 Pulsars

Most of the formulae in this section are from Lyne & Graham-Smith (2006). If not, references are spelled out explicitly.

- ‘Dispersion measure’ (DM):

$$DM = \int_0^L n_e dl,$$

where $L$ is the distance to the pulsar in parsecs and $n_e$ the electron density. The physics behind has been described just above.
• Relations between pulsar period, $P$, rotation frequency, $\nu$, and their time derivatives are:

\[
P = \frac{1}{\nu}, \quad \dot{P} = -\frac{\dot{\nu}}{\nu^2}, \quad \ddot{P} = \frac{2\ddot{\nu}^2}{\nu^3} - \frac{\dddot{\nu}}{\nu^2},
\]

\[
\nu = \frac{1}{\dot{P}}, \quad \dot{\nu} = -\frac{\dot{P}}{\nu^2}, \quad \ddot{\nu} = 2\frac{\ddot{P}^2}{\nu^3} - \frac{\dddot{\nu}}{\nu^2}.
\]

• The spin-down law:

\[
\dot{\Omega} = -k\Omega^n,
\]

where $k$ is a constant, $n$ is the ‘braking index’ and $\Omega$ is the angular velocity.

• Characteristic age of a pulsar

\[
\tau_c = \frac{1}{n - 1}\frac{P}{\dot{P}}
\]

here $n$ is the ‘braking index’ (cf. below). As discussed below, $n$ is measured only for a few pulsars, hence for most of pulsars people use $n = 3$ as follows from standard theory to estimate $\tau_c$. This is why $\tau_c$ is commonly written:

\[
\tau_c = \frac{P}{2\dot{P}} = \frac{\nu}{2\dot{\nu}}
\]

• For the ‘braking index’:

\[
n = \frac{\nu\ddot{\nu}}{\dot{\nu}^2} = 2 - \frac{P\ddot{P}}{\dot{P}^2}.
\]

• The ‘braking index’: theory and observations.

An interesting thing showing that classical theory and observations do not agree is the measurement of pulsar braking index $n$. According to classical theory the pulsar braking index is $n = \nu\ddot{\nu}/\dot{\nu}^2$ and is supposed to equal 3, if pulsars radiate as perfect magnetic dipoles. Here $\dot{\nu}$ and $\ddot{\nu}$ are the frequency time derivatives. The same assumption is usually used in estimates of the characteristic age, $\tau_c$. Up to date, about 1700 pulsars are known, but braking indices were measured only for six of them including PSR B0540-69.3. As can be seen from Table 4.1, for all these six pulsars the measured braking index is less then 3. A value of $n < 3$ leads to a higher characteristic age of the pulsars.

Table 4.1: Spin period and inferred parameters for pulsars with measured ‘braking index’ $n$, ordered by spin-down age (Livingstone et al. 2006 and references therein).

<table>
<thead>
<tr>
<th>Pulsar</th>
<th>$\nu$, $s^{-1}$</th>
<th>$\tau_c$, yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>J1846-0258</td>
<td>2.65±0.01</td>
<td>3.07</td>
</tr>
<tr>
<td>B0531+21 (Crab psr)</td>
<td>2.51±0.01</td>
<td>30.2</td>
</tr>
<tr>
<td>B1509-58</td>
<td>2.839±0.003</td>
<td>6.63</td>
</tr>
<tr>
<td>J1119-6127</td>
<td>2.91±0.05</td>
<td>2.45</td>
</tr>
<tr>
<td>B0540-69</td>
<td>2.140±0.009</td>
<td>19.8</td>
</tr>
<tr>
<td>B0833-45 (Vela psr)</td>
<td>1.4±0.2</td>
<td>11.2</td>
</tr>
</tbody>
</table>

Livingstone et al. (2007) showed that in addition to dipole magnetic radiation torque, pulsars experience an extra torque which affects the pulsar spin down. There were several attempts to explain the nature of this additional
torque. It could be that the relativistic pulsar wind affects the spin-down (Michel & Tucker 1969), or if magnetic dipole will be treated not like a point but have finite size, then $2 < n < 3$, and approaching 3 as the pulsar ages (Melatos 1997). One more model was recently proposed to solve the $n < 3$ problem. In this plasma fills the pulsar magnetosphere, giving an extra torque to the pulsar and it spins down faster (see, for example Spitkovsky 2005; Timokhin 2006; Contopoulos & Spitkovsky 2006).

- Slowdown rate and magnetic dipole momentum.

$$\frac{d\left(\frac{1}{2}I\Omega^2\right)}{dt} = I\dot{\Omega} = \frac{2}{3}M^2\sin^2\alpha\Omega^4c^{-3}$$

The left part of the ratio represents the time derivative of the angular kinetic energy of a rotating body. Here $I$ is the moment of inertia. The right part of the ratio shows that this energy goes to radiation of a wave at angular frequency $\Omega$, i.e. $\Omega = 2\pi\nu$. Here $M$ is a magnetic dipole momentum and $\alpha$ is angle between the dipole and rotation axis.

Note that this ratio also has some limitations, e.g., it does not take into account an outflow of particles. The ratio is, however, very useful for a rough estimate.

- Surface magnetic field strength and pulsar period.

In standard theory, when we assume an orthogonal rotator, the magnetic dipole and spin axis are aligned, and the pulsar has radius of 10 km and moment of inertia of $10^{45}$ g cm$^2$, then the ratio between surface magnetic field strength and pulsar period can be written as

$$B_s = 3.2 \times 10^{19}(P\dot{P})^{1/2} \text{ G(auss)}.$$  

Note that there are exceptions to when this ratio holds. The expression should only be used to get a rough estimate of $B_s$.

- The efficiency factors in radio, optical and X-rays bands,

$$\eta_R \equiv \frac{L_R}{\dot{E}}, \quad \eta_O \equiv \frac{L_O}{\dot{E}}, \quad \eta_X \equiv \frac{L_X}{\dot{E}},$$

where the dissipation rate of rotational kinetic energy is

$$\dot{E} = -\frac{dE_{\text{rot}}}{dt} \equiv 4\pi^2I\dot{P}/P^3.$$  

4.1.2 SNRs

- Approximate time needed for a pulsar to cross its SNR ejecta, assuming that the SNR shell is spherical, can be estimated as

$$t_{\text{cross}} = 44\left(\frac{E_{SN}}{10^{51} \text{ ergs}}\right)^{1/3}\left(\frac{n_0}{1 \text{ cm}^{-3}}\right)^{-1/3}\left(\frac{V_{PSR}}{500 \text{ km s}^{-1}}\right)^{-5/3} \text{kry}.$$  

Here $E_{SN}$ is the energy of the SN explosion, $n_0$ the density of ambient medium, $V_{PSR}$ the space velocity of the pulsar (Gaensler & Slane 2006).